MATH/CSCI 2113

Solutions to Assigment 4

1. Give a formula for the coefficients a_n if the sequence $\{a_n\}$ has the given generating function $a(x)$:

   (a) $a(x) = \frac{5}{x-3}$

   $a_n = -5\left(\frac{1}{3}\right)^{n+1}$ for $n \geq 0$.

   (b) $a(x) = \frac{3-7x}{1-5x+6x^2}$ (*use partial fractions*)

   $$a(x) = \frac{1}{1-2x} + \frac{2}{1-3x},$$

   so $a_n = 2^n + 2 \cdot 3^n$ for $n \geq 0$.

   (c) $a(x) = \frac{x+1}{(1-2x)^2}$

   $a_n = (n+1)2^n + n \cdot 2^{n-1} = (\frac{3n}{2} + 1)2^n$ for $n \geq 1$, $a_0 = 1$.

   For the following problems, let $a(x)$ be the generating function of the recursively defined sequence $\{a_n\}$. Find an equation satisfied by $a(x)$, and solve for $a(x)$. (*You do not have to find a direct formula for a_n.*)

2. $a_0 = 3$, and $a_n = -a_{n-1} + 2$ for $n \geq 1$.

   $$a(x) = \frac{3-x}{1-x^2}.$$

3. $a_0 = 2$, $a_1 = 1$, and $a_n = a_{n-1} - 3a_{n-2}$ for $n \geq 2$.

   $$a(x) = \frac{2-x}{3x^2 - x + 1}.$$

4. For this problem, do the same as in the previous two, but also find a direct formula for a_n: $a_0 = -1$, $a_1 = 0$, and $a_n = -a_{n-1} + 2a_{n-2}$

   $$a(x) = \frac{-(1+x)}{(1-x)(1+2x)} = \frac{-2/3}{1-x} + \frac{-1/3}{1+2x},$$

   so $a_n = -2/3 - (1/3)(-2)^n$.

5. Do Exercise 11.1.2, page 483 of the text book.

Note: many different correct answers possible!

(a) *bcabcd* (the walk must have a repeated edge)
(b) *befged* (the trail cannot have a repeated edge, but must have a repeated vertex)
(c) *bcd* (no repeated vertices or edges)
(d) *bacdcb* (must have a repeated edge)
(e) *bcdegfeb* (no repeated edge, but repeated vertex)
(f) *bacdeb* (no repeated vertices or edges)

6. Do Exercise 11.1.6, page 483 of the text book.

Distances:

	c	e	f	g	h	i	j	k	ℓ	m
d	1	1	1	2	3	4	3	2	3	3

7. Do Exercise 11.1.10, page 484 of the text book.

Any tree works.

8. Do Exercise 11.1.14, page 484 of the text book.

(a) (a) 3 (b) 5 (c) 5.
(b) (i) n if $n \neq 4$, 5 if $n = 4$. (ii) $n + 1$.