MATH/CSCI 2113

Assigment 7

Solutions

1. Do problem 12.2.18 on page 570 of the text book.

   The chain letter mailing can be modelled by two complete 5-ary trees (one for Rizzo and one for Frenchie), each of 7 levels. Each node of the tree, except the root, represents a chain level. Each tree has $1+5+5^2+\ldots+5^7 = 5^8-1$ nodes, so $2(\cdot 5^8-1)-2$ chain letters have been mailed. The chain letters mailed on the last three Sunday correspond to the nodes on the levels $5, 6$ and 7, so there are $5^5+5^6+5^7$ letter mailed.

2. Given is the following adjacency matrix of a graph $G = (V, E)$, where $V = \{v_1, \ldots, v_{10}\}$ (rows are columns correspond to vertices in order of increasing indices) :

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

   (a) Find a depth-first spanning tree and a breadth-first spanning tree for this graph, with the given vertex order.

   (b) Find a breadth-first spanning tree for this graph when the vertices are ordered $v_{10}, v_9, v_8, \ldots, v_1$.

   Very similar example done in class.

3. Do problem 12.3.2 on page 574 of the text book.

   Very similar example done in class.

4. Use Huffman coding to construct an optimal prefix code for the symbols $a, b, \ldots i, j$ that occur with respective frequencies $79, 4, 49, 21, 31, 122, 38, 29, 17, 78$. Calculate the average length of your Huffman code.

   Code:
   NOTE: depending on your arrangement of the symbols, a different code may

be obtained. However, the length of each codeword in your code should be the same as in mine, and, if you would draw the tree of my code and yours, they should be isomorphic.

a: 110
b: 01010
c: 000
d: 0100
e: 0010
f: 10
g: 011
h: 0011
i: 01011
j: 111

Average length:

$$\frac{79 \cdot 3 + 4 \cdot 5 + 49 \cdot 3 + 21 \cdot 0100 + 31 \cdot 4 + 122 \cdot 2 + 38 \cdot 3 + 29 \cdot 4 + 17 \cdot 5 + 78 \cdot 3}{468}$$

5. For each of the following sequences of lengths, does there exist a prefix code with the codewords of the following prescribed lengths? If so, give such a code. If not, give proof that it doesn't exist.

(a) $6, 5, 4, 4, 3, 1$.

Exists: 0,100,1010,1011,11000,110010.

(b) $1, 2, 3, 3, 3$.

$2^2 + 2^1 + 2^0 + 2^0 + 2^0 = 9 > 2^3$, so such a code does not exist.

(c) $2, 2, 3, 3, 3, 4, 4$.

Exists: 00,01,100,101,110,1110,1111.