Review of functions:

A function $f : A \to B$ is one-to-one, if no two elements of A have the same image. So if $a \neq a'$, then $f(a) \neq f(a')$. In other words, every element of B has *at most one* **pre-image.**

A function $f : A \to B$ is onto, if every element of B has at least one pre-image. So for every element $b \in B$, there exists an element $a \in A$ so that $f(a) = b$.

A function $f : A \to B$ is a bijection if it is one-to-one and onto. Note that this implies that every element in B has *exactly one* **pre-image.**

For a bijection we can form the inverse function $f^{-1} : B \to A$, where $f^{-1}(b) = a$ precisely when $f(a) = b$.

If $f : A \to B$ is a bijection, then $|A| = |B|$

Isomorphism

An (undirected) graph $G_1 = (V_1, E_1)$ **is** *isomorphic* **to another graph** $G_2 = (V_2, E_2)$ **if there exists a bijection** $f : V_1 \to V_2$ **so that, for all** $a, b \in V_1$**,**

$$\{a, b\} \in E_1 \Leftrightarrow \{f(a), f(b)\} \in E_2.$$

The bijection f **is called a graph isomorphism.**

Similarly:

A directed graph $G_1 = (V_1, E_1)$ **is** *isomorphic* **to another digraph** $G_2 = (V_2, E_2)$ **if there exists a bijection** $f : V_1 \to V_2$ **so that, for all** $a, b \in V_1$**,**

$$(a, b) \in E_1 \Leftrightarrow (f(a), f(b)) \in E_2.$$