Given an undirected graph $G = (V, E)$**.**

Two vertices u **and** v **are** *adjacent* **if there is an edge in** E **of which they are the endpoints.**

The *degree* **of a node** v **is the number of nodes adjacent to** v**, or alternatively, the number of edges in** E **that have** v **as an endpoint. Notation:** $d(v)$**.**

For all graphs $G = (V, E)$**, where** $V = \{v_1, \ldots, v_n\}$**:**

$$\sum_{i=1}^{n} d(v_i) = 2|E|.$$

Sketch of proof: every edge contributes to the degree of two nodes.

The distance between two nodes in an undirected graph is the length of the shortest path between them.

Given a directed graph $G = (V, E)$.

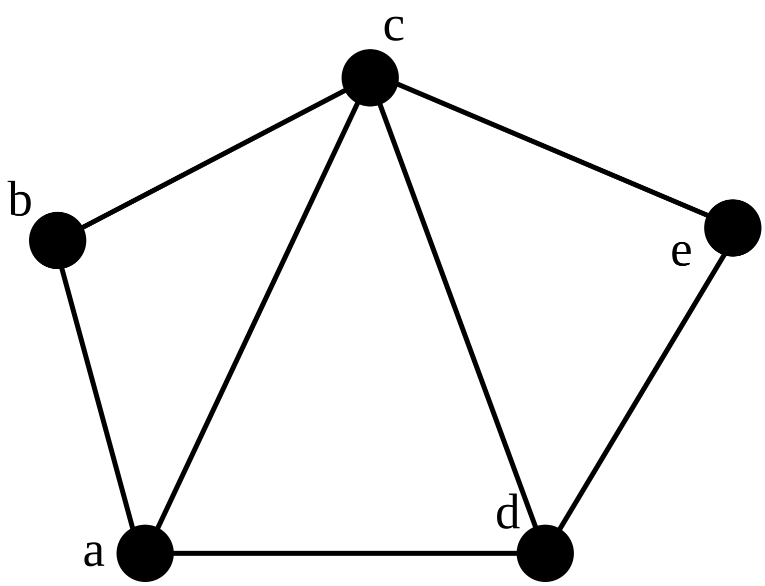
The *in-degree* **of a node v is the number of edges in E that have v as their origin.**
Notation: $d^-(v)$.

The *out-degree* **of a node v is the number of edges in E that have v as their terminus.**
Notation: $d^+(v)$.

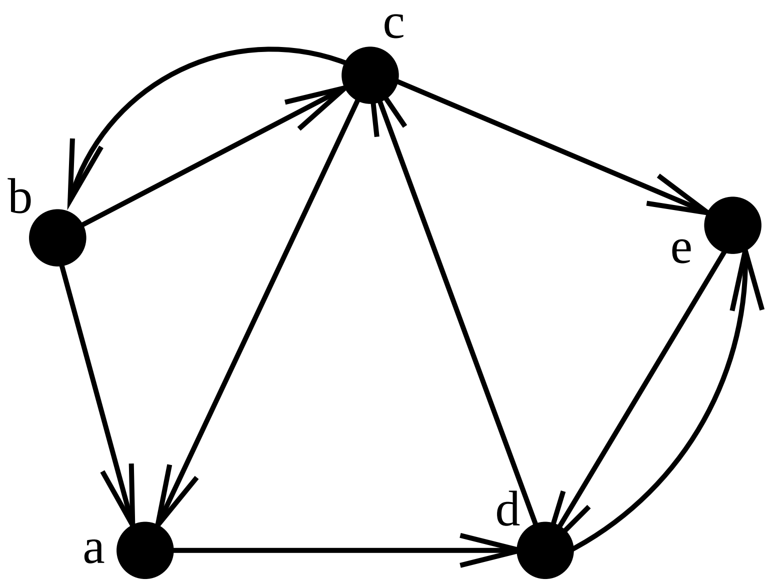
For all directed graphs $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$:

$$\sum_{i=1}^{n} d^+(v_i) = \sum_{i=1}^{n} d^-(v_i) = |E|.$$

Adjacency matrix



$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

If A is the adjacency matrix of a graph G, then entry (i, j) of A^k gives the number of walks from node i to node j in G.

Sketch of proof (induction):

Basic step: obviously, entry (i, j) of A gives the number of walks (zero or one) of length 1 from node i to node j.

Induction step:

Let k be a positive integer. Assume that the statement is true for A^k. Since $A^{k+1} = A^k \cdot A$, it follows from the rules of matrix multiplication that:

$$A^{k+1}_{i,j} = \sum_{t=1}^{n} A^k_{it} \, A_{tj} = \sum_{t,\, tj \in E} A^k_{it}.$$

The result follows from the observation that the number of paths of length $k + 1$ from i to j equals the sum, over all neighbours t of j, of the number of paths of length k from i to t.