Trees

A *tree* is an acyclic, connected graph.

A *forest* is an acyclic graph.

A *spanning tree* of a graph G is a spanning subgraph of G which is a tree.

A *spanning forest* of a graph G is a spanning subgraph of G which is a forest.

A *pendant vertex* in a tree or graph is a vertex of degree 1.

A *cut-edge* in a graph is an edge so that, if you remove this edge, the graph becomes disconnnected.

Every tree T has at least **2** pendant vertices.

Proof sketch: Consider a longest path in T. The endpoints of this path can have no neighbours outside the path, because the path is longest. They can have only one neighbour on the path, otherwise there would be a cycle. So they have degree 1.

Every tree T with n vertices has $n-1$ edges.

Sketch of proof by induction on n:

Basic step: $n = 1$.

Inductive step: Pick $k \geq 1$. Assume that any tree with k vertices has $k-1$ edges.

Let T be a tree with $k+1$ vertices. Remove a pendant vertex from T. The remaining graph is a tree with k vertices, and we can apply induction.

Every connected graph has a spanning tree.

Every graph with a spanning tree is connected.

Every connected graph with n vertices has at least $n - 1$ edges.

A tree is a minimally connected graph.

If we remove any edge from a tree, then the remaining graph is disconnected.

A connected graph is a tree if and only if every edge is a cut-edge.

Rooted Trees

A *directed tree* is a directed graph for which the underlying undirected graph is a tree.

A directed tree is called a *rooted tree* if there is a unique vertex r, called the *root* so that r has in-degree 1, and all other vertices has in-degree 1.

A *leaf* in a rooted tree is a vertex with outdegree zero.

A *branch node* is a vertex which is not a leaf.

Level k of a rooted tree is the level containing all vertices at distance k from the root.

If there is an edge (u, v) in a rooted tree, then v is a *child* of u, and u is the *parent* of v.

An *ancestor* of a vertex v is a vertex on the path from r to v. Vertex v is a *descendant* of u if u is an ancestor of v.

Two vertices with a common parent are called *siblings*.

The *subtree* at vertex v is the subgraph induced by the root v and all of its descendants.

A *binary* rooted tree is such that every vertex has at most two children. It is a *complete* binary tree if every branch node has exactly two children.