Depth-first Search

INPUT: **A connected graph $G = (V, E)$, $V = \{v_1, \ldots, v_n\}$.**
OUTPUT: **A (depth-first) spanning tree T of G.**

(*Step 1*) **Initialize $T = (V_T, E_T)$: $V_T = \{v_1\}$ and $E_T = \emptyset$. Set $current := v_1$.**

(*Step 2*) **Select the neighbour of $current$ with the lowest index which is not yet visited. If no such neighbour exist, continue to *Step 3*. If it does exist, say v_i, set $current := v_i$ and return to *Step 2*.**

(*Step 3*) **If $current \neq v_1$, then find the parent v_j of $current$, and set $current = v_j$ (backtracking). Go to *Step 2*.**

(*Step 4*) **If $current = v_1$, then stop.**

Breadth-first Search

INPUT: **A connected graph** $G = (V, E)$**,** $V = \{v_1, \ldots, v_n\}$**.**
OUTPUT: **A (breadth-first) spanning tree** T **of** G**.**

(*Step 1)* **Initialize** $T = (V_T, E_T)$**:** $V_T = \{v_1\}$ **and** $E_T = \emptyset$**. Insert** v_1 **in a queue** Q**.**

(*Step 2)* **Set** *current* **to be the vertex at the front of the queue** Q**, and delete this vertex from** Q**. Add all neighbours of** *current* **that have not been visited to the rear of the queue, in order of increasing index. If no such neigbours exist, repeat** *Step 2***.**

(*Step 3)* **If** Q **is empty, then stop.**

Vertex set: $\{a, b, c, d, e, f, g, h\}$.
Adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Merge sort:

INPUT: **A list L of n unordered numbers**
OUTPUT: **A list OL of the same numbers in increasing order.**

(*Step 1:*) **Form a binary tree T of height t, where $t = \lceil \log_2(n) \rceil$. Label its leaves with the elements of L. Label all intermediate nodes as unfinished, all leaves as finished.**

(*Step 2:*) **Find an unfinished intermediate node v with two finished children with lists L_1 and L_2:**

(*Step2a:*) **Merge L_1 and L_2 into L: take the smallest of the first elements of L_1 and L_2, remove it from its list and put it at the end of L. Repeate this until L_1 and L_2 are empty.**

(*Step 2b:*) **Assign L to v, and label v as finished.**

Repeat this step until all nodes are finished.

(*Step 3:*) **Let OL be the list assigned to the root.**

A *relation* from set A to set B is a subset of $A \times B$.

Notation: Given a relation $R \subseteq A \times B$,

$$xRy \Leftrightarrow (x, y) \in R$$

A relation R on a set A is a relation from A to A. Such a relation can have the following properties (universe of the quantifiers is A):

Reflexive:

$$\forall_x \, (x, x) \in R.$$

Symmetric:

$$\forall_x \forall_y \, (x, y) \in R \rightarrow (y, x) \in R.$$

Antisymmetric:

$$\forall_x \forall_y \, [(x, y) \in R \land (y, x) \in R] \rightarrow x = y$$

Transitive:

$$\forall_x \forall_y \forall_z [(x, y) \in R \land (y, z) \in R] \rightarrow (x, z) \in R$$

A	xRy **if:**	**refl.**	**sym.**	**antis.**	**tr**
people	x **is taller than** y			**X**	
reals	$x < y$			**X**	
reals	$x \geq y$	**X**		**X**	
integers	$x\|y$	**X**		**X**	
integers	$x + y$ **is even**	**X**	**X**		
people	x **has the same age as** y	**X**		**X**	
people	x **has a parent in common with** y	**X**		**X**	
integers	$y = x^2$				
sets	$x \subseteq y$	**X**		**X**	
integers	x **and** y **are relatively prime**		**X**		
integers	x **and** y**, divided by 5 have the same remainder**	**X**	**X**		
bit strings of length 7	x **and** y **have the same number of 1's**	**X**	**X**		
web pages	x **has a hyperlink to** y				

Partial order: a relation that is reflexive, anti-symmetric and transitive.

Examples:

- **The relation** R **on** $\mathbb{Z}$ **where** xRy **if** $x \geq y$**,**
- **The relation** R **on the collection of alls sets where** xRy **if** $x \subseteq y$**,**
- **The relation** R **on** $\mathbb{Z}$ **where** xRy **if** $x|y$**.**

Equivalence relation: a relation that is reflexive, symmetric and transitive.

Examples:

- **The relation** R **on** $\mathbb{Z}$ **where** xRy **if** $x - y$ **is divisible by 5.**
- **The relation** R **on the set of all people where** xRy **if** x **and** y **have the same age.**

A relation R on a set A can be represented as a *directed graph* $G_R = (A, E)$ **as follows:**

The vertices of G_R are the elements of A
The edge set $E = \{(a, b) \in A \times A | aRb\}$.

A relation R on a set $A = \{a_1, a_2, \ldots, a_n\}$ can be represented by an $n \times n$ *relation matrix* A **as follows:**

$$A_{i,j} = \begin{cases} 1 & \textbf{if } a_i R a_j \\ 0 & \textbf{otherwise} \end{cases}$$

Note that the relation matrix of R is the adjacency matrix of the directed graph representing R.