Partial order: a relation that is reflexive, anti-symmetric and transitive.

Examples:

- **The relation R on $\mathbb{Z}$ where xRy if $x \geq y$,**
- **The relation R on the collection of alls sets where xRy if $x \subseteq y$,**
- **The relation R on $\mathbb{Z}$ where xRy if $x|y$.**

Equivalence relation: a relation that is reflexive, symmetric and transitive.

Examples:

- **The relation R on $\mathbb{Z}$ where xRy if $x - y$ is divisible by 5.**
- **The relation R on the set of all people where xRy if x and y have the same age.**

A relation R **on a set** A **can be represented as a** *directed graph* $G_R = (A, E)$ **as follows:**

The vertices of G_R **are the elements of** A
The edge set $E = \{(a, b) \in A \times A | aRb\}$**.**

A relation R **on a set** $A = \{a_1, a_2, \ldots, a_n\}$ **can be represented by an** $n \times n$ *relation matrix* A **as follows:**

$$A_{i,j} = \begin{cases} 1 & \textbf{if } a_i R a_j \\ 0 & \textbf{otherwise} \end{cases}$$

Note that the relation matrix of R **is the adjacency matrix of the directed graph representing** R**.**

Partial Orders

Given a partial order R on a set A:

An element $x \in A$ is *maximal* if

$$\forall_{a \in A} xRa \rightarrow x = a.$$

An element $x \in A$ is *minimal* if

$$\forall_{a \in A} aRx \rightarrow x = a.$$

An element $x \in A$ is a *greatest* element if

$$\forall_{a \in A} aRx.$$

An element $x \in A$ is a *least* element if

$$\forall_{a \in A} xRa.$$

Let $B \subseteq A$.

An element $x \in A$ is a *lower bound* **of B if**

$$\forall_{b \in B} x R b.$$

An element x is a *greatest lower bound (glb)* **of B if x is a lower bound of B, and for every lower bound x' of B, $x' R x$.**

Let $B \subseteq A$.

An element $x \in A$ is a *upper bound* **of B if**

$$\forall_{b \in B} b R x.$$

An element x is a *least upper bound (lub)* **of B if x is an upper bound of B, and for every upper bound x' of B, $x R x'$.**