Combinations with repititions:

The number of ways to distribute objects of n different types over r different containers equals:

$$C(n + r - 1, r)$$

(assuming there are enough objects of each type)

Examples:

The number of ways to choose three integers i, j, k, where $0 \leq i \leq j \leq k \leq 100$

The number of times the print statement is executed in the program:

for $i := 1$ **to** 20 **do**
  for $j := i$ **to** 20 **do**
  for $k := j$ **to** 20 **do**
    print $(i * j + k)$

Self-test

1. **How many ways to give 10 different books to Mary, William and Charles so that Mary receives 3 books, William 5, and Charles 2?**
2. **How many ways to distribute 10 identical pens to William, Mary and Charles?**
3. **How many ways to distribute 3 types of pens over 10 different people?**
4. **How many ternary strings of length 10 are there with three zeros and five ones?**
5. **When six dice are cast, how many different outcomes are there? Are they all equally probable?**

Lexicographic order

A string $x_1x_2\dots x_n$ **is lexicographically less than** $y_1y_2\dots y_n$ **if for some** i**,** $1 \le i \le n$

- $x_j = y_j$ **for all** j **so that** $1 \le j < i$**, and**
- $x_i < y_i$

For example, 21345 is lexicographically less than 21534, and 12345 is lexicographically less than all other permutations of 1,2,3,4,5.

Generating permutations

Input: n.
Output: all permutations of $1, 2, \ldots, n$**, listed in lexicographic order.**

1. **Start with the permutation** $s_1 = 1, s_2 = 2, \ldots, s_n = n$.
2. **Find the rightmost element** s_i **so that** $s_{i-1} < s_i$.
3. **Find the smallest element** s_j **so that** $s_j > s_i$ **and** $j > i$.
4. **Swap** s_i **and** s_j.
5. **Rearrange the elements** $s_{i+1} \ldots s_n$ **in increasing order.**
6. **Repeat steps 2–5 until no element** s_i **can be found in step 2 (so all** s_i**'s are in decreasing order).**