Derangements

A derangement is an arrangement of the integers $1 \ldots n$ so that none of the integers stays in its natural position.

Example: all derangements of 1234

2143	**3142**	**4123**
2341	**3412**	**4312**
2413	**3421**	**4321**

The number of derangements of $1 \ldots n$ is denoted by d_n, and can be calculated with the inclusion/exclusion formula:

$$\begin{aligned} d_n &= n! - \tbinom{n}{1}(n-1)! + \tbinom{n}{2}(n-2)! - \ldots \\ &= \textstyle\sum_{t=0}^{n} (-1)^n \tbinom{n}{t} (n-t)! \\ &= n! \textstyle\sum_{t=0}^{n} (-1)^n \frac{1}{t!} \end{aligned}$$

Maclaurin series for the exponential function:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{t=0}^{\infty} \frac{x^t}{t!}.$$

For $x = -1$:

$$e^{-1} = 1 + (-1) + \frac{1}{2!} - \frac{1}{3!} + \ldots = \sum_{t=0}^{\infty} (-1)^t \frac{1}{t!}.$$

The Travelling Salesman Problem

INPUT: **A set of points V, and a travel cost w_{uv} for each pair of points u, v in V.**

OBJECTIVE: **find a tour through all the points of V so that the total travel cost is minimized.**

Example.

Travel cost:

	A	B	C	D	E
A	$-$	45	98	43	27
B		$-$	38	53	24
C			$-$	62	89
D				$-$	55