MATH 3330 — Applied Graph Theory

Assignment 1

Due Tuesday, January 15,2006 (before class)

(1.1.19) Determine, with the methods shown in class, whether each of the following sequences is graphic. If it is, draw a graph that realizes the sequence.
a. (7,6,6,5,4,3,2,1) b.(5,5,5,4,2,1,1,1)
c. (7,7,6,5,4,4,3,2) d.(5,5,4,4,2,2,1,1)

(1.1.26,27) A pair of sequences $< a_1, \ldots, a_n >$ and $< b_1, \ldots, b_n >$ is *digraphic* if there exists a simple digraph (digraph with no multi-edges or self-loops) with vertex-set $\{v_1, \ldots, v_n\}$ so that $outdegree(v_i) = a_i$ and $indegree(v_i = b_i)$ for $i = 1, \ldots, n$.

(a) Develop a method to determine whether a pair of sequences is digraphic, similar to the one for determining whether a sequence is graphic. Explain in logical detail why the method works.

(b) Use your method to determine whether the pair of sequences $< 3, 1, 1, 0 >$ and $< 1, 1, 1, 2 >$ is digraphic. Show your work.

(1.2.2) What is the maximum possible number of edges in a simple bipartite graph on m vertices? (Explain your answer)

(1.2.28) Show that every simple graph is an intersection graph by describing (in general) how to construct a family of sets which it represents.

(1.4.21–24) Determine the diameter, radius, and central vertices of the following graphs:

(a) Path graph P_n

(b) Cycle graph C_n

(c) Complete graph K_n

(d) Complete bipartite graph $K_{n,m}$

(e) Petersen graph