MATH 4320/5320
Combinatorial Optimization
Assignment 1

Due Tuesday, Jan. 13

In each of the following problems, a set $S \subset \mathbf{Z}^X$ of combinatorial objects is described. Find an inequality description that defines a polyhedron P so that $S = P \cap \mathbf{Z}^X$.

1. An *dominating set* in a graph $G = (V, E)$ is a set of nodes of G so that each node of G is either in the set, or a neighbour of a node in the set. Let S be the collection of all dominating sets, represented as 0/1 vectors in $\mathbf{Z}^V$.

2. Given a directed graph $G = (V, E)$ and a node $r \in V$, a rooted spanning tree with root r is a tree T in G so that for each node $v \in V$, there is a directed path in T from v to r. Let S be the collection of all rooted spanning trees, represented as 0/1 vectors in $\mathbf{Z}^E$.

3. A *coclique* in a graph $G = (V, E)$ is a collection of nodes so that no edge of G has more than one endpoint in the collection. Let S be the collection of all cocliques, represented as 0/1 vectors in $\mathbf{Z}^V$.

4. (only for 5320 students) Given a directed graph $G = (V, E)$, and two nodes $s, t \in V$, let S be the set of all directed paths from s to t in G, represented as 0/1 vectors in $\mathbf{Z}^E$.

Due Thursday, Jan. 15

1. Prove that the maximum of a linear function over a polytope is always attained at a vertex.

2. Show that any maximization problem $max\{c^T x : Ax \leq b\}$, can be transformed into an equivalent problem $max\{d^T x : A'x = b'\}$. Precisely, both problems must achieve the same maximum, and there must be one-to-one correspondence between their feasible solutions. Give this correspondence.