Combinatorial Optimization

MATH 4320/5320

Combinatorial optimization is a lively field of applied mathematics, combining techniques from combinatorics, linear programming, and the theory of algorithms, to solve optimization problems over discrete structures. The study of the polyhedra associated with these problems has proved to be a most powerful, coherent, and unifying tool throughout cominatorial optimization. Not only has it led to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. This course will provide an introduction to the problems of combinatorial optimization and the application of polyhedral methods.

This will be a theoretical course, with emphasis on the use of techniques from combinatorial optimization to prove theorems about and gain insight in combinatorial problems. Parting from the integer formulation of a combinatorial problem, we will look at the Linear Programming (LP) relaxation and its dual, as well as the corresponding dual combinatorial problem. We will also study the polytope which is the convex hull of the (integer) solutions to the combinatorial problem, and its relation to the polytope which is formed by the inequalities of the LP relaxation.

We will loosely follow the course text, with an emphasis on Chapters 5 through 8. We will also be reading relevant papers and sections from other books. The course will be held as a typical "topics" course, with lots of in-class discussion and presentations, and hopefully some real research at the end!

Instructor:

Jeannette Janssen. Office: Chase 205. Email: janssen@mathstat.dal.ca.
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Time and place:

TR 11:30-13:00, Seminar room, Chase building (second floor)

Course text:

Combinatorial Optimization: Cook, Cunningham, Pulleyblank and Schrijver, Wiley Interscience 1998.
Other recommended texts will be presented in class and made available through the instructor.

Topics:

Linear programming and the duality theorem. Integer programming formulations of combinatorial problems and their LP relaxations. Bipartite matchings. Minimum spanning trees. Polytopes and polyhedra that represent combinatorial objects. The bipartite matching polytope. Network flows. Matchings in non-bipartite graphs. Matroids. Independent sets and graph colouring. The perfect graph theorem. The Travelling Salesman Problem. Total Unimodularity. Recent results.

Marking scheme:

Assignments: 40%
Presentations: 30%
Final project: 30%