January 2009 Questions

1) Suppose a and b are real, positive numbers such that $a+b=1$. Show the following:

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right) \geq 9$$

2) We define the "floor" operation as follows: for any real number t, $\lfloor t \rfloor$ is the greatest integer which is less than or equal to t. For example:

$$\begin{aligned} \lfloor 12 \rfloor &= 12, \\ \lfloor 0.5 \rfloor &= 0, \\ \lfloor \pi \rfloor &= 3, \\ \lfloor -7.643 \rfloor &= -8. \end{aligned}$$

Show that there is no real number x which satisfies the following equation:

$$\lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 4x \rfloor + \lfloor 8x \rfloor + \lfloor 16x \rfloor + \lfloor 32x \rfloor = 12345.$$

Submit all solutions by 23.59 January 31, 2009.