

November 2008 Questions

- 1) Suppose we are given a list of positive integers with such that
- (i) there is at least one integer in the list;
 - (ii) all integers in the list are less than or equal to 2008;
 - (iii) repetition of integers is allowed, i.e. an integer can appear more than once in the list;
 - (iv) the sum of all the integers in the list is 2008.

Some such possible lists are $[1, 1, \dots, 1]$ where the integer 1 appears 2008 times, $[4, 5, 16, 16, 57, 120, 208, 1582]$, and $[2008]$.

Suppose in our list that there are n_1 integers equal to 1, n_2 integers equal to 2, \dots , n_{2008} integers equal to 2008. What is the largest possible value of

$$n_2 + 2n_3 + 3n_4 + \dots + 2007n_{2008}?$$

- 2) Let $p(x)$ be a polynomial with the following properties:

- (i) $p(1) = 1$;
- (ii) $\frac{p(2x)}{p(x+1)} = 8 - \frac{56}{x+7}$ for all values x where both sides are defined.

Show the following:

- (a) the degree of $p(x)$ is 3.
- (b) $p(-1) = -\frac{5}{21}$.

Submit all solutions by 23.59 November 30, 2008.