Math 1000 Pre-Cal Practice Solutions

1. (Working with functions)

   (a) Given that $f(x) = (x+3)^2 + x$, find $f(a+h)$ and simplify.

$$f(a+h) = ((a+h)+3)^2 + (a+h) = (a+h+3)^2 + a + h = (a+h+3)(a+h+3) + a + h$$
$$= (a^2+ah+3a+ah+h^2+3h+3a+3h+9) + a + h = a^2+2ah+6a+6h+h^2+9+a+h =$$
$$a^2 + 2ah + 7a + 7h + h^2$$

   (b) Given that $g(x) = \sin(x)$ and $f(x) = 2x + 1$, find $g(f(x))$ and $f(g(x))$.

$$f(g(x)) = f(\sin(x)) = 2\sin(x) + 1$$
$$g(f(x)) = g(2x+1) = \sin(2x+1)$$

   (c) Given that $f(x) = \frac{\sqrt{x}}{x^2+1}$, find $f(2)$, $f(4)$, and $f(u^2)$.

$$f(2) = \frac{\sqrt{2}}{2^2+1} = \frac{\sqrt{2}}{5}$$
$$f(4) = \frac{\sqrt{4}}{4^2+1} = \frac{2}{17}$$
$$f(u^2) = \frac{\sqrt{u^2}}{(u^2)^2+1} = \frac{u}{u^4+1}$$

   (d) Given that $f(x) = x^3$, evaluate

$$\frac{f(2+h) - f(2)}{h}$$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^3 - (2)^3}{h} = \frac{(2+h)(4+2h+h^2) - 8}{h}$$
$$= \frac{8+4h+2h^2+4h+2h^2+h^3-8}{h} = \frac{4h^2+8h+h^3}{h} = \frac{h(4h+8+h^3)}{h} = h^3+4h+8$$

2. (Equations of lines)

   (a) Find the equation of the line if the slope is 3 and it goes through the point $(1, 1)$.

$$y - 1 = 3(x-1) \Longrightarrow y = 3x - 3 + 1 \Longrightarrow y = 3x - 2$$

   (b) Find the equation of the line if the slope is -1 and it goes through the point $(0, 1)$.

$$y - 1 = -1(x - 0) \Longrightarrow y = -x + 1$$

(c) Find the equation of the line if the slope is $\frac{-1}{2}$ and it goes through the point $(2, -2)$.

$$y - (-2) = \frac{-1}{2}(x - 2) \Longrightarrow y + 2 = \frac{-1}{2}x + 1 \Longrightarrow y = \frac{-1}{2}x + 1 - 2 \Longrightarrow y = \frac{-1}{2}x - 1$$

3. (Laws of logarithms) Use Laws of Logs to rewrite the following:

(a)

$$\ln(x^2) = 2\ln(x)$$

(b)

$$\frac{\ln(8)}{2} = \frac{\ln(2^3)}{2} = \frac{3\ln(2)}{2}$$

(c)

$$\ln\frac{(x+1)^3(3x^2+5)^4}{x^5} = \ln\left((x+1)^3(3x^2+5)^4\right) - \ln(x^5)$$
$$= \ln(x+1)^3 + \ln(3x^2+5)^4 - \ln(x^5) = 3\ln(x+1) + 4\ln(3x^2+5) - 5\ln(x)$$

(d)

$$\ln(8)\ln(2^(1/3)) = \ln(2^3)\ln(2^(1/3)) = 3\ln(2)\frac{1}{3}\ln(2) = \ln(2)\ln(2) = 2\ln(2) = \ln(4)$$

4. (Working with exponents)

(a)

$$x^{-5}x^{-4} = x^{-5+(-4)} = x^{-9}$$

(b)

$$(x^2)^3 + x^6 = x^{2\cdot 3} + x^6 = x^6 + x^6 = 2x^6$$

(c)

$$(x^2)^3 + x^4 = x^{2\cdot 3} + x^4 = x^6 + x^4$$

(d)

$$\frac{x^6}{x^4} = x^{6-4} = x^2$$

(e)

$$\frac{x^{1/2}}{x^2} = x^{(1/2)-2} = x^{-3/2}$$

(f)

$$8^{2/3} = (2^3)^{2/3} = \left((2^3)^{1/3}\right)^2 = 2^2 = 4$$

5. (Trigonometry)

(a) What is $\cos(\frac{\pi}{2})$?

$$\cos(\frac{\pi}{2}) = 0$$

(b) What is the cos(sin(0))?

$$\cos(\sin(0)) = \cos(0) = 1$$

(c) Simplify $\frac{\tan(2\theta)}{\sin(2\theta)}$.

$$\frac{\tan(2\theta)}{\sin(2\theta)} = \frac{1}{\sin(2\theta)}\frac{\sin(2\theta)}{\cos(2\theta)} = \frac{1}{\cos(2\theta)} = \sec(2\theta)$$

(d) Simplify $\frac{\cot(3\theta)}{\sin(2\theta)}$.

$$\frac{\cot(3\theta)}{\sin(2\theta)} = \frac{1}{\sin(2\theta)}\frac{\cos(3\theta)}{\sin(3\theta)}$$

Can't be simplified any more than this.