Instruction

- **You are allowed 2 hours (120 minutes) for this exam. Please pace yourself accordingly.**
- **You may not use any calculator, notes, or other assistance on this exam.**
- **In order to receive full credit, you must show your work and carefully justify your answers. The correct answer without any work will receive little or no credit.**
- **Please write neatly. Illegible answers will be assumed to be incorrect.**
- **Read the question carefully, if the method is specified you can not use other methods.**

Method of variation method:(Do not ask me what $W(x)$ is)

$$y_p = -y_1(x)\int \frac{y_2(x)f(x)}{W(x)} + y_2(x)\int \frac{y_1(x)f(x)}{W(x)}$$

Trigonometric identity you might need for the question

$$\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$

$$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

1. **(40 points) Find the general solution of the following first order differential equations.**
   - **(a)** $\frac{dy}{dx} = y\sin(x)$
   - **(b)** $2xy' - 3y = 9x^3$
   - **(c)** $xyy' = x^2 + 2y^2$
   - **(d)** $xy' + 6y + 3xy^{\frac{4}{3}} = 0$
   - **(e)** $x^2y'' + 3xy' = 2$

2. **(10 points) Solve the following initial problem.**

$$3y^{(3)} + 2y'' = 0$$
$$y(0) = -1, \quad y'(0) = 0, \quad y''(0) = 1$$

3. **(10 points) Find a particular solution of**

$$y'' + 9y = \sin(3x)$$

4. **(10 points) Use the method of variation of parameters to find a particular solution of**

$$y'' + 9y = 2sec(3x)$$

5. **(15 points) First determine whether $\lambda = 0$ is an eigenvalue, then find the positive eigenvalues and associated eigenfunctions.**

$$y'' + \lambda y = 0; \quad y'(-\pi) = 0 \quad y'(\pi) = 0$$

6. **(10 points) Apply the Convolution Theorem to find the inverse Laplace transform of the function.**

$$F(s) = \frac{s^2}{(s^2+4)^2}$$

7. **(10 points) Use Laplace transform to solve the initial problem:**

$$x'' + x = \sin(2t), \quad x(0) = 0, \quad x'(0) = 0$$

8. **(15 points) Transform the given differential equation to find a nontrivial solution such that $x(0) = 0$.**

$$tx'' + (3t-1)x' + 3x = 0$$