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*Counting Base Phi Representations*,  
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**Abstract**

In a base phi representation, a natural number is written as a sum of powers of the golden mean  $\varphi$ . There are many ways to do this. How many? Even if the number of powers of  $\varphi$  is finite, then any number has infinitely many base phi representations. By not allowing an expansion to end with the digits 0,1,1, the number of expansions becomes finite, a solution proposed by Ron Knott. Our first result is a recursion to compute this number of expansions. This recursion is closely related to the recursion given by Neville Robbins to compute the number of Fibonacci representations of a number, also known as Fibonacci partitions. We propose another way to obtain finitely many expansions, which we call the natural base phi expansions. We prove that these are closely connected to the Fibonacci partitions.