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Extension of the Equation $\sum_{j=1}^{k} jF_j^p = F_n^q$ *to a Family of Lucas Sequences*,

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Abstract

We solve the equation $\sum_{j=1}^{k} jU_j(x,y)^p = U_n(x,y)^q$ for positive integers x, p, q, k, n, with $y = \pm 1$ and $\max\{p,q\} \leq 11$, where $U_m(x,y) = \frac{\alpha^m - \beta^m}{\alpha - \beta}$ and α and β are the roots of the polynomial $t^2 - xt + y$. This generalizes existing results on similar equations, wherein the sequence was fixed as the Fibonacci or Pell numbers. In addition, we find all solutions when $k = 2$ and $y = \pm 1$.