1967 AS THE SUM OF SQUARES

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With the coming of a new year, there is always a tendency to find out whether the number that indicates it has some special mathematical properties. 1967 is, of course, no square. But there is a theorem that states that any integer can be represented as the sum of at most four squares. Let us investigate the minimum number of squares that will add up to 1967.

First, we note that the square of every even number is divisible by 4 and the square of every odd number on being divided by 4 gives a remainder of one. If 1967 is to be the sum of the squares of two numbers, one must be odd and one even, otherwise there could not be an odd sum. But the sum of two such squares on being divided by 4 would give a remainder of one, while 1967 on being divided by 4 gives a remainder of 3. Thus 1967 cannot be the sum of two squares.

For three squares, 1967 would have to be the sum of the squares of three odd numbers since the remainder on division by 4 is 3. Now the squares of the odd numbers ending in 1, 3, 5, 7, 9 end respectively in 1, 9, 5, 9, and 1. Taking these endings three at a time, it can be easily shown that only the combinations 1, 5, 1 and 9, 9, 9 give a last digit of 7. So one systematic way to proceed is to consider the various cases corresponding to numbers ending first of all in 5, namely: 5, 15, 25, 35. Subtracting out 25, 225, 625, and 1225 gives remainders of 1942, 1742, 1342, 742. We can proceed by table as follows:

	742	1342	1742	1942
1	741	1341	1741	1941
9	661	1261	1661	1861
11	621	1221	1621	1821
19	381	981	1381	1581
21	301	901	1301	1501
29		501	901	1101
31		381	781	981

We can stop at 31 since this brings us to the halfway point with the largest number 1942. Since no squares appear in the table, this disposes of the possibility 1, 5, 1 as endings. Similar considerations apply for 9, 9, 9. Thus, the only possibility is four squares. One such representation is:

$$1967 = 6^2 + 9^2 + 25^2 + 35^2 \quad .$$

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