In Praise of Loops

Pavol Hell, Simon Fraser University

Dalhousie by Zoom, November 17, 2021

From Various Papers

Includes joint results with

- Tomás Feder
- César Hernández Cruz
- Jing Huang
- Jephian C.-H. Lin
- Ross McConnell
- Jarik Nešetřil
- Arash Rafiey

Graphs

Graphs

An interval graph

The intersection graph $G(\mathcal{I})$ of a family $\mathcal{I}$ of intervals has vertices $I, I \in \mathcal{I}$, and adjacencies $I \sim I' \iff I \cap I' \neq \emptyset$

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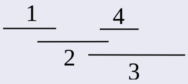

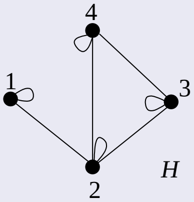

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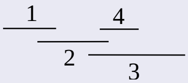

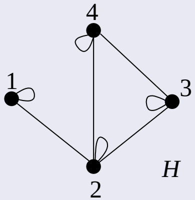

A **reflexive** graph

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A 2-DR graph

Intersection graph of a family of UP rays vs a family of RIGHT rays

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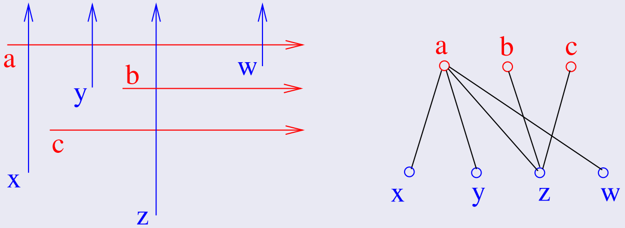

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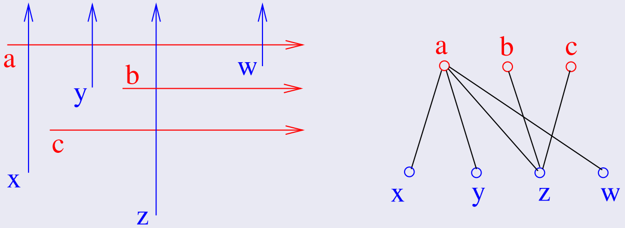

An **irreflexive** graph

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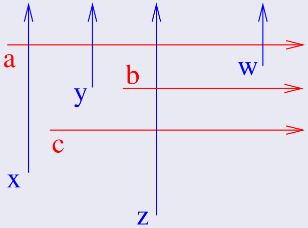
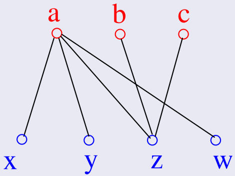
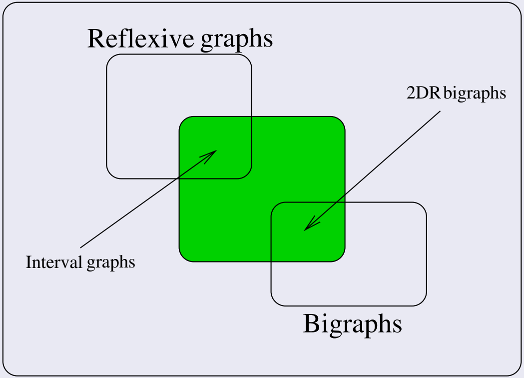




A **bigraph**

Graphs with Possible Loops

A new unifying concept



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Dominating set

A set S of vertices in a graph G is a *dominating set* in G if each vertex not in S has a neighbour in S

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In a reflexive graph a total dominating set is just a dominating set.

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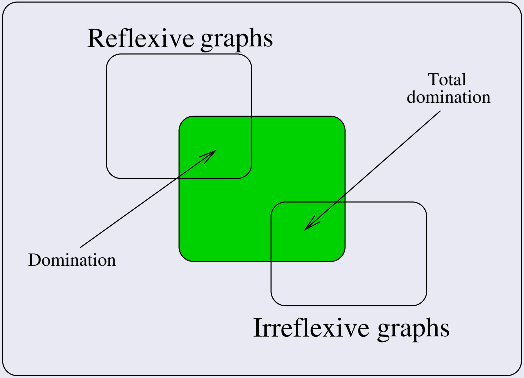
Same notion, applied in a **reflexive** versus an **irreflexive** graph

A new unifying concept



Cops and robbers

A cop trying to capture a robber on a graph G: at each move, each player can stay where it is, or move along an edge

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Applied in an **irreflexive graph**

A cop-win graph

A **reflexive graph** in which the cop wins

Is there an irreflexive version?

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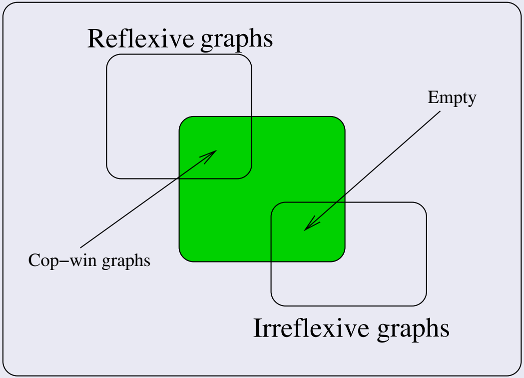
(The robber can shaddow the cop)

Cop-win graphs



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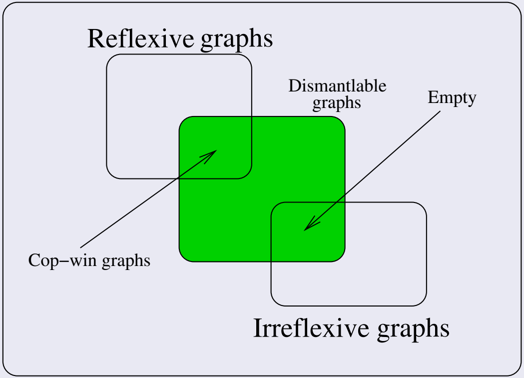

Connected Graphs with Possible Loops

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Reducible to a single vertex by a sequence of *folds*

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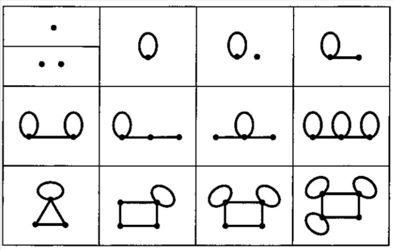
Examples of dismantlable graphs G with possible loops



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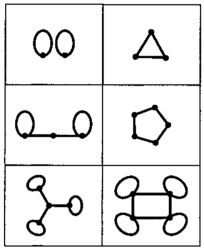

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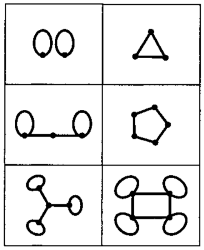

Dismantlable graphs with possible loops

The cop has a winning strategy in $G \iff G$ is dismantlable

Nowakowski+Winkler 1983, Quilliot 1983, Brightwell+Winkler 2000

Homomorphism Problems

Given a fixed graph H

Is there a homomorphism of input G to H?

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Dichotomy for undirected graphs H

The problem is polynomial if H is bipartite, and NP-complete otherwise

H+Nešetřil 1990

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Gödel Prize 2021 to Bulatov (counting)

Graphs with Possible Loops

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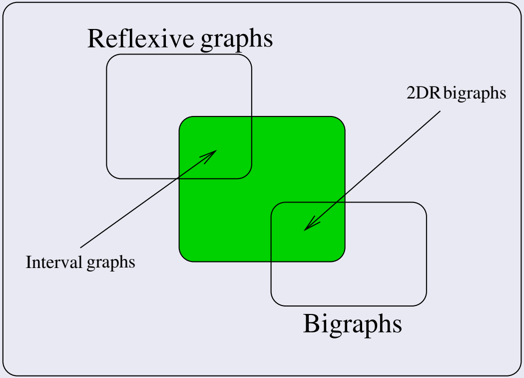
A unifying concept for interval graphs and 2DR graphs



Strongly Chordal Graphs

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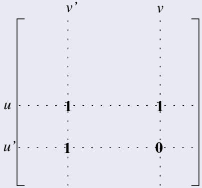

Characterization Theorems

For a reflexive graph H

- H is strongly chordal $\iff$
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- H does not contain an induced cycle > 3 or an induced trampoline

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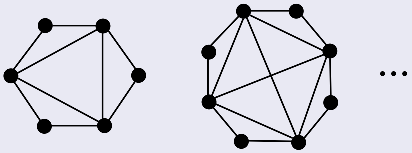

Farber 1983, Anstee+Farber 1984

Chordal Bigraphs

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A chordal bigraph $H = (U, V)$: there are orderings $<_U, <_V$

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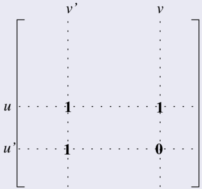
Bi-ADJ(H) can be permuted to avoid the Γ matrix



Golumbic+Goss 1978

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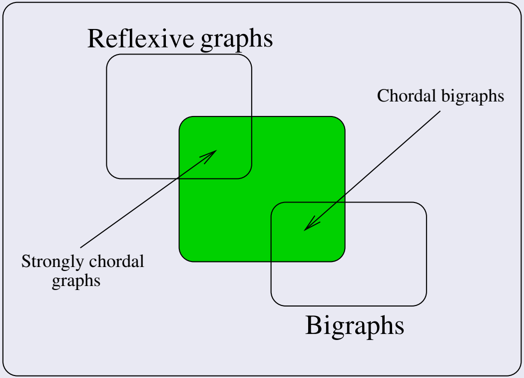
Golumbic+Goss 1978

A new unifying concept



Graphs With Possible Loops

A strongly chordal graph H **with possible loops**:

The adjacency matrix can be permuted to avoid Γ

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Special cases

- For reflexive graphs, as before, the strongly chordal graphs

Graphs With Possible Loops

A strongly chordal graph H with possible loops:

The adjacency matrix can be permuted to avoid Γ

Special cases

- For reflexive graphs, as before, the strongly chordal graphs
- For irreflexive graphs H, we have
  H is strongly chordal $\iff$ H is chordal bipartite

Strongly Chordal Graphs with Possible Loops

For graphs H with possible loops

- H is strongly chordal

Strongly Chordal Graphs with Possible Loops

For graphs H with possible loops

- H is strongly chordal $\iff$
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Strongly Chordal Graphs with Possible Loops

For graphs H with possible loops

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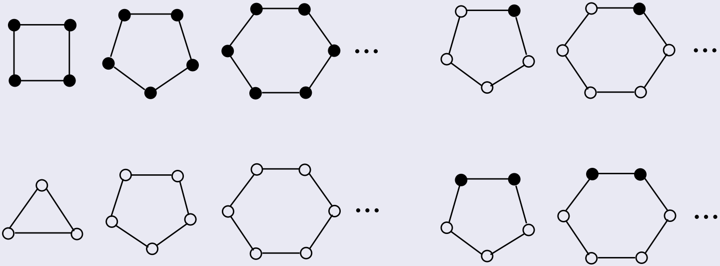

Strongly Chordal Graphs with Possible Loops

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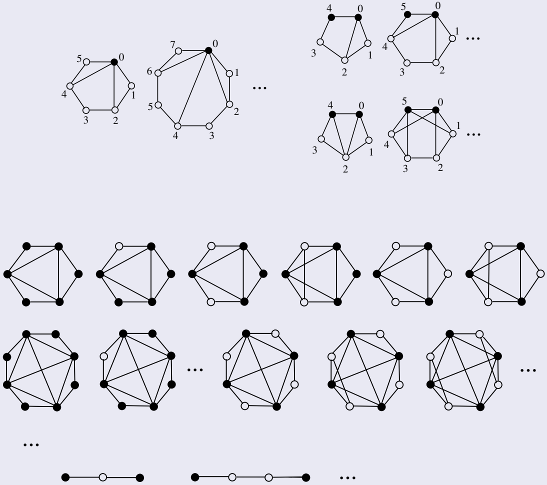

H+Hernandez-Cruz+Huang+Lin 2020

Domination

Dominating set in a reflexive graph

Each vertex is in the set or has a neighbour in the set

Total dominating set in an irreflexive graph

Each vertex has a neighbour in the set

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General dominating set in a graph with possible loops

Each vertex has a neighbour in the set

Domination

Existing linear-time algorithms

- Minimum dominating set in reflexive strongly chordal graphs

  (Farber 84)

- Minimum total dominating set in a chordal bipartite graph

  (Damaschke+Mueller+Kratsch 1990)

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New result

Linear-time algorithm to find a smallest general dominating set in strongly chordal graphs with possible loops

H+Hernandez-Cruz+Huang+Lin 2020

Domination in strongly chordal graphs with possible loops

C = a set of vertices with disjoint neighbourhoods
D = a general dominating set

Duality

Max $|C|$ = Min $|D|$

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Algorithm / Proof

Repeat until all vertices are labeled:

- Find, in $<$, the first vertex x without the label N
- Find, in $<$, the last neighbour y of x
- Label x by C and y by D, then label all neighbours of y by N

H+Hernandez-Cruz+Huang+Lin 2020

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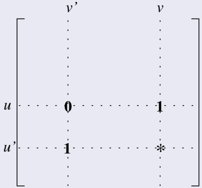

Graphs with Possible Loops

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Threshold tolerance (tt-) graphs

Each vertex v can be assigned a *weight* w_v and a *threshold* t_v so that

$$u \sim v \iff w_u + w_v \geq \min(t_u, t_v)$$

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Co-tt graphs = complements of tt-graphs

H is a co-tt graph $\iff$ there exist real functions ℓ, r on $V(H)$ such that

$$u \sim v \iff \ell(u) \leq r(v) \text{ and } \ell(v) \leq r(u)$$

Monma+Reed+Trotter 1988

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$$u \not\sim v \iff \ell(u) > r(v) \text{ or } \ell(v) > r(u)$$

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Positive (blue) and negative (red)

- Positive intervals (blue vertices) have $\ell(v) \leq r(v)$
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Possible loops

- Blue vertices have loops
- Red vertices have no loops

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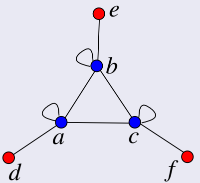
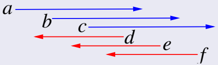
Adjacency rules

- Blue u and blue v have $u \sim v \iff I_u \cap I_v \neq \emptyset$
- Red u and blue v have $u \sim v \iff I_u \subseteq I_v$

Co-tt Graphs

Example





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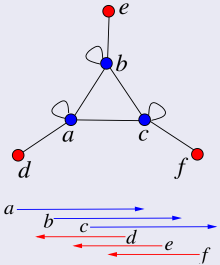

A co-tt model

- Blue = interval graph
- Red = independent set

Graphs with Possible Loops

A graph H with possible loops

- H is a co-tt graph $\iff$
- ADJ(H) can be permuted to avoid a Σ matrix

H+Huang+McConnell+Rafiey 2019

ADJ(H) can be permuted to avoid a Σ matrix

- $\iff$ H is an interval graph (if H is a reflexive graph)
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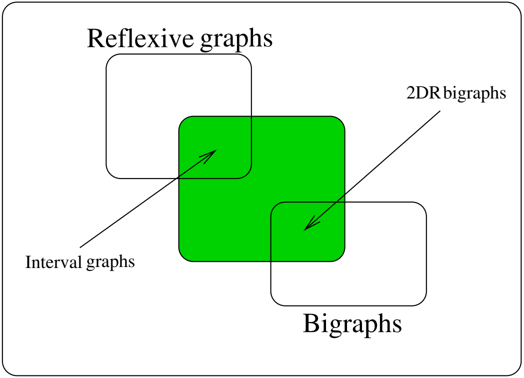

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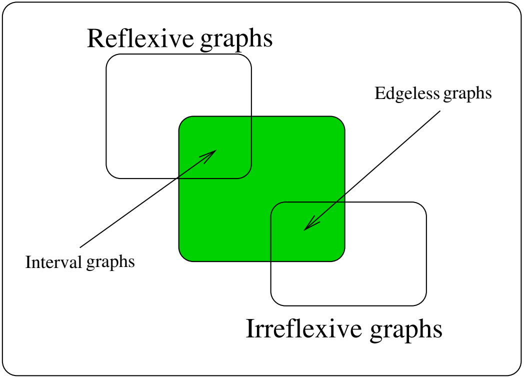

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A bigraph H

A bipartite red-blue digraph with all edges from red to blue

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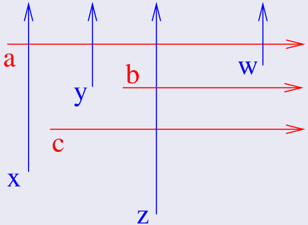

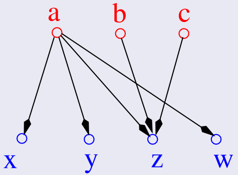

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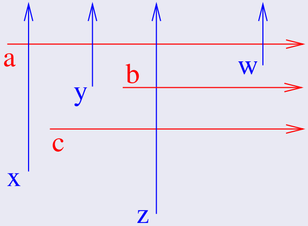

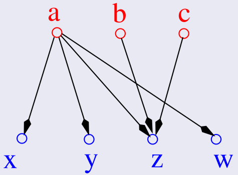

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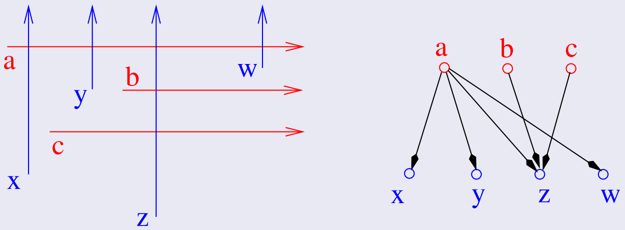

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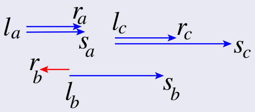
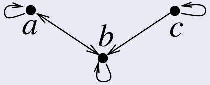
Example signed-interval model of a digraph H





H+Huang+McConnell+Rafiey 2019

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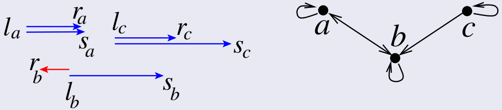
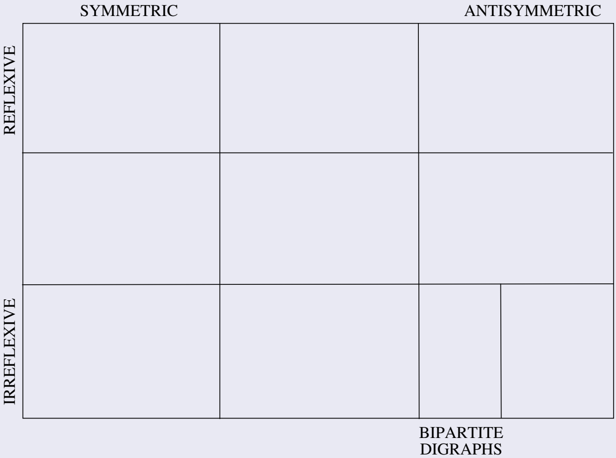


H+Huang+McConnell+Rafiey 2019

Polynomial recognition Rafiey+Rafiey 2022?

Digraphs

Digraphs



Signed-interval Digraphs

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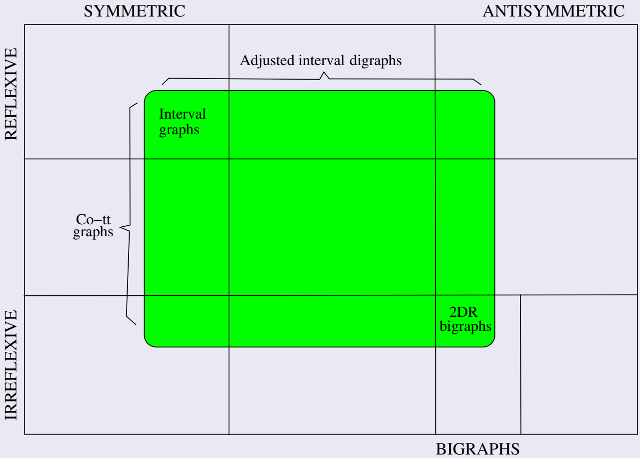

Reflexive Digraphs

An adjusted-interval digraph

Vertices can be represented by pairs of *adjusted* intervals I_v, J_v,

$$v \to w \iff I_v \cap J_w \neq \emptyset$$

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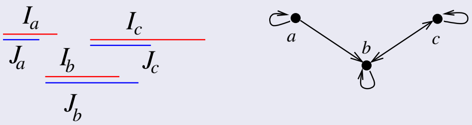

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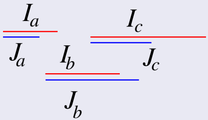

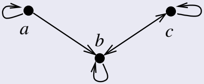

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Feder+H+Huang+Rafiey 2012

Obstruction Characterization

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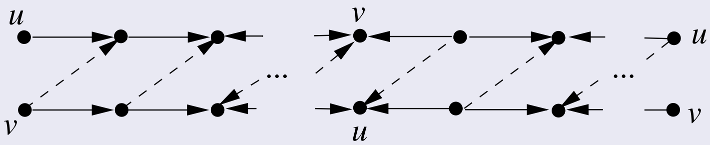
it has no invertible pair



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Adjusted-interval Digraphs

Similarities to interval graphs

- similar geometric representations
- similar obstructions
- similar ordering characterization

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$O(n^4)$ recognition algorithm

Open

A more efficient recognition algorithm?