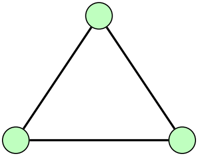
Quintic graphs with every edge in a triangle

James Preen, Cape Breton University

Atlantic Graph Theory Seminar Series: 1st December 2021

Regular graphs with every edge in a triangle

The 2-regular connected graph with the triangle property:



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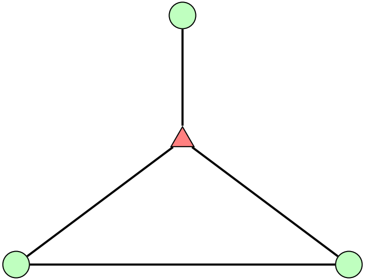
A 3-regular connected graph with the triangle property:



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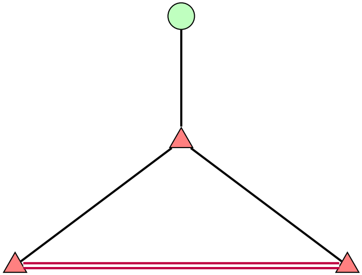

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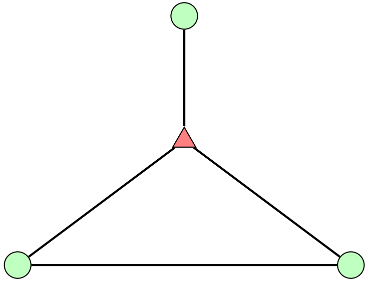

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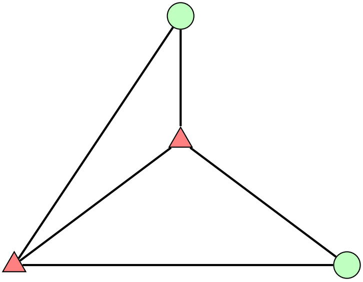

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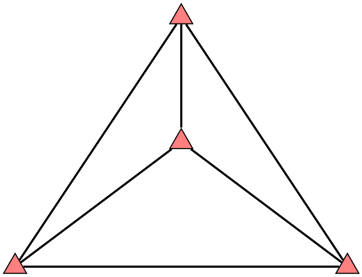
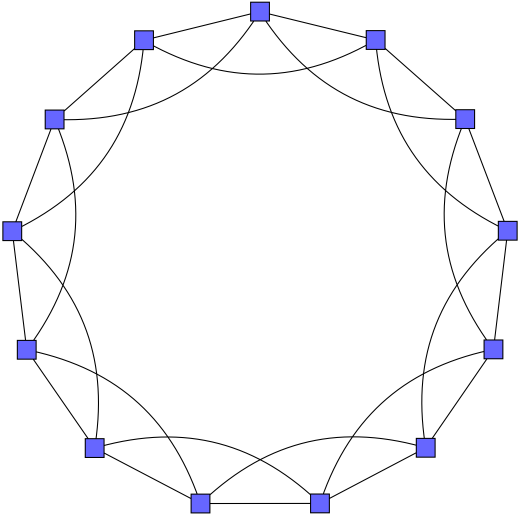


4-regular with the triangle property: Squares of cycles



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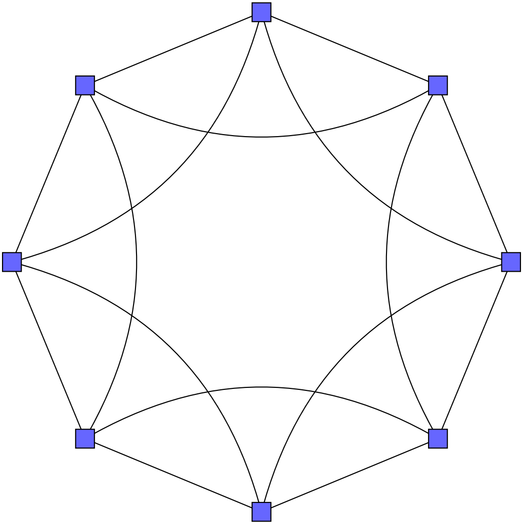

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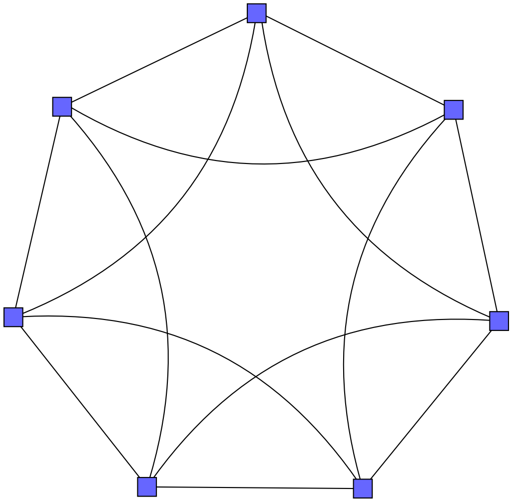

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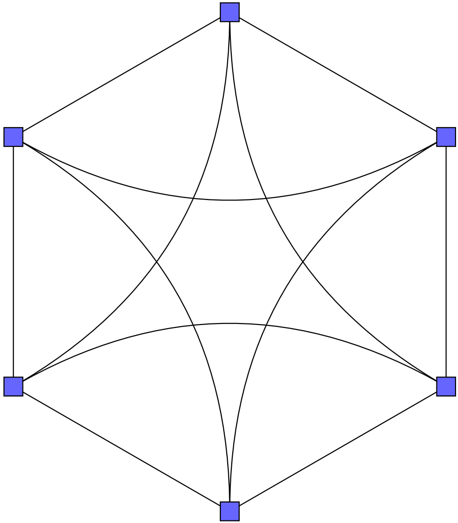

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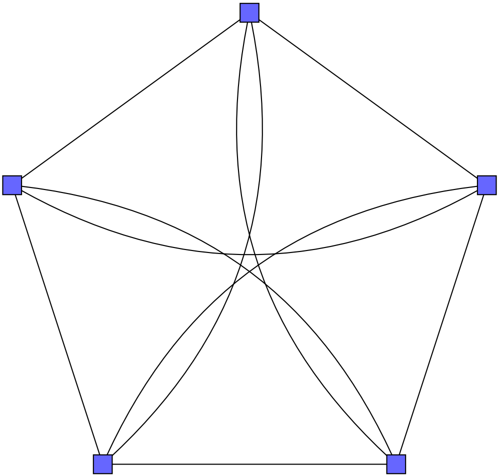

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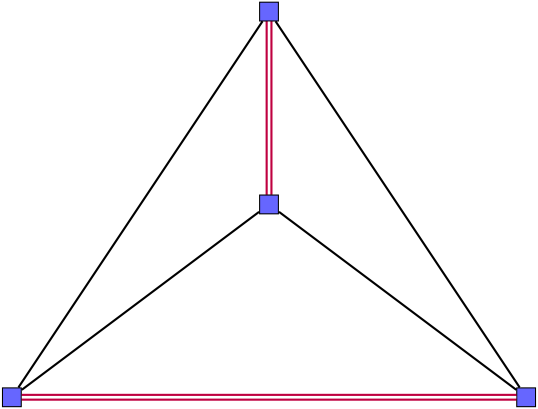

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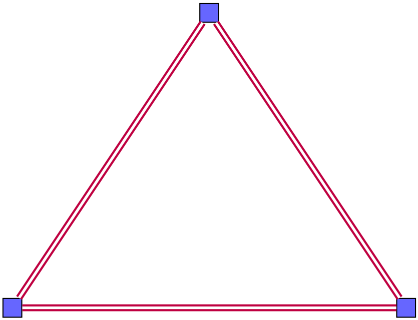
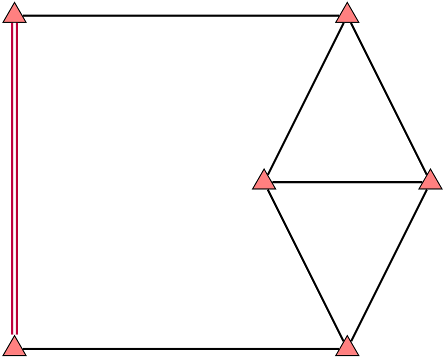


4-regular graphs with the triangle property: Line (multi-)graphs



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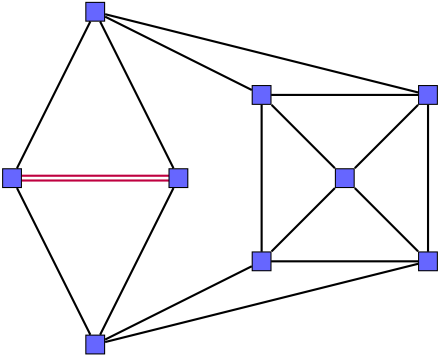
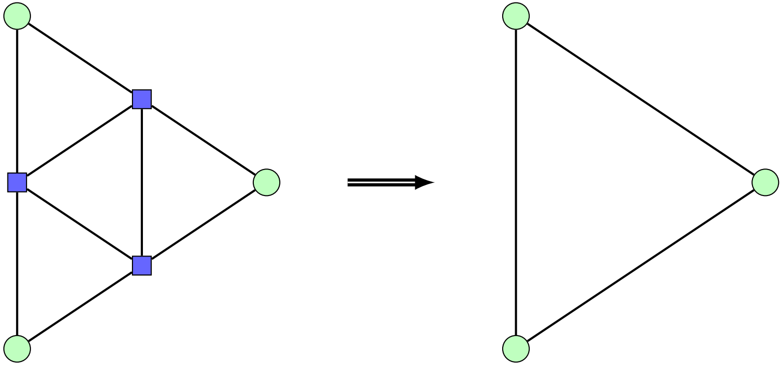


4-regular graphs with the triangle property: Reductions



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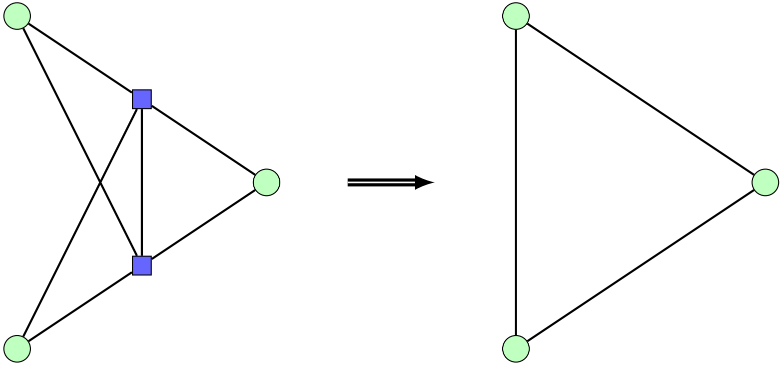

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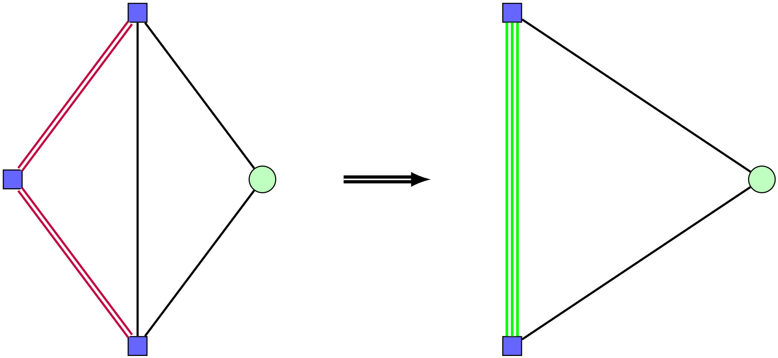

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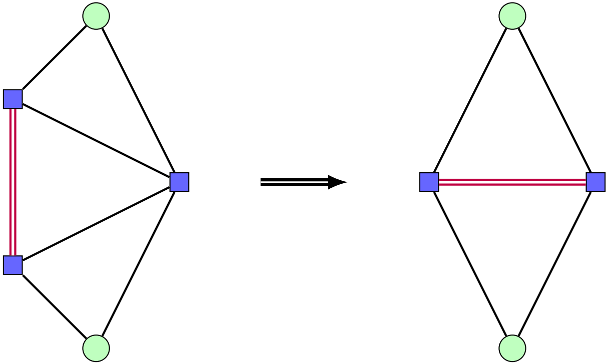

4-regular graphs with the triangle property: Proof Genesis

mathoverflow *Asked January 3rd 2012 by Gordon Royle.*

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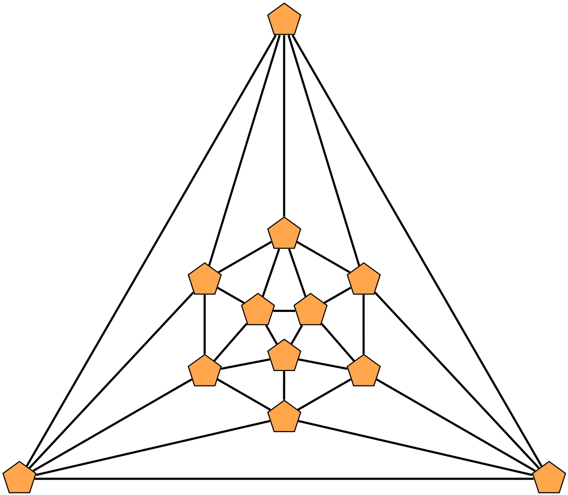
I'm tempted to just say that this is an uncontrollable mess, but perhaps someone can figure out what to do

Icosahedron is 5-regular, all edges in 2 triangles



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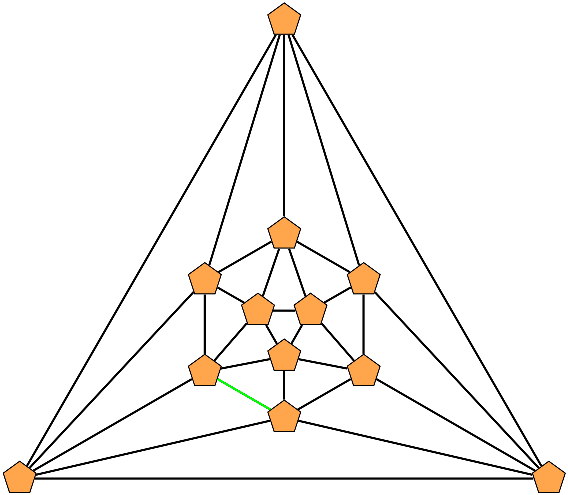

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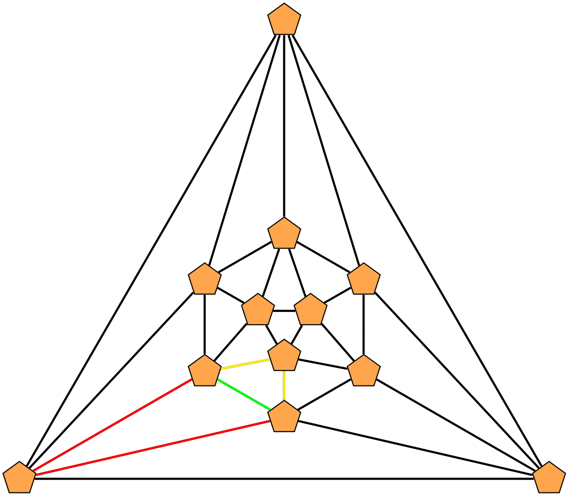
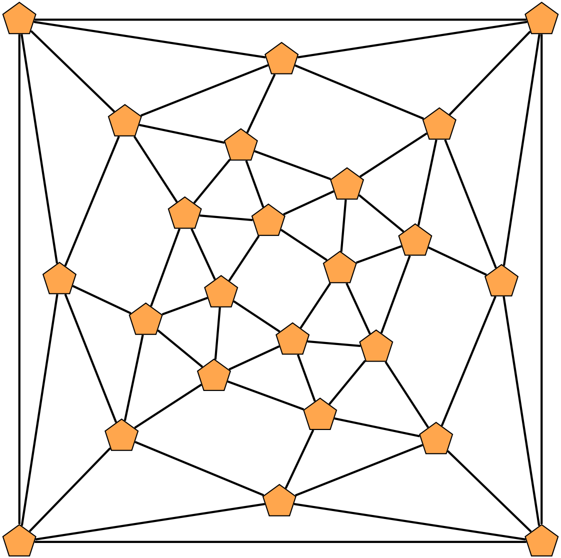


A larger 5-regular graph with all edges in 1 or 2 triangles



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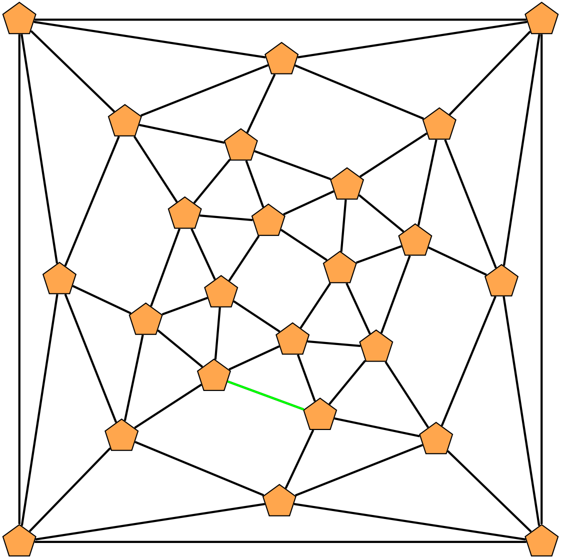

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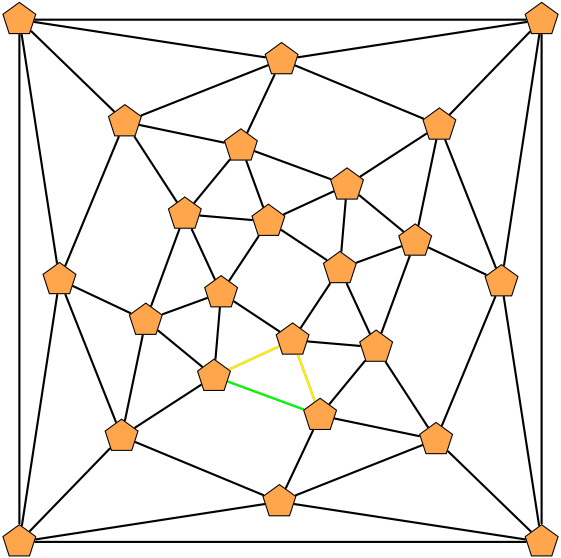
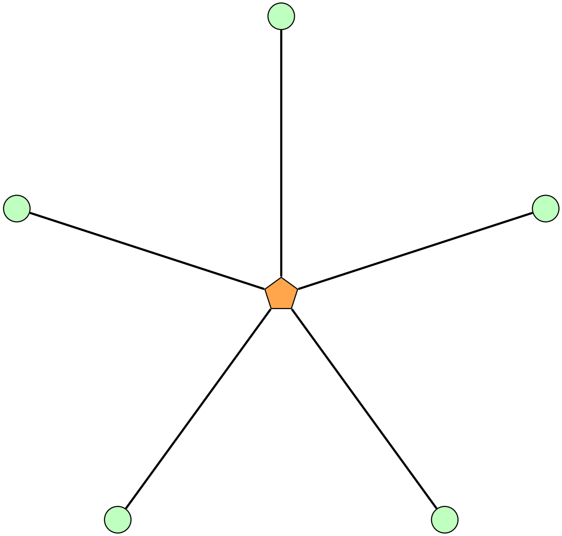


Vertex in no double edges but every edge in a triangle



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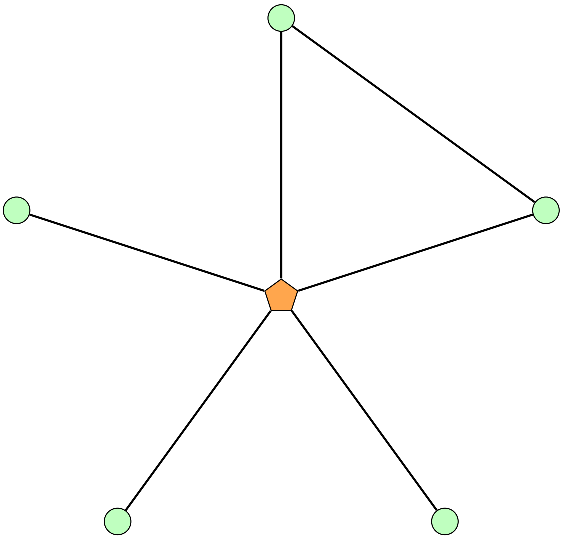

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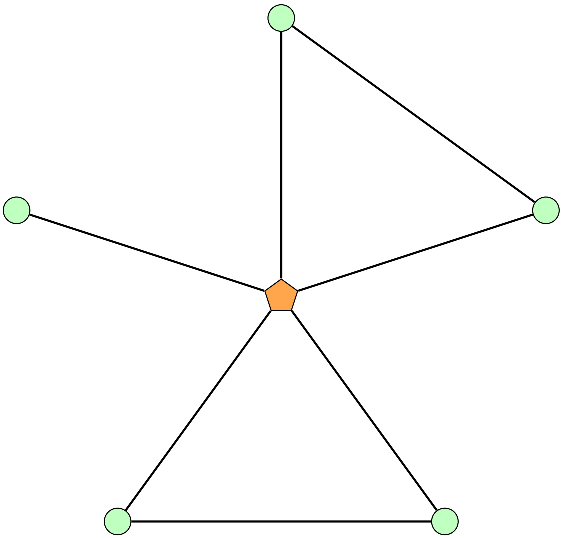

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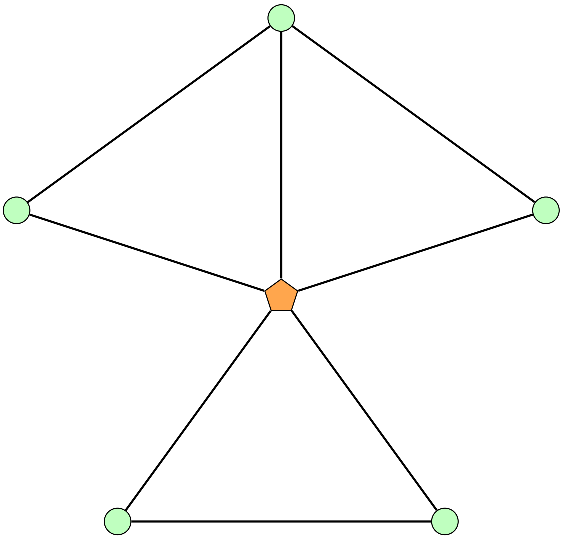
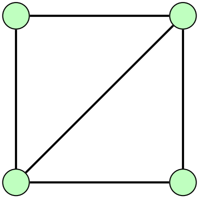


Z-box reduction



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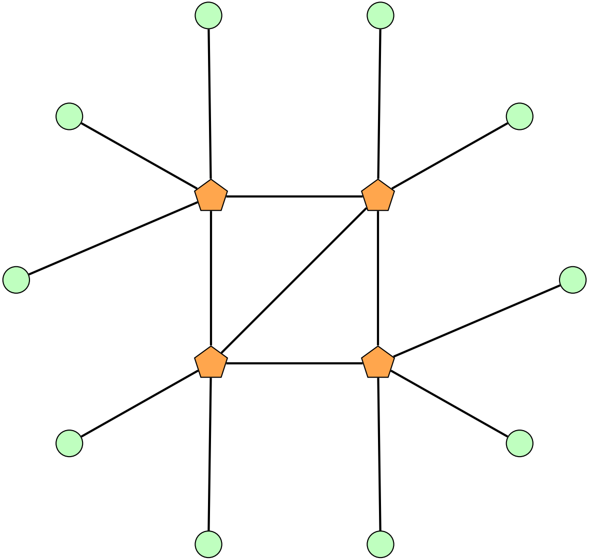

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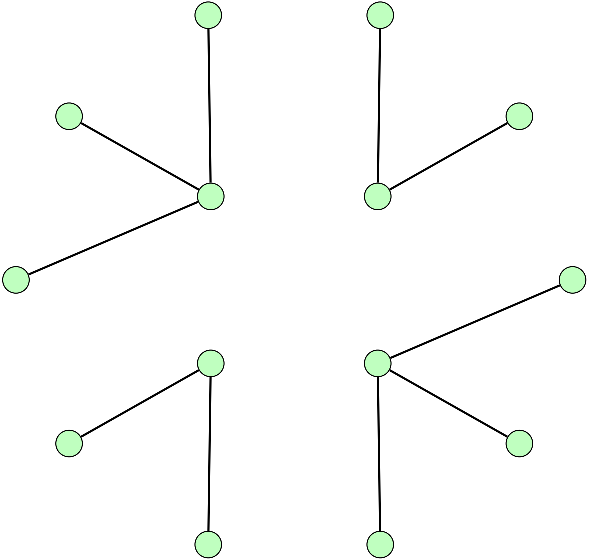

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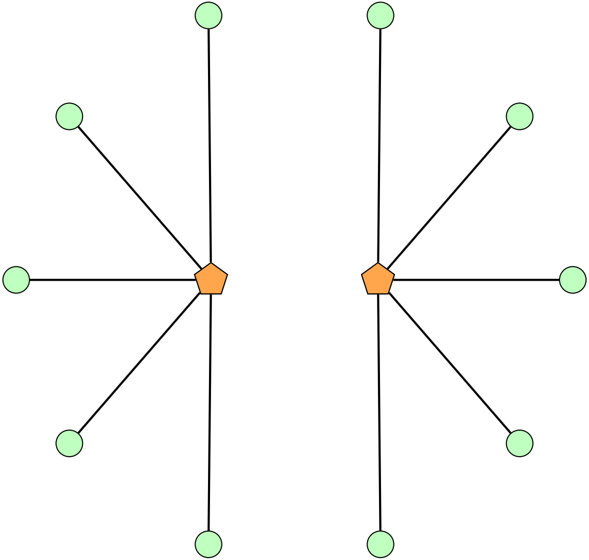

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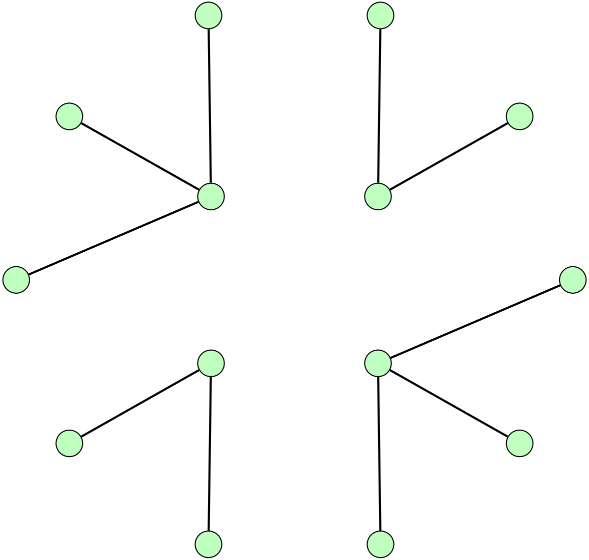

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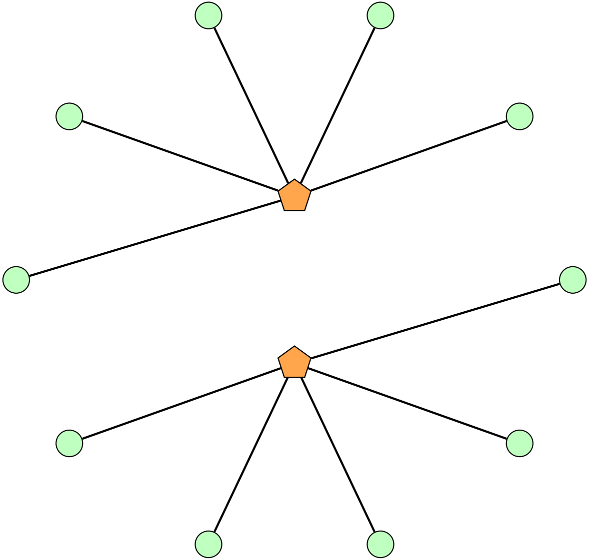
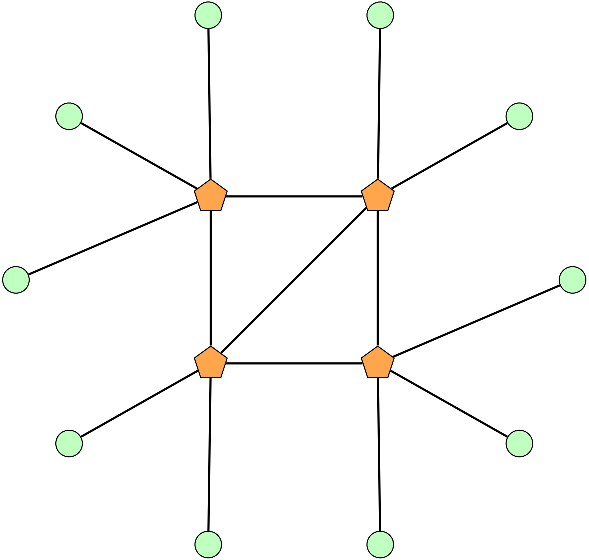


How can Z-box reduction fail?



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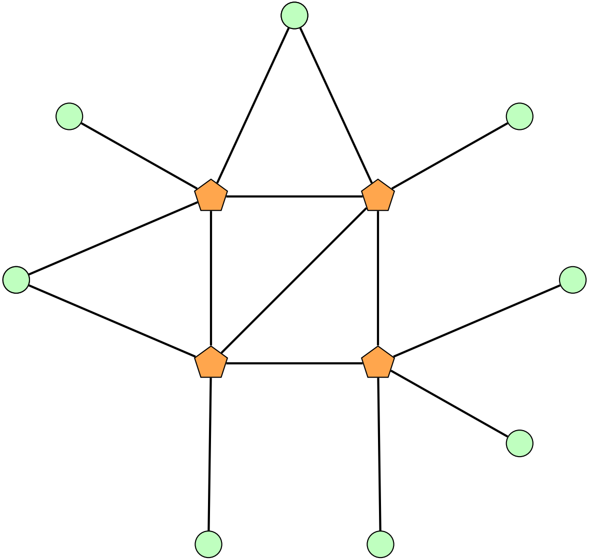

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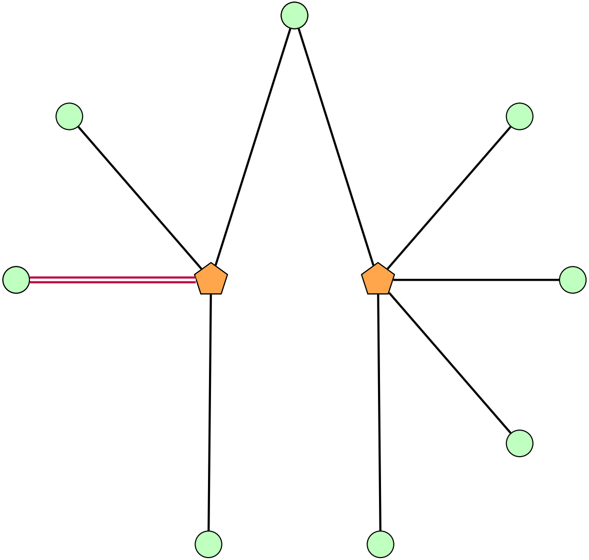
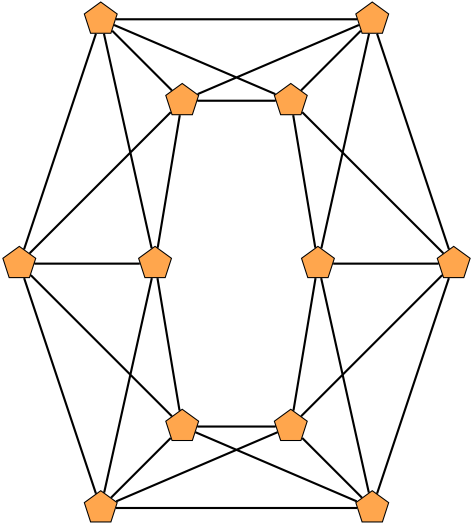


Some graphs have no Z-box



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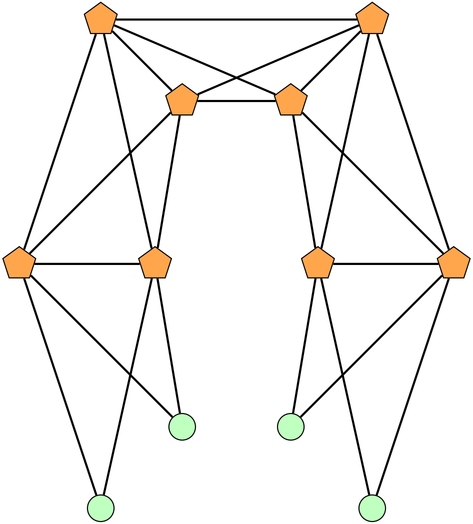

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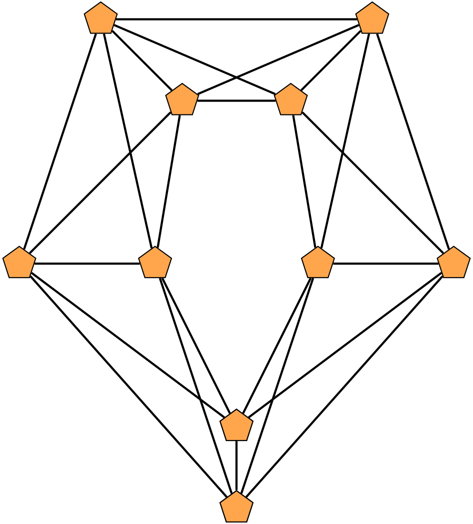

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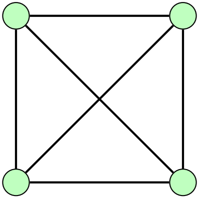

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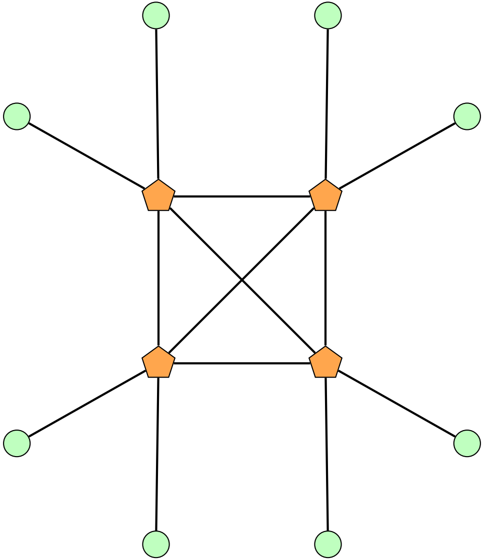

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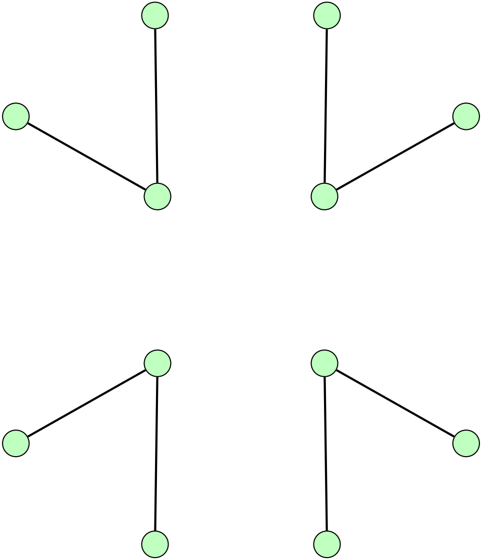

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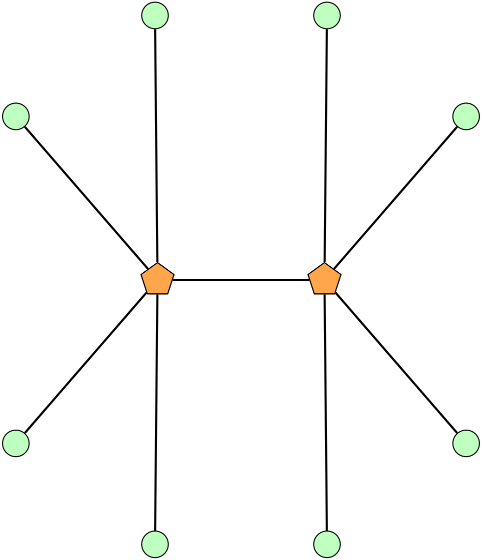

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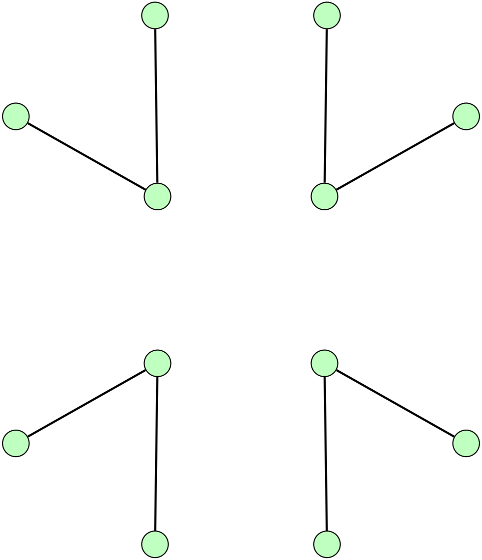

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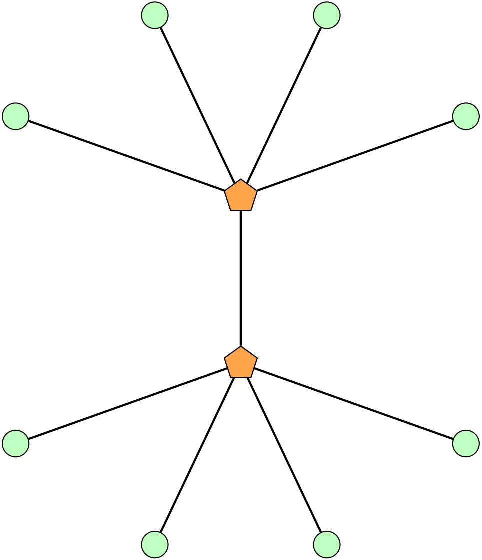

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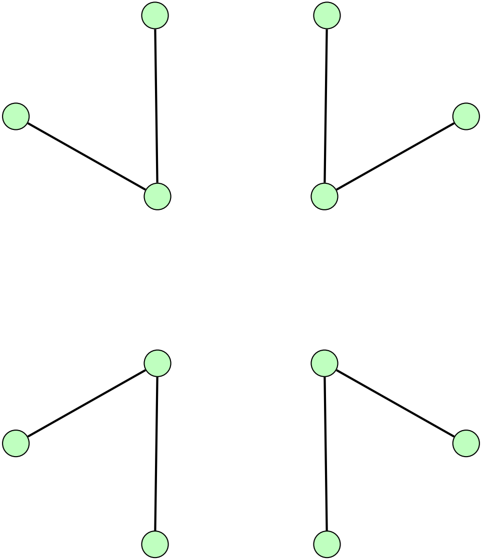

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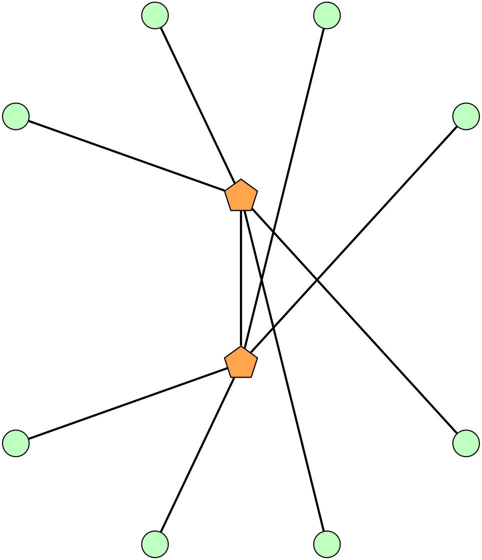

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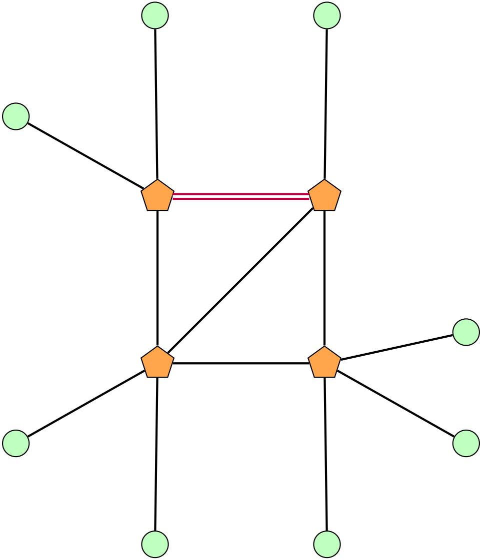

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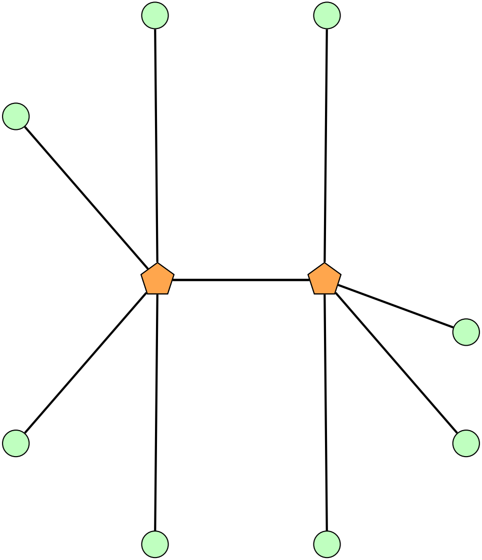
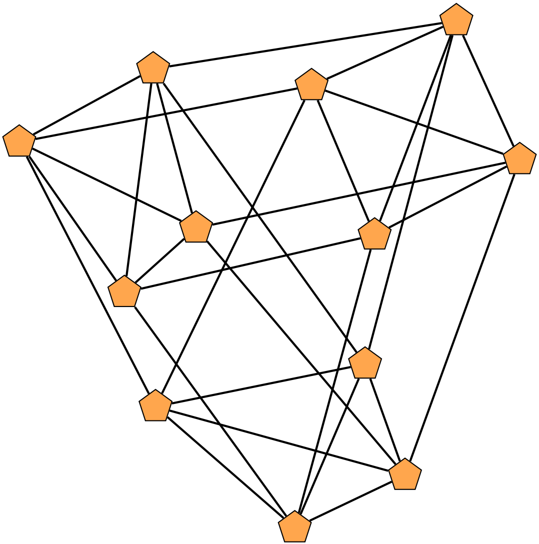


Graphs which are Irreducible using either Z-box or X-box



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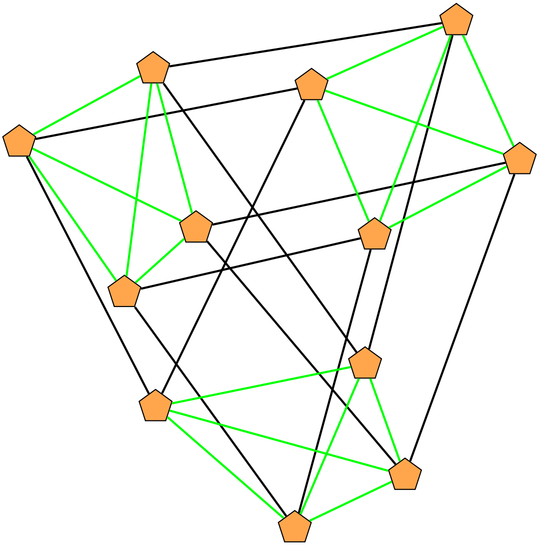

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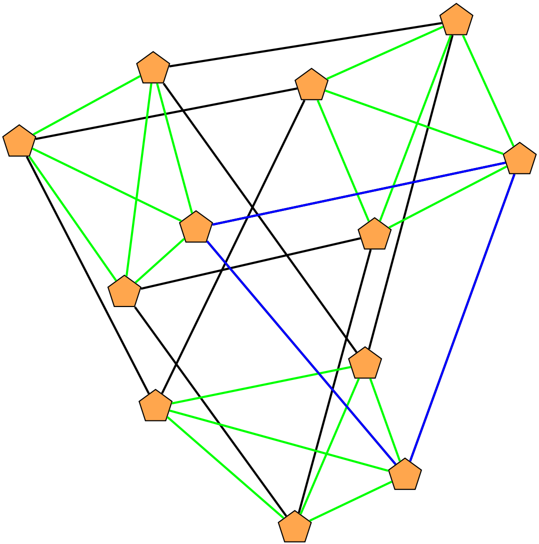

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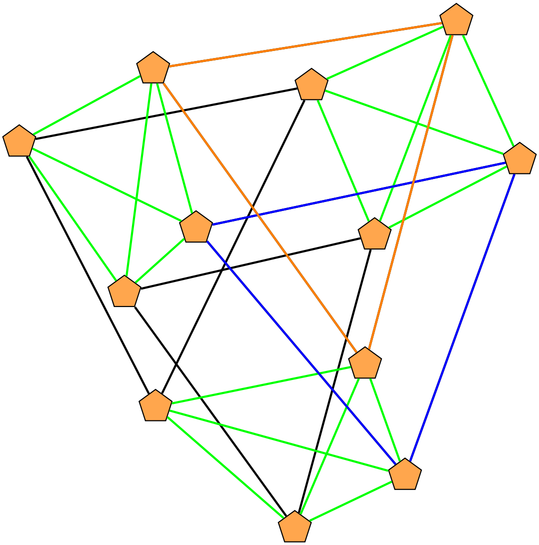

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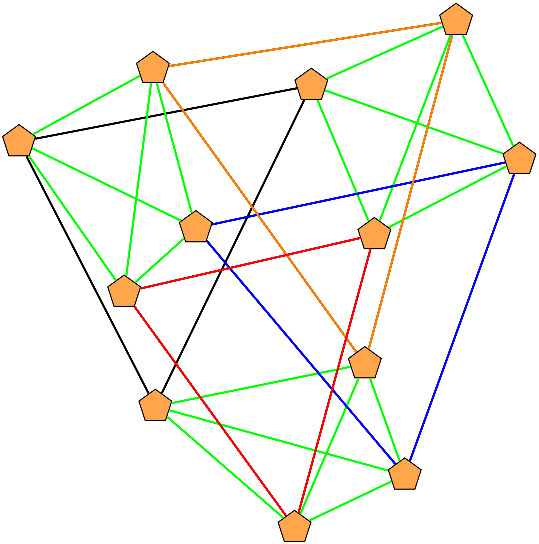

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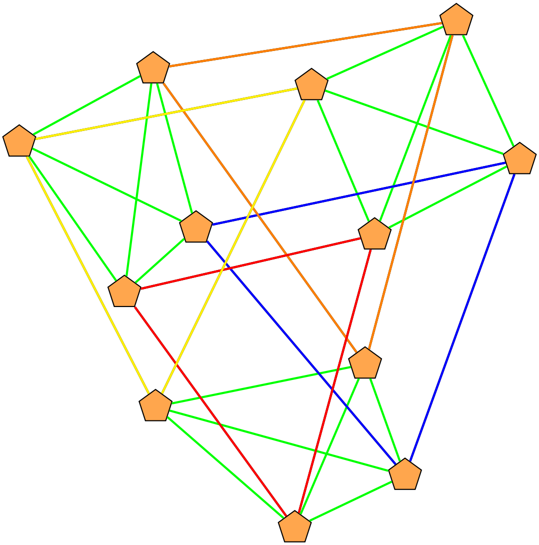
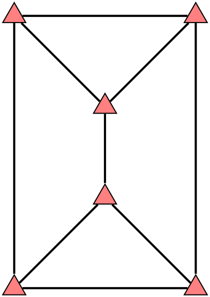


Generalising Line Graphs using Biregular Bipartite Graphs



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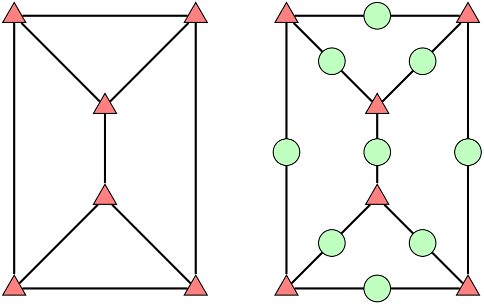

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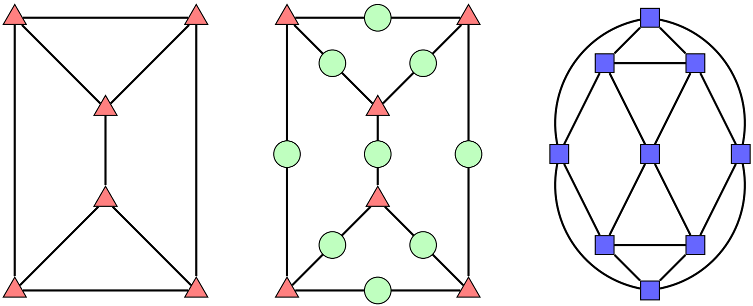

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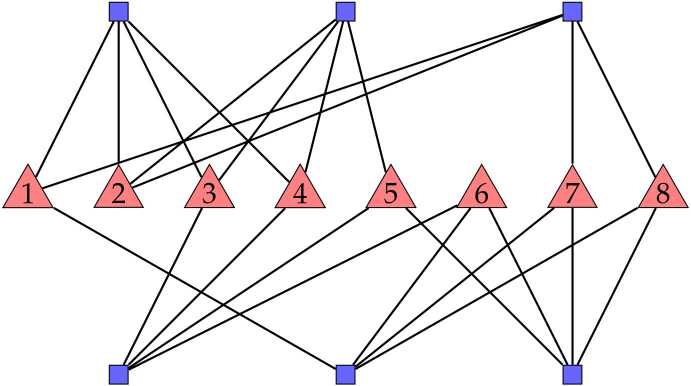

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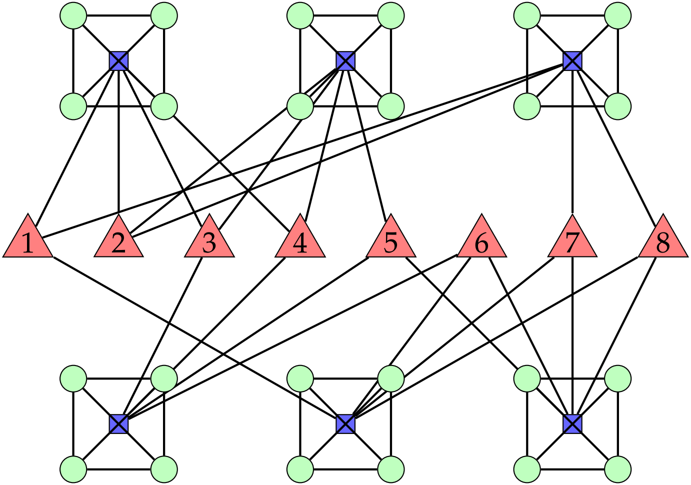

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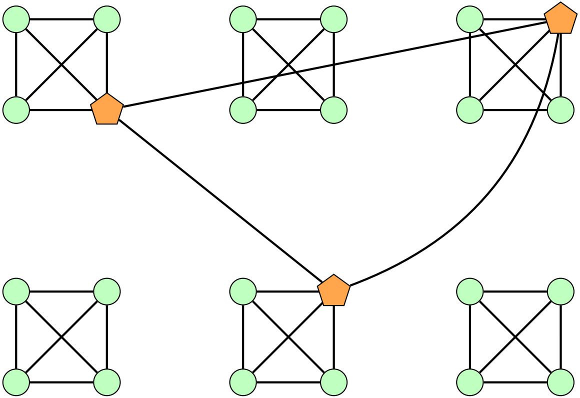

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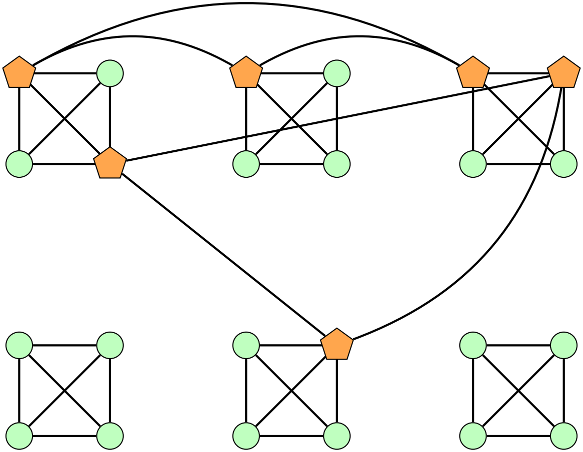

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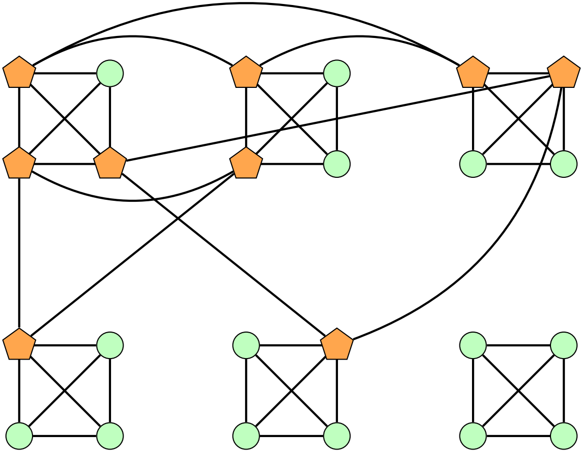

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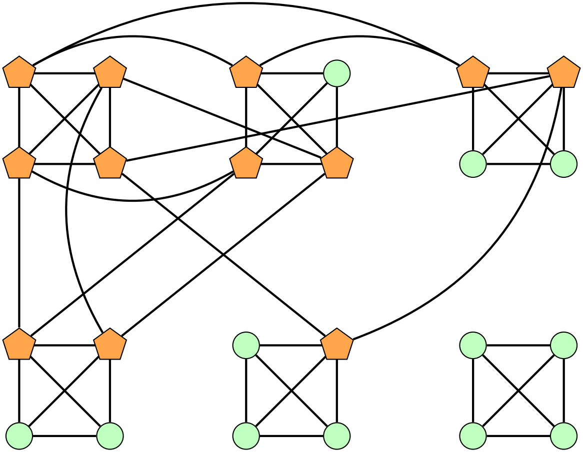

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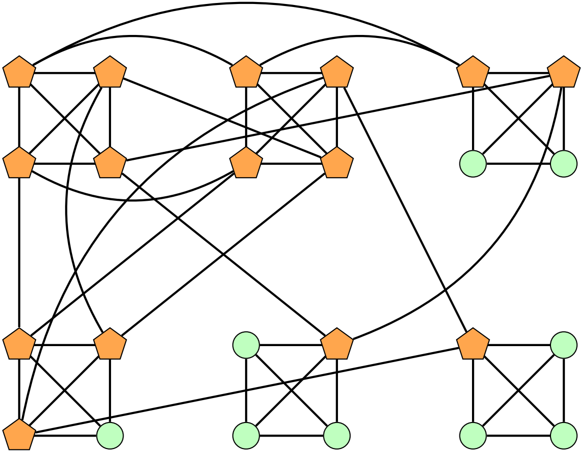

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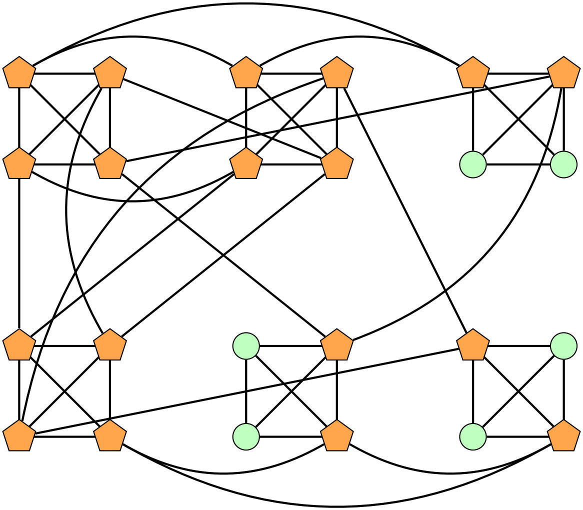

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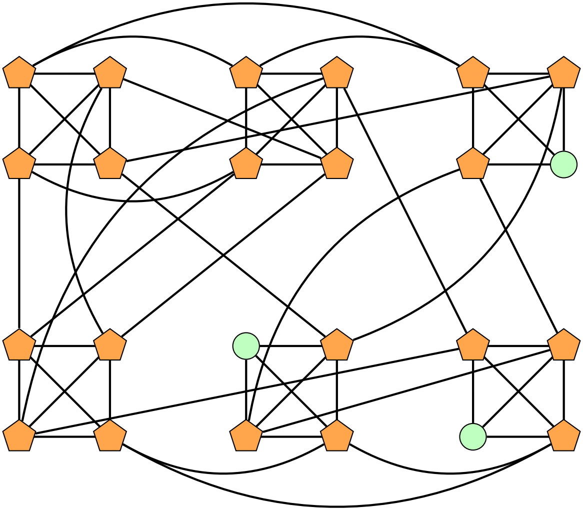

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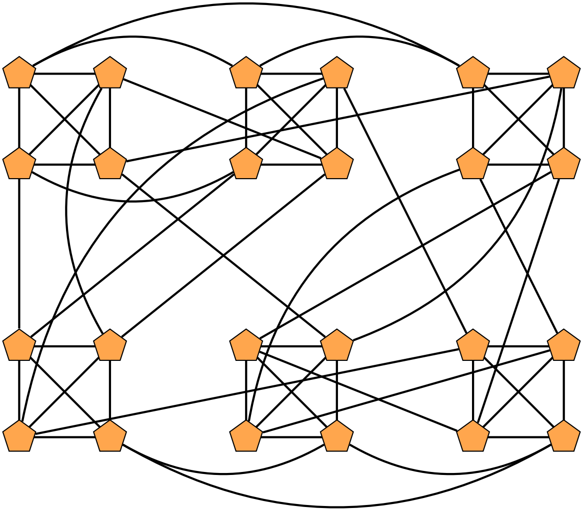
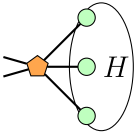


Cut vertex reductions



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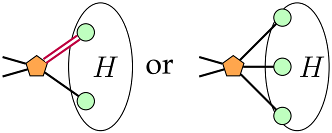

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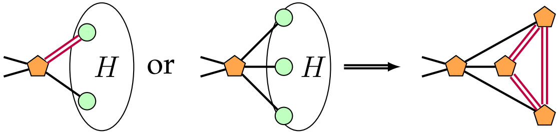

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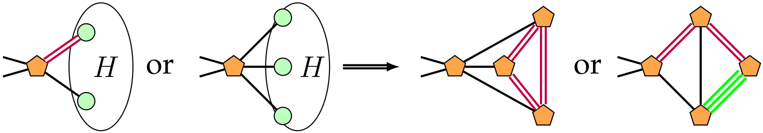

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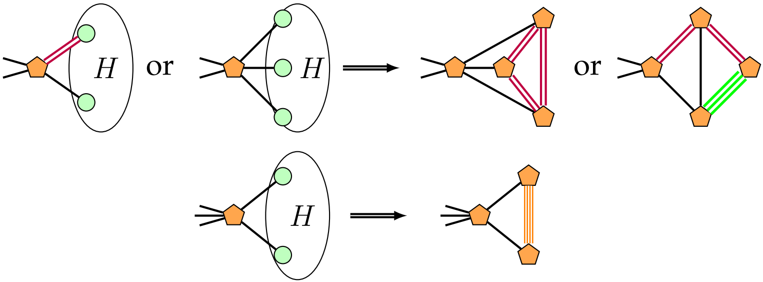
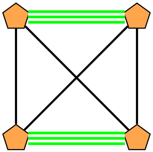


Smallest 5-regular graphs with the triangle property



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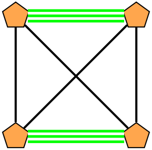

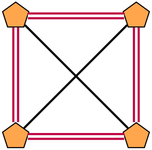

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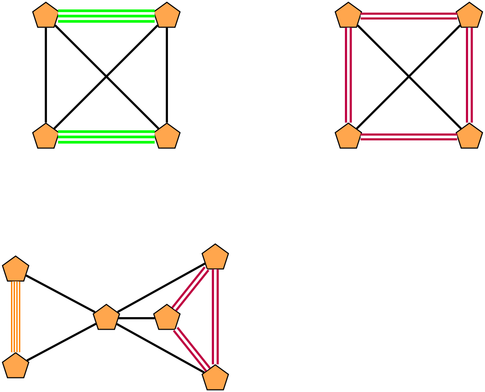

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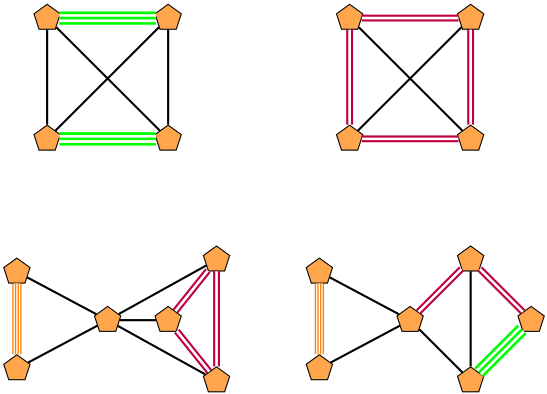
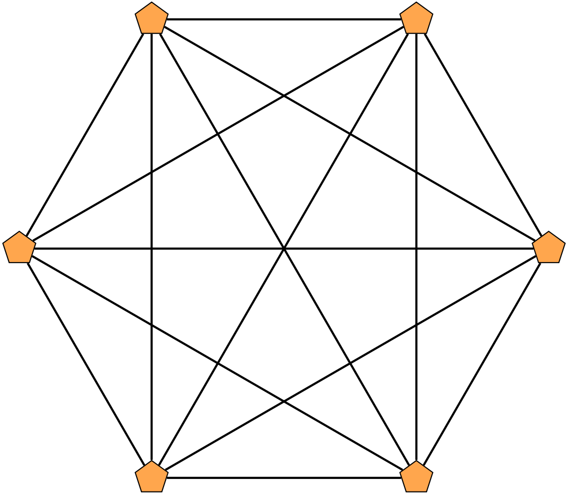


Complete graph on 6 vertices



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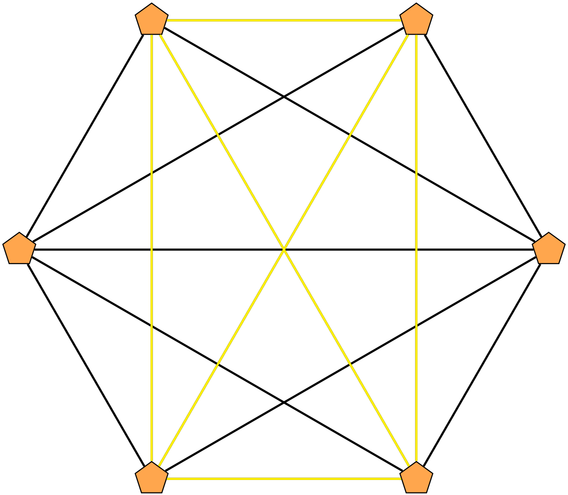

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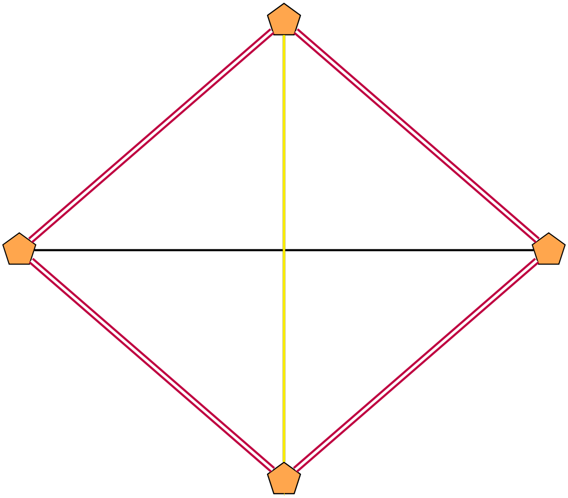
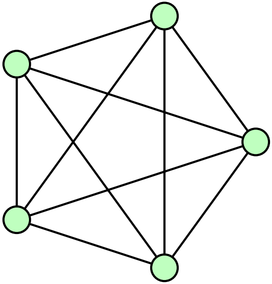


Clique Number 5



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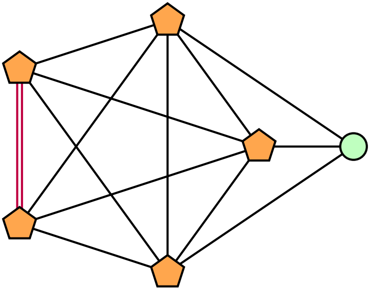

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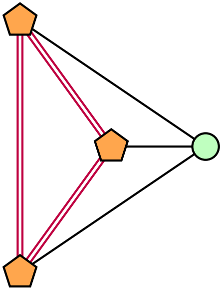

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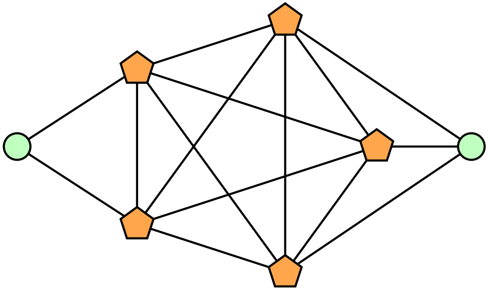

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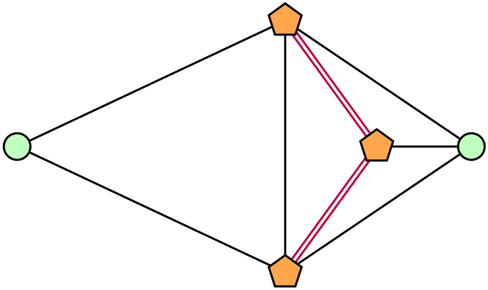
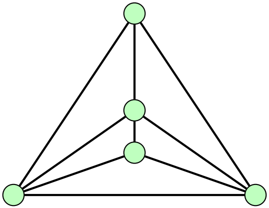


Clique Number 4: If K_5 minus an edge uv is a subgraph:



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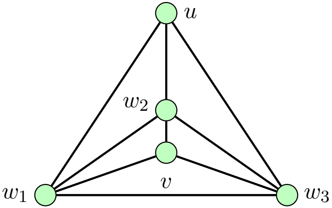

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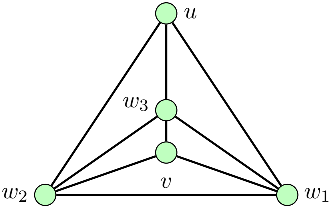

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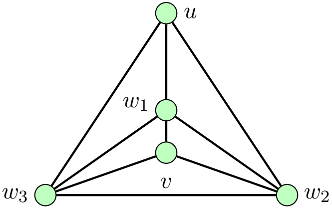

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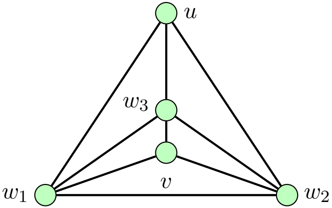

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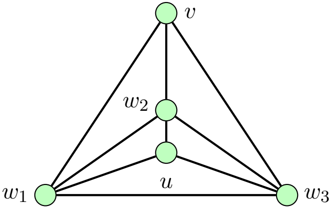

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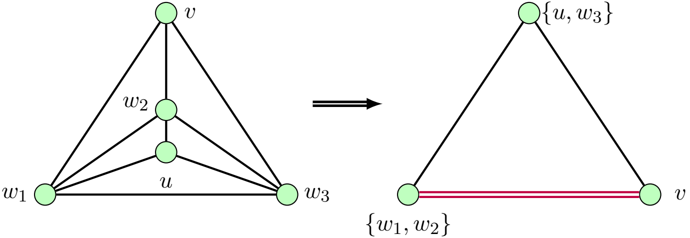

Clique Number 4: Counting the vertices adjacent to edges:

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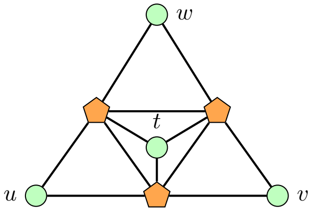

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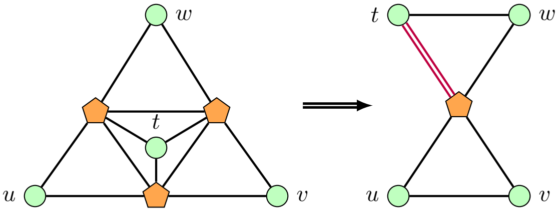
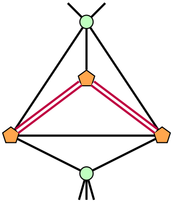


Clique Number 4: Problems when $1 \leq s_{K_4} \leq 2$



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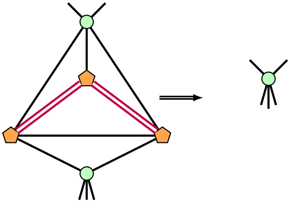

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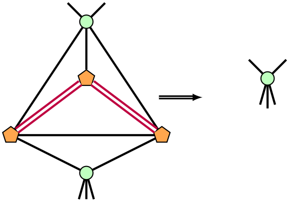

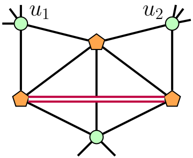

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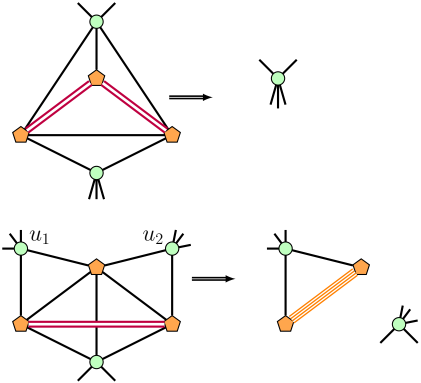
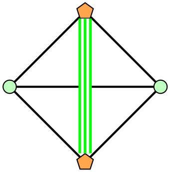


Clique Number 4: Problems when $s_H = 0$



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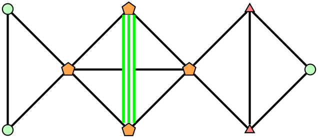

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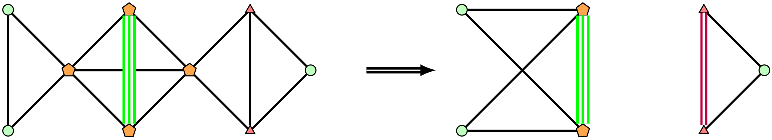

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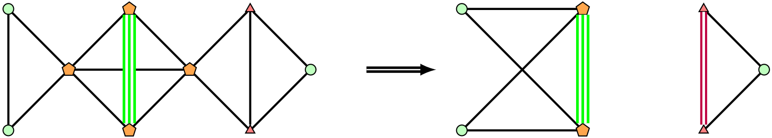

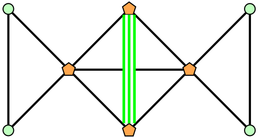

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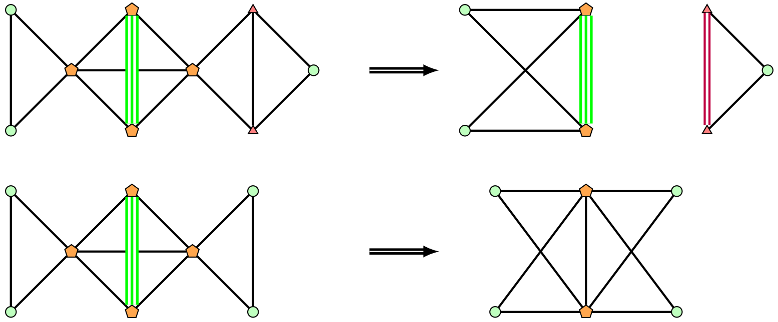
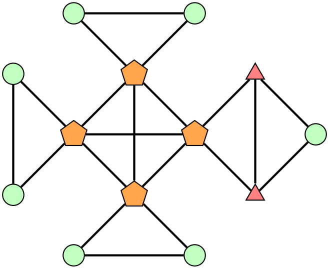


Clique Number 4: When K_4 has three aloof triangles:



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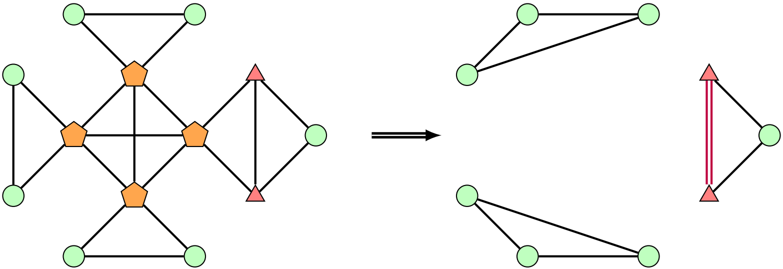
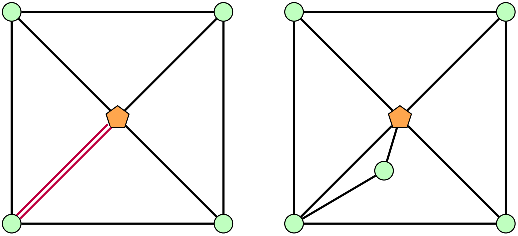


Clique number 3: the wheel with 5 vertices



Many triangles incident with an edge

How many vertices adjacent to both ends of an edge?

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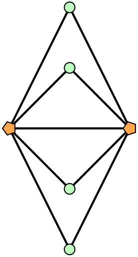

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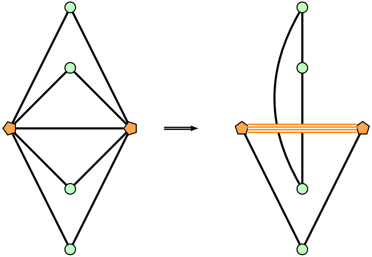

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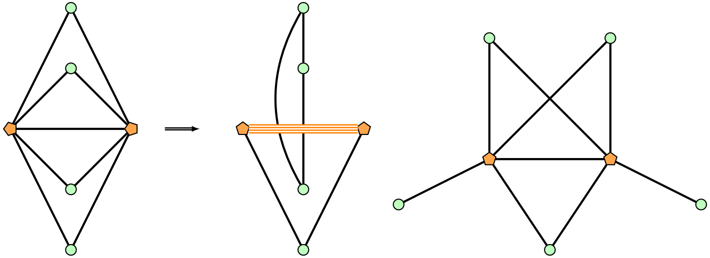
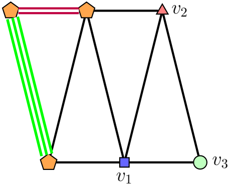


Triple edges part 1



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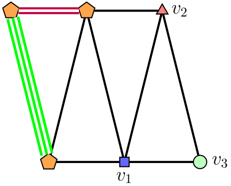

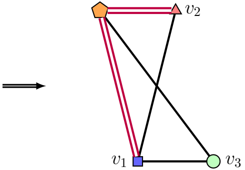

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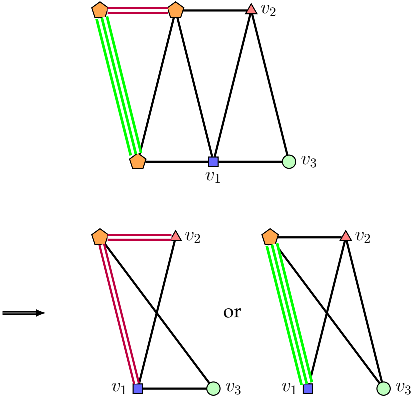

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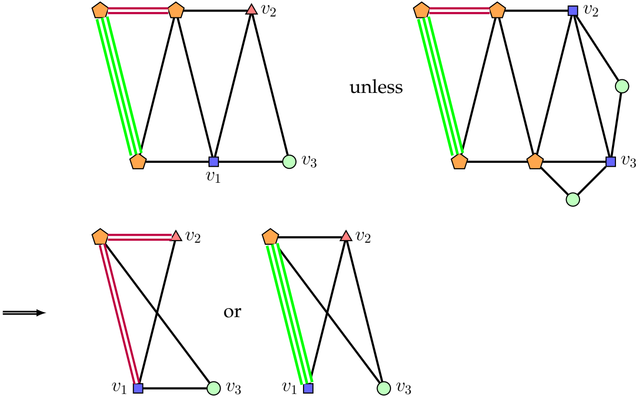

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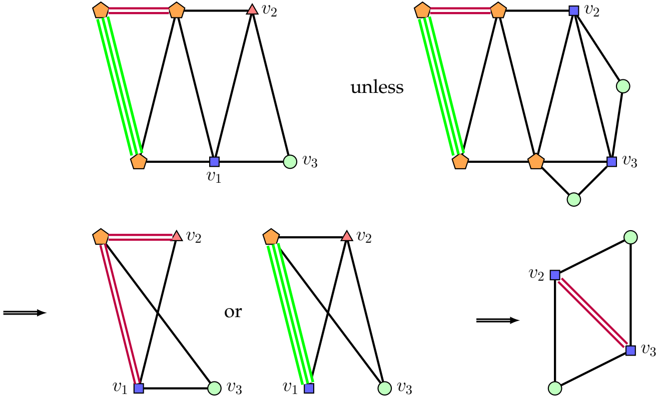
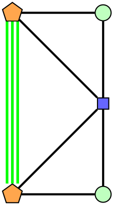


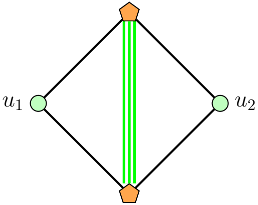
Triple edges part 2

Similarly, we can deal with



Triple edges part 2

However,



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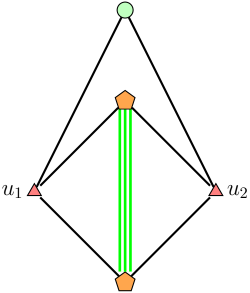

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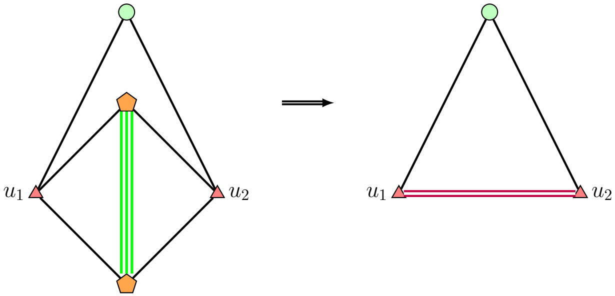

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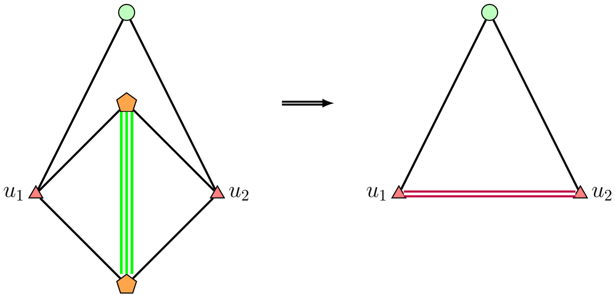
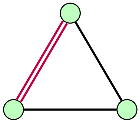


but if there are no common neighbours of u_1 and u_2, we cannot reduce.

Aloof triangles with a double edge



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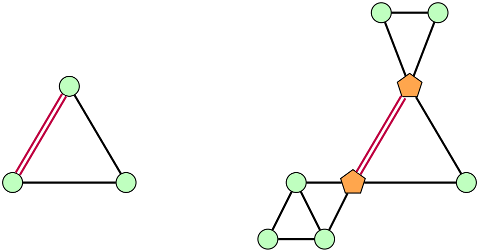

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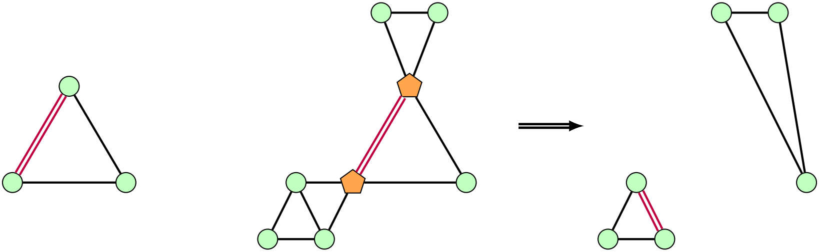
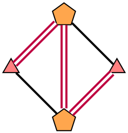


Double edges in non-aloof triangles



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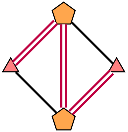

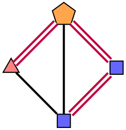

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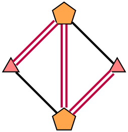
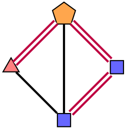
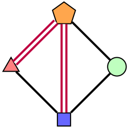
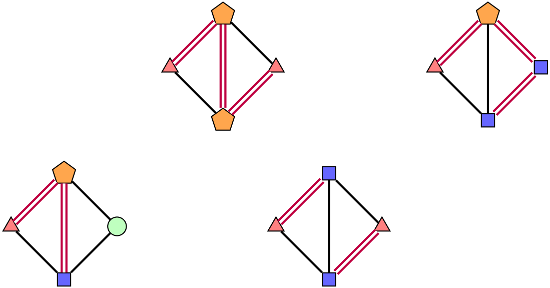


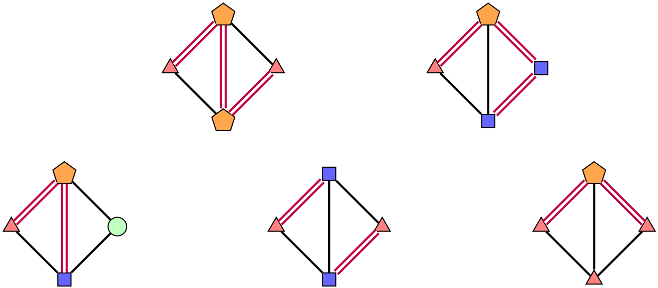




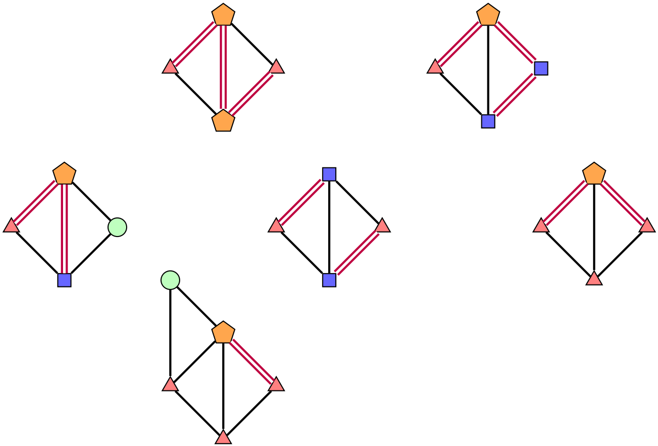
Double edges in non-aloof triangles



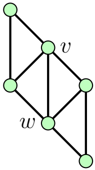
Double edges in non-aloof triangles



Double edges in non-aloof triangles



No double edges and two triangles adjacent to a Z-box



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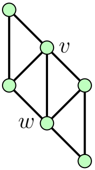
No double edges and two triangles adjacent to a Z-box



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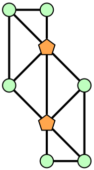
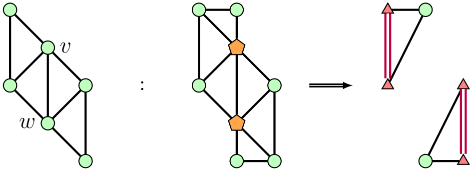
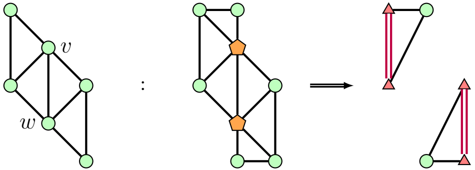
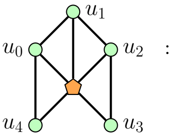


No double edges and two triangles adjacent to a Z-box

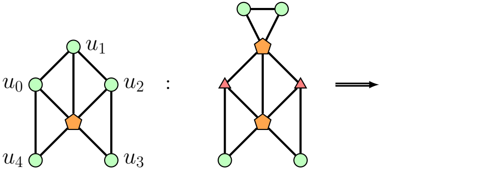


No double edges and two triangles adjacent to a Z-box

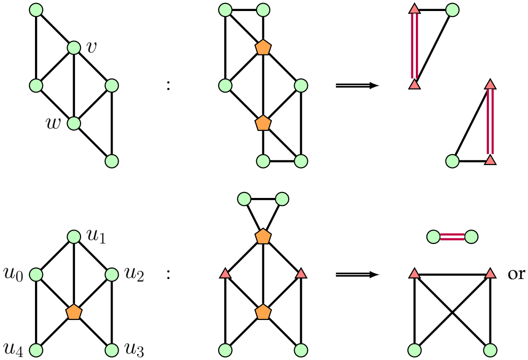




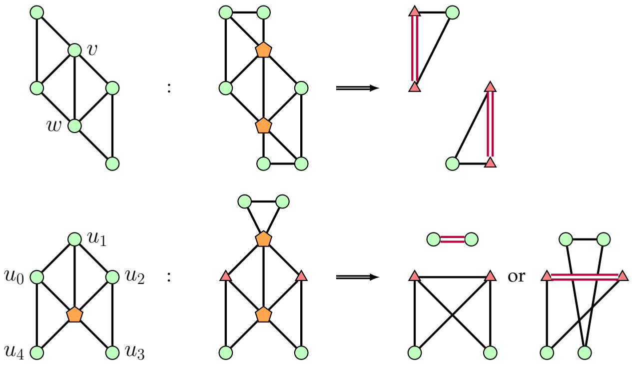
No double edges and two triangles adjacent to a Z-box



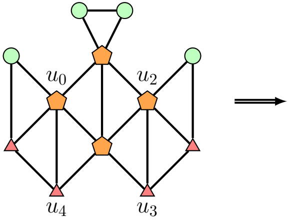
No double edges and two triangles adjacent to a Z-box



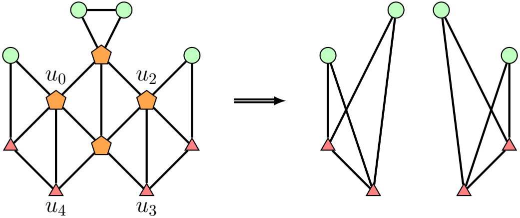
No double edges and two triangles adjacent to a Z-box



Pentagonal wheel reduction

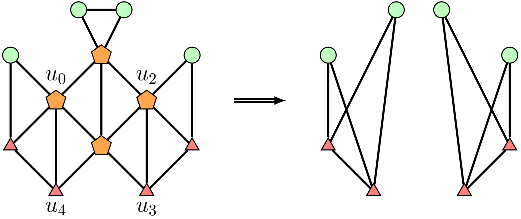


Pentagonal wheel reduction

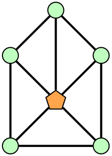


unless

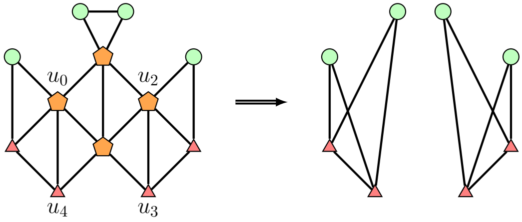
Pentagonal wheel reduction



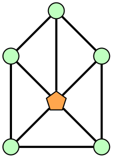
unless

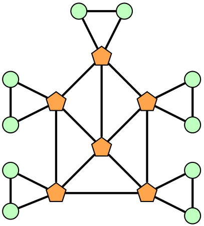


Pentagonal wheel reduction

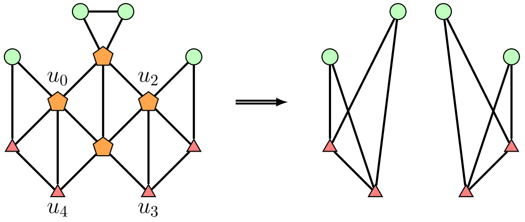


unless

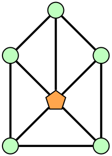


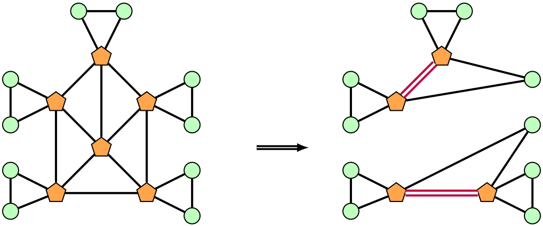


Pentagonal wheel reduction

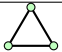


unless

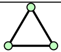
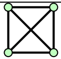




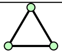
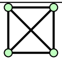
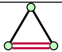
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3

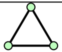
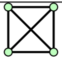
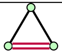
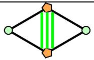
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0

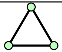
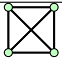
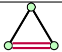
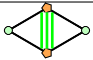
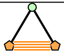
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0
A_3		0	2	1

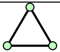
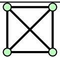
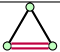
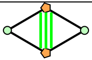
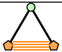
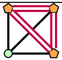
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0
A_3		0	2	1
A_7		2	0	2

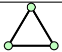
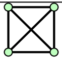
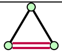
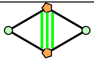
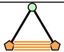
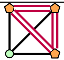
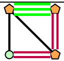
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0
A_3		0	2	1
A_7		2	0	2
A_4		2	0	1

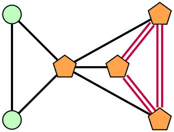
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0
A_3		0	2	1
A_7		2	0	2
A_4		2	0	1
A_{10}		3	1	0

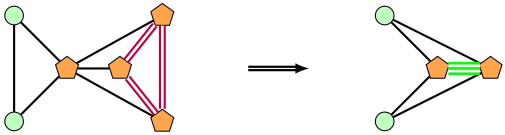
Atoms which remain

Atom	Configuration	Degree 5	Degree 3	Degree 2
A_1		0	0	3
A_2		0	4	0
A_3		0	2	1
A_7		2	0	2
A_4		2	0	1
A_{10}		3	1	0
A_{11}		3	1	0

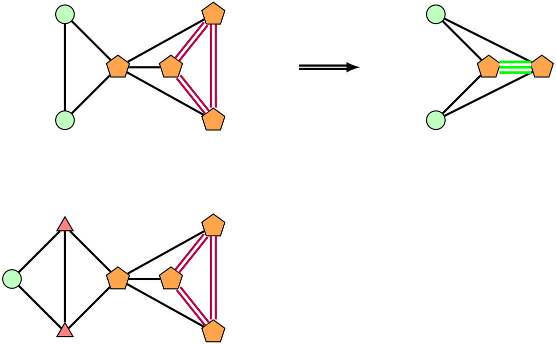
Reductions for A_{10} (or A_{11} analagously)



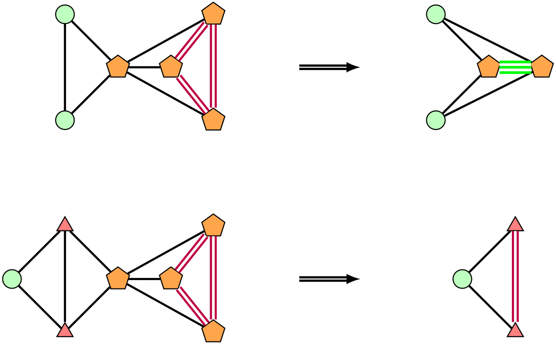
Reductions for A_{10} (or A_{11} analagously)



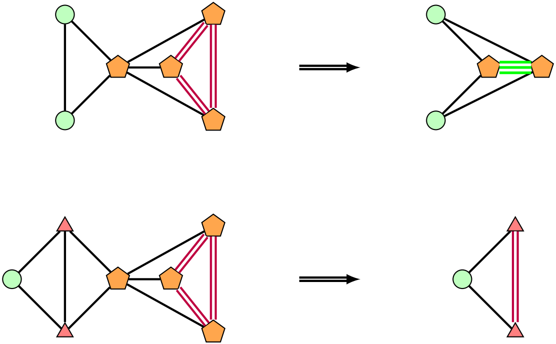
Reductions for A_{10} (or A_{11} analagously)



Reductions for A_{10} (or A_{11} analagously)

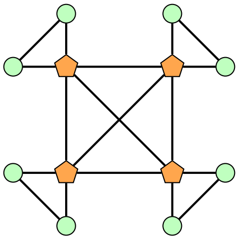


Reductions for A_{10} (or A_{11} analagously)

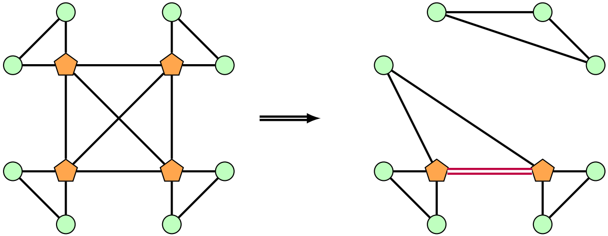


Reductions for A_4 are more numerous, but similar.

Reduction for K_4 with aloof triangles

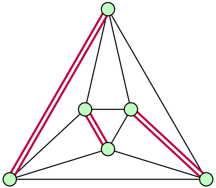


Reduction for K_4 with aloof triangles



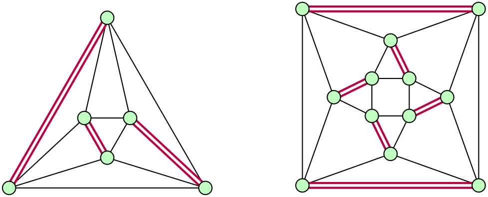
Small Foundational Graphs

All foundational connected quintic graphs with the triangle property and at least eight vertices are constructed from a line graph of a cubic graph H, with a perfect matching M, by adding a second edge to H for every edge in M.

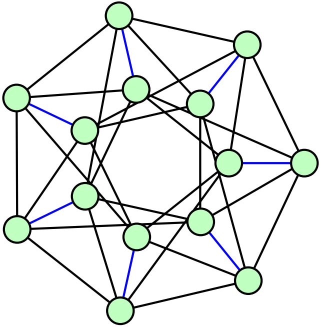


Small Foundational Graphs

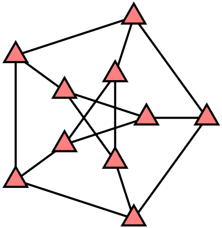
All foundational connected quintic graphs with the triangle property and at least eight vertices are constructed from a line graph of a cubic graph H, with a perfect matching M, by adding a second edge to H for every edge in M.



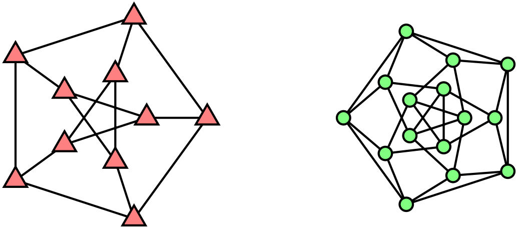
A simple graph with the property but fewest possible triangles



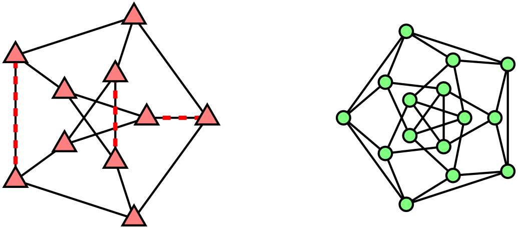
Edges in Petersen at distance 3 giving a minimal 6 regular graph



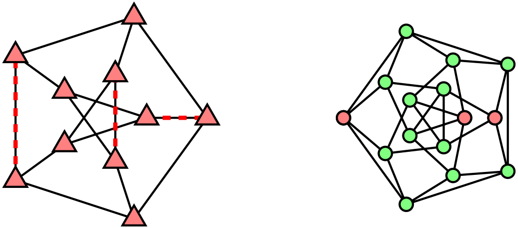
Edges in Petersen at distance 3 giving a minimal 6 regular graph



Edges in Petersen at distance 3 giving a minimal 6 regular graph



Edges in Petersen at distance 3 giving a minimal 6 regular graph



Edges in Petersen at distance 3 giving a minimal 6 regular graph