The Cheating Robot and Insider Information

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(Joint work with Richard J. Nowakowski)

December 2, 2020
Atlantic Graph Theory Seminar

Changing how we move...

Combinatorial Game Theory (CGT)

- Two players
- Perfect Information
- No Chance Devices
- Alternating Play

Simultaneous CGT

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The two players are typically called Left and Right.

Cheating Robot

- Simultaneous play, but the robot player can react to the moves of their opponent.

- Philosophy: Players know when they have moves available to them.

Robots making headlines in the news...

Superfast rock-paper-scissors robot 'wins' every time

By Matthew Wall
Technology reporter, BBC News

4 November 201

TECHNOLOGY 11/04/2013 10:00 EST | **Updated** 11/04/2013 11:11 EST

How This Robot Wins Rock-Paper-Scissors Every Single Time (It Cheats)

Betsy Isaacson
The Huffington Post

This obnoxiously talented robot will beat you in rock-paper-scissors every single time

Danielle Muoio, Tech Insider Sep. 22 2015, 3:33 PM

Rock Paper Scissors robot wins 100% of the time

By Graham Templeton (https://www.extremetech.com/author/gtempleton) on September 18, 2015 at 12:00 pm

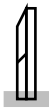
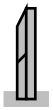
Photocredit: BBC news

Something to think about...

Consider playing cheating robot TIC-TAC-TOE on a large grid.

The robot can always tie the game, but can they ever win?

Ruleset for CR-TOPPLING DOMINOES

- Board: A row of black , white  and gray  dominoes.

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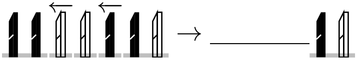
  -  (same direction)

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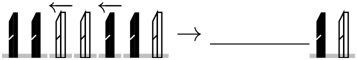
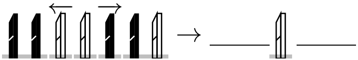
  -  (same direction)
  -  (different direction; away)

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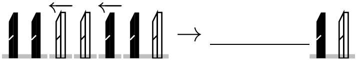
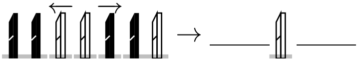
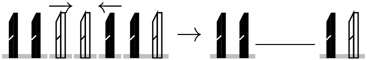
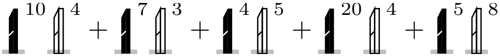
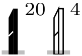
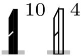
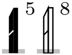
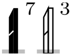
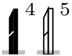
  -  (same direction)
  -  (different direction; away)
  -  (different direction; toward)

Case Study: CR-TOPPLING DOMINOES

$G =$ 

Case Study: CR-TOPPLING DOMINOES

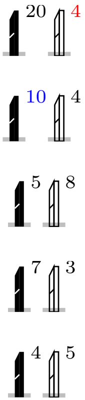
Ordering the components by sum, we obtain the following:

Position	Baseline Strategy
	16
	6
	-3
	4
	-1

Case Study: CR-TOPPLING DOMINOES

Left must play in order, otherwise Right can do much better:

Position



Baseline Strategy: 22

Alternate Strategy: 6

Assuming Left is playing in order, other options for Right:

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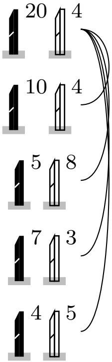

16

12

17

15 same as baseline; omit

Case Study: CR-TOPPLING DOMINOES

Repeat for all others below to obtain...

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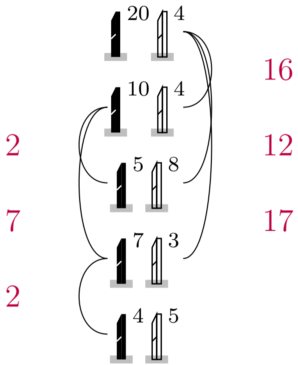
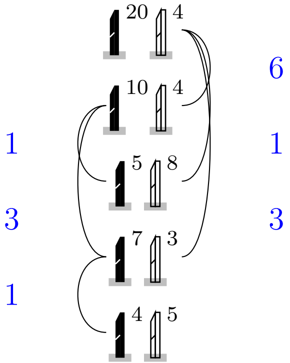


Case Study: CR-TOPPLING DOMINOES

Now, we examine this as an optimization problem for Right using gains for him...



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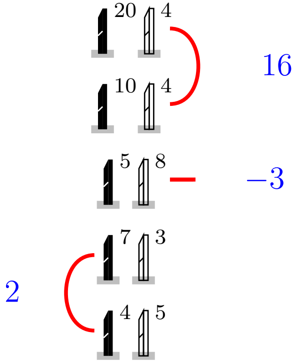

Baseline strategy: 22

Optimization strategy: 15

Right prefers the result of the optimization strategy!

The Cheating Robot: Outcomes

- $o_{CR}(G) = \mathcal{L}$ if Left can force a win.
- $o_{CR}(G) = \mathcal{R}$ if Right can force a win.
- $o_{CR}(G) = \mathcal{D}$ if both players can prevent the opponent from winning.

Outcomes are ordered: $\mathcal{L} > \mathcal{D} > \mathcal{R}$.

Notation

A game G is represented by its Left, Right, and simultaneous options. Formally,

$$G = \{G^{\mathcal{L}} \mid G^{\mathcal{S}} \mid G^{\mathcal{R}}\}$$

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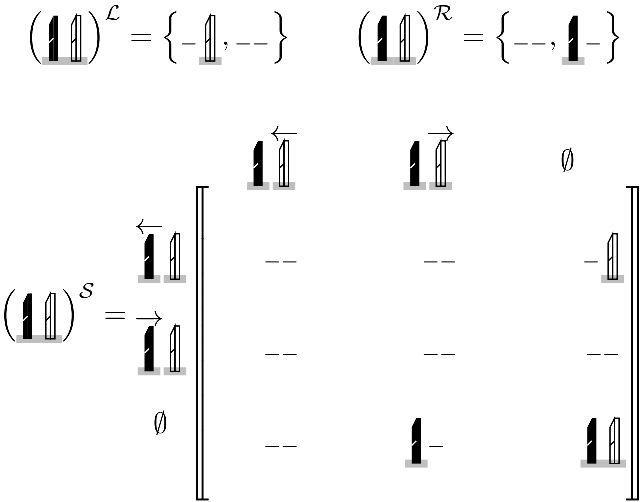
$$G = \{G^{\mathcal{L}} \mid G^{\mathcal{S}} \mid G^{\mathcal{R}}\}$$

Why do we need $G^{\mathcal{L}}$ and $G^{\mathcal{R}}$?
Consider a sum of two games $G + H$. Then

$G + H =$

$$\{G^{\mathcal{L}} + H, G + H^{\mathcal{L}} \mid G^{\mathcal{S}} + H, G + H^{\mathcal{S}}, G^{\mathcal{L}} + H^{\mathcal{R}}, G^{\mathcal{R}} + H^{\mathcal{L}} \mid G^{\mathcal{R}} + H, G + H^{\mathcal{R}}\}$$

Example



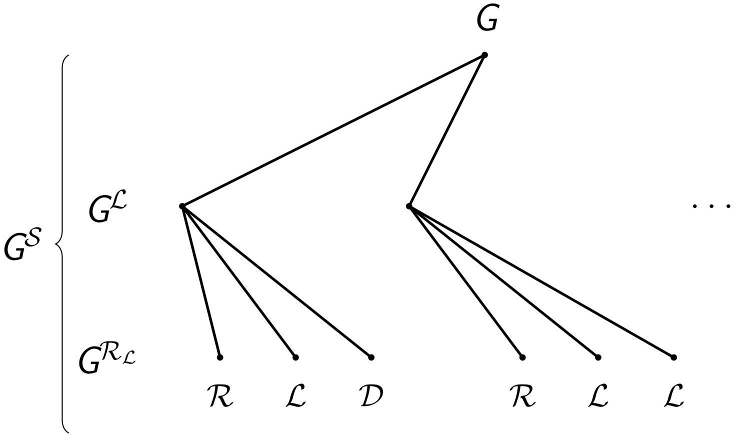
The Cheating Robot: Game Trees

Definition

- If $G = \{\cdot \mid \cdot \mid \cdot\}$, then $o_{CR}(G) = \mathcal{D}$.
- If $G = \left\{G^{\mathcal{L}} \neq \emptyset \mid \cdot \mid \cdot\right\}$, then $o_{CR}(G) = \mathcal{L}$.
- If $G = \left\{\cdot \mid \cdot \mid G^{\mathcal{R}} \neq \emptyset\right\}$, then $o_{CR}(G) = \mathcal{R}$.
- If $G^{\mathcal{S}} \neq \emptyset$ ($\implies G^{\mathcal{L}} \neq \emptyset \neq G^{\mathcal{R}}$)...

The Cheating Robot: Game Trees

- If $\forall G^{\mathcal{L}}$, $\exists G^{\mathcal{R}}$ such that $o_{CR}(G^{\mathcal{R}_{\mathcal{L}}}) = \mathcal{R}$, then $o_{CR}(G) = \mathcal{R}$.



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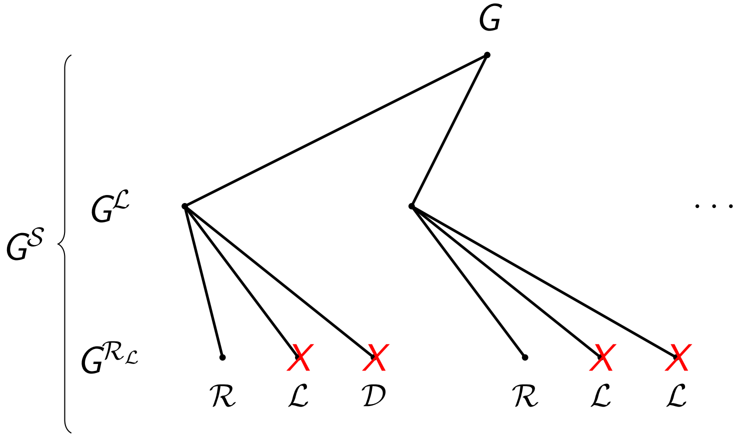

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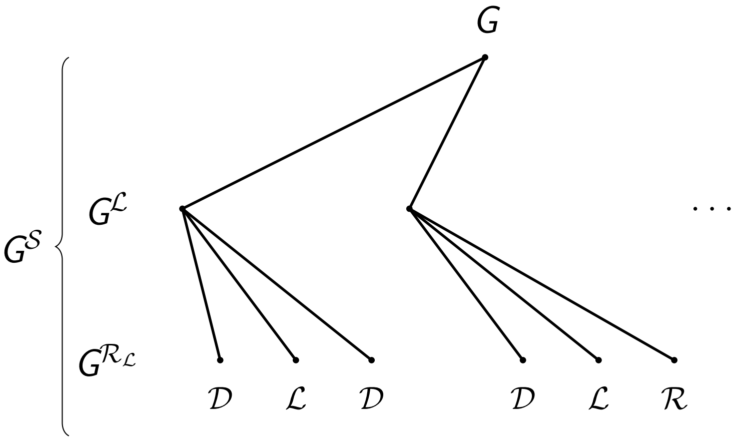
- If $\exists G^{\mathcal{L}}, \forall G^{\mathcal{R}}$ such that $o_{CR}(G^{\mathcal{R}_{\mathcal{L}}}) \geq \mathcal{D}$ and $\exists o_{CR}(G^{\mathcal{R}_{\mathcal{L}}}) = \mathcal{D}$, then $o_{CR}(G) = \mathcal{D}$.



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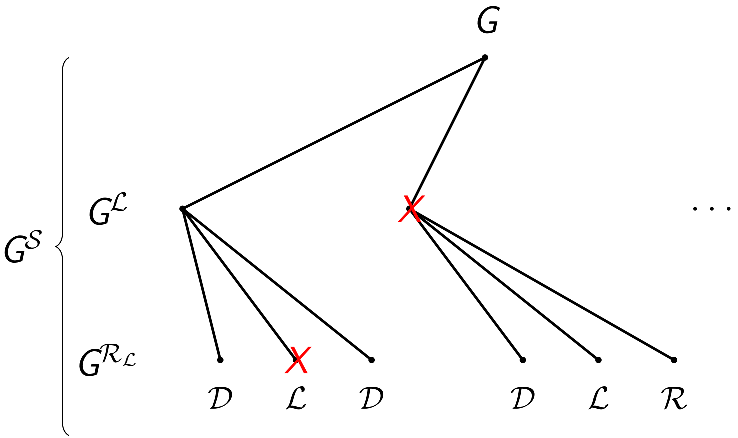
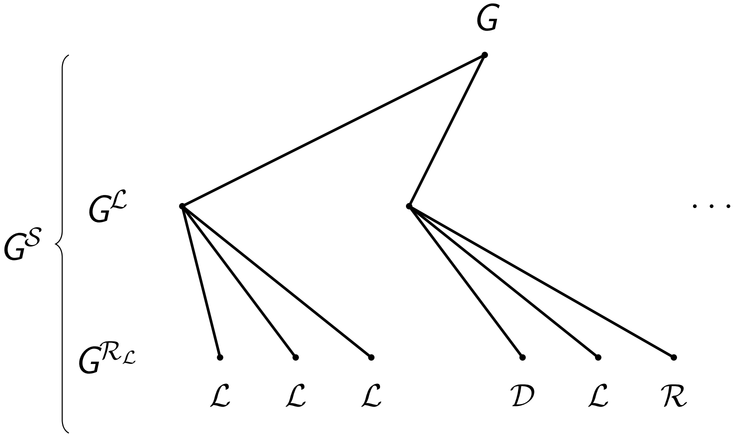


The Cheating Robot: Game Trees

- Otherwise $o_{CR}(G) = \mathcal{L}$.



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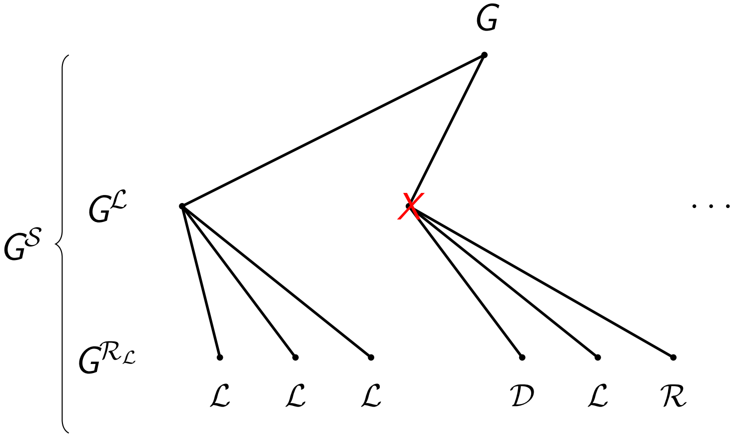
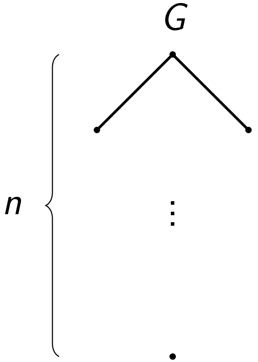


The Cheating Robot: Zero is unique

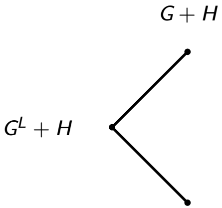
Theorem

If $G \equiv 0$ *then* $G = \{\cdot \mid \cdot \mid \cdot\}$.

sketch of proof



Let $H = \{-n-1 \mid 0 \mid -n-1\}$. Then



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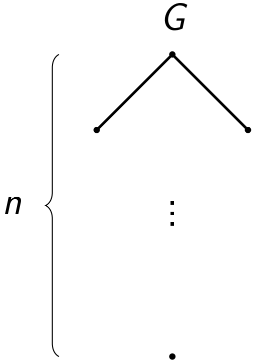

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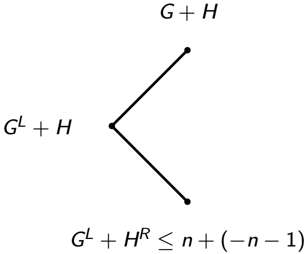

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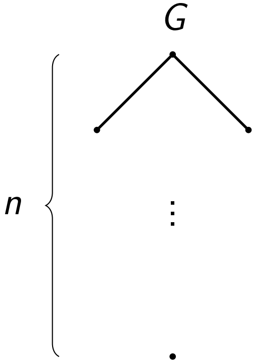

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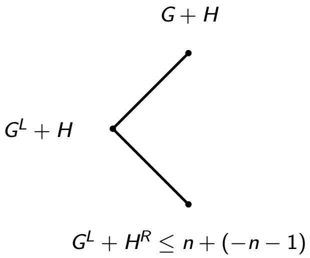

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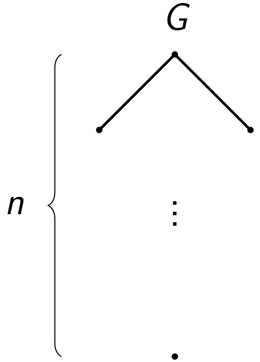

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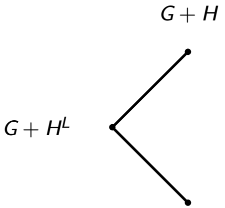

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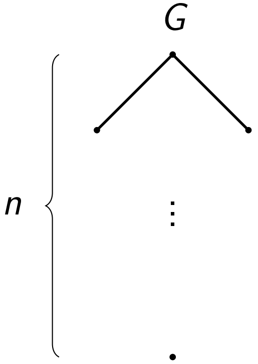

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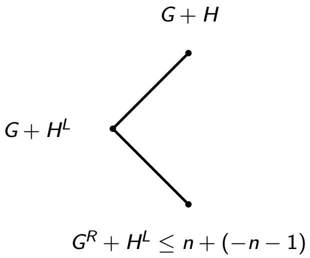

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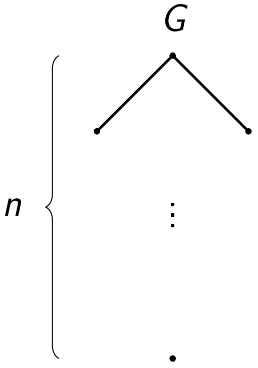
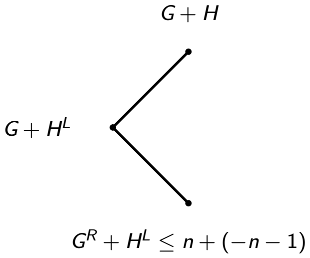




Hence, Right wins.

The Cheating Robot: $-G$

Conjugate of G: $-G = \left\{-G^{\mathcal{R}} \mid -G^{\mathcal{S}} \mid -G^{\mathcal{L}}\right\}$, but Right is still the Cheating Robot.

The Cheating Robot: $G - G < 0$.

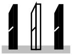
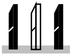
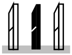
Theorem

For all $G \neq 0$, $G - G < 0$.

Proof.

Suppose $G \neq 0$ then $G - G$ has a Right option and a Left option. $o_{CR}(G - G + X) \leq o_{CR}(X)$ since if Left plays in $(G - G)$ Right mimics in the other component. So $G - G + X$ is no worse for Right than X. Thus $G - G \leq 0$. By the previous theorem $G - G \not\equiv 0$ so $G - G < 0$. □

Example: CR-TOPPLING DOMINOES

Let $G =$ . Consider $G - G =$  $+$ .

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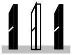
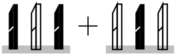
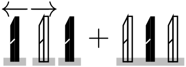
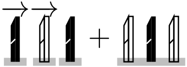
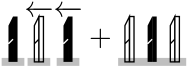
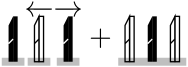
Let $G =$ . Consider $G - G =$ .

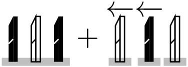
Best responses for every $G^{\mathcal{L}}$.

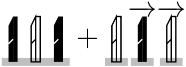
(i) 

(ii) 

(iii) 

(iv) 

(v) 

(vi) .

Bounding simultaneous CGT: A deterministic approach

- Right is the robot: Bound from below
- Left is the robot: Bound from above

Future work

- Simultaneous CGT
- CR-theory: We are currently working on the general theory; including canonical form and comparability.
- CR-TOPPLING DOMINOES: what about more complicated positions?
- Exploring other game universes.

Thank you!

Melissa Huggan
Postdoctoral Fellow
Melissa.Huggan@ryerson.ca