Chapter 3

Geographic Position and Time

Geographic terms

In celestial navigation, the earth is regarded as a sphere. Although this is an approximation, the geometry of the sphere is applied successfully, and the errors caused by the flattening of the earth are usually negligible (chapter 9). A circle on the surface of the earth whose plane passes through the center of the earth is called a **great circle**. Thus, a great circle has the greatest possible diameter of all circles on the surface of the earth. Any circle on the surface of the earth whose plane does **not** pass through the earth's center is called a **small circle**. The **equator** is the only great circle whose plane is perpendicular to the **polar axis**, the axis of rotation. Further, the equator is the only **parallel of latitude** being a great circle. Any other parallel of latitude is a small circle whose plane is parallel to the plane of the equator. A **meridian** is a great circle going through the **geographic poles**, the points where the polar axis intersects the earth's surface. The **upper branch** of a meridian is the half from pole to pole passing through a given point, e. g., the observer's position. The **lower branch** is the opposite half. The **Greenwich meridian**, the meridian passing through the center of the transit instrument at the **Royal Greenwich Observatory**, was adopted as the **prime meridian** at the International Meridian Conference in 1884. Its upper branch is the reference for measuring longitudes (0°...+180° east and 0°...–180° west), its lower branch (180°) is the basis for the **International Dateline** (*Fig. 3-1*).
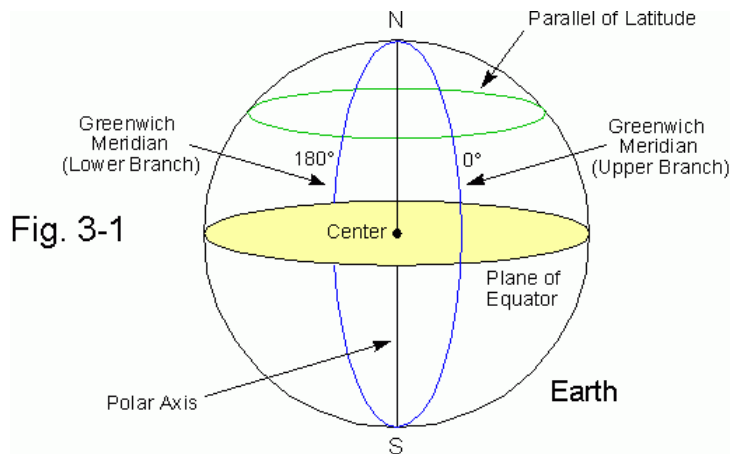

Fig. 3-1

Each point of the earth's surface has an imaginary counterpart on the surface of the celestial sphere obtained by central projection. The projected image of the observer's position, for example, is the zenith. Accordingly, there are two **celestial poles**, the c**elestial equator**, **celestial meridians**, etc.

The equatorial system of coordinates

The **geographic position of a celestial body**, **GP**, is defined by the **equatorial system of coordinates** (*Fig. 3-2*). The **Greenwich hour angle**, **GHA**, is the angular distance of GP from the upper branch of the Greenwich meridian (0°), measured **westward** from 0° through 360°. **Declination**, **Dec**, is the angular distance of GP from the plane of the equator, measured northward through +90° or southward through –90°. GHA and Dec are **geocentric coordinates** (measured at the center of the earth). The great circle going through the poles and GP is called **hour circle** (*Fig. 3-2*).
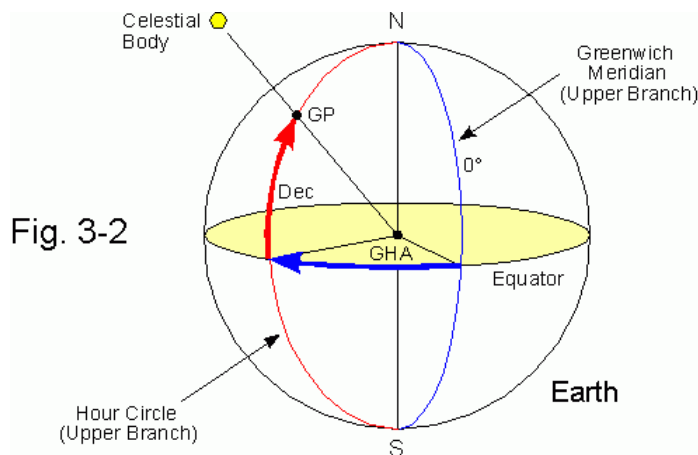

Fig. 3-2

GHA and Dec are equivalent to geocentric longitude and latitude with the exception that **longitude is measured westward through –180° and eastward through +180°.**

Since the Greenwich meridian rotates with the earth from west to east, whereas each hour circle remains linked with the almost stationary position of the respective body in the sky, the Greenwich hour angles of all celestial bodies increase by approx. 15° per hour (360° in 24 hours). In contrast to stars (15° 2.46' /h), the GHA's of sun, moon, and planets increase at slightly different (and variable) rates. This is caused by the revolution of the planets (including the earth) around the sun and by the revolution of the moon around the earth, resulting in additional apparent motions of these bodies in the sky. In many cases, it is useful to measure the angular distance between the hour circle of a celestial body and the hour circle of a reference point in the sky instead of the Greenwich meridian because the angle thus obtained is independent of the earth's rotation. The **sidereal hour angle**, **SHA**, is the angular distance of a body from the hour circle (upper branch) of the **first point of Aries** (also called **vernal equinox**, see below), measured **westward** from 0° through 360°. Thus, the GHA of a body is the sum of the SHA of the body and **GHA$_{Aries}$**, the GHA of the first point of Aries, :

$$GHA = SHA + GHA_{Aries}$$

(If the resulting GHA is greater than 360°, subtract 360°.)

The angular distance of a body **eastward** from the hour circle of the vernal equinox, measured in time units (24h = 360°), is called **right ascension**, **RA**. Right ascension is mostly used by astronomers whereas navigators prefer SHA.

$$RA[h] = 24h - \frac{SHA[°]}{15} \qquad \Leftrightarrow \qquad SHA[°] = 360° - 15 \cdot RA[h]$$

Fig. 3-3 illustrates how the various hour angles are interrelated.


Fig. 3-3
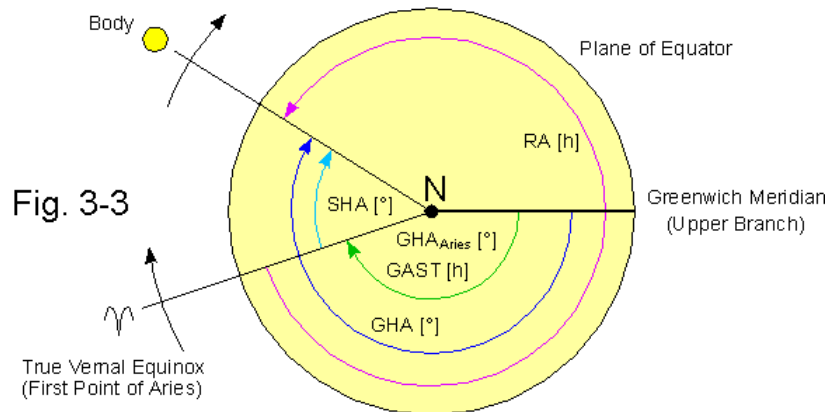
Declinations are not affected by the rotation of the earth. The declinations of sun and planets change primarily due to the **obliquity of the ecliptic**, the inclination of the earth's equator to the **ecliptic**. The latter is the plane of the earth's orbit and forms a great circle on the celestial sphere. The declination of the sun, for example, varies periodically between ca. +23.5° at the time of the summer solstice and ca. -23.5° at the time of the winter solstice. (*Fig.3-4*).

Fig. 3-4
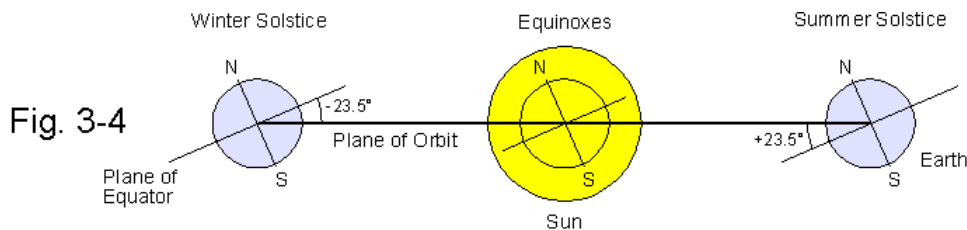
The two points on the celestial sphere where the great circle of the ecliptic intersects the celestial equator are called **equinoxes**. The term equinox is also used for the time at which the apparent sun, moving westward along the ecliptic, passes through either of these points, approximately on March 21 and on September 23. Accordingly, there is a **vernal equinox** (**first point of Aries, vernal point**) and an **autumnal equinox**. The former is the reference point for measuring sidereal hour angles (*Fig. 3-5*).

When the sun passes through either of the equinoxes (Dec ≈ 0), day and night have (approximately) the same length, regardless of the observer's position (Lat. *aequae noctes* = equal nights).

Fig. 3-5
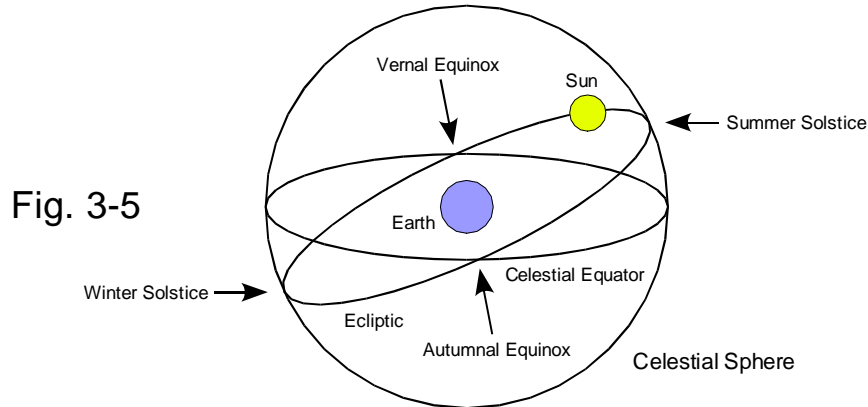
Declinations of the planets and the moon are also influenced by the inclinations of their own orbits to the ecliptic. The plane of the moon's orbit, for example, is inclined to the ecliptic by approx. 5° and makes a tumbling movement (precession, see below) with a cycle of 18.6 years (Saros cycle). As a result, the declination of the moon varies between approx. -28.5° and +28.5° at the beginning and at the end of the Saros cycle, and between approx. -18.5° and +18.5° in the middle of the Saros cycle.

Further, sidereal hour angles and declinations of all bodies change slowly due to the influence of the **precession** of the earth's polar axis. Precession is a slow, tumbling movement of the polar axis along the surface of an imaginary double cone. One revolution takes about 26000 years (Platonic year). As a result, the equinoxes move westward along the celestial equator at a rate of approx. 50" per year. Thus, the sidereal hour angle of each star decreases at about the same rate. In addition, the polar axis makes a small elliptical oscillation, called **nutation**, which causes the equinoxes to travel along the celestial equator at a periodically changing rate, and we have to distinguish between the ficticious **mean equinox of date** and the **true equinox of date** (see time measurement). Accordingly, the declination of each body oscillates. The same applies to the rate of change of the sidereal hour angle and right ascension of each body.

Even stars are not fixed in space but move individually, resulting in a slow drift of their equatorial coordinates (**proper motion**). Finally, the apparent positions of bodies are influenced by the limited speed of light (**light time, aberration**), and, to a small extent, by **annual parallax** [16]. The accurate prediction of geographic positions of celestial bodies requires complicated algorithms. Formulas for the calculation of low-precision **ephemerides** of the sun (accurate enough for celestial navigation) are given in chapter 15.

Time measurement in navigation and astronomy

Due to the rapid change of Greenwich hour angles, celestial navigation requires accurate time measurement, and the time at the instant of observation should be noted to the second if possible. This is usually done by means of a chronometer and a stopwatch. The effects of time errors are dicussed in chapter 16. On the other hand, the earth's rotation with respect to celestial bodies provides an important basis for astronomical time measurement.

Coordinates tabulated in the **Nautical Almanac** refer to **Universal Time**, **UT**. UT has replaced **Greenwich Mean Time**, **GMT**, the traditional basis for civil time keeping. Conceptually, UT (like GMT) is the hour angle of the fictitious m**ean sun**, expressed in hours, with respect to the **lower branch** of the Greenwich meridian (mean solar time, *Fig. 3-6*).

Fig. 3-6
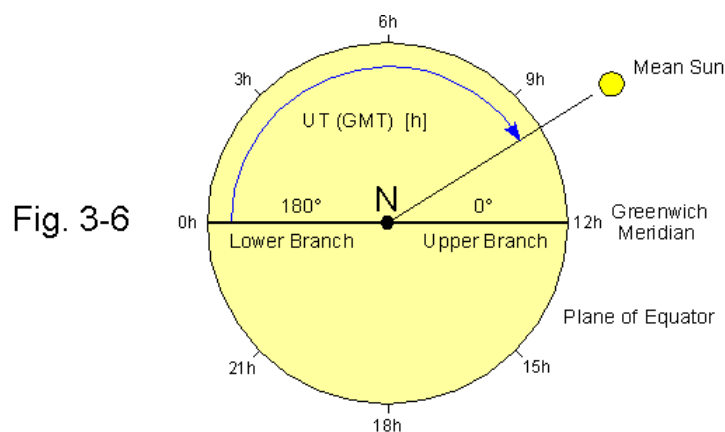
UT is calculated using the following formula:

$$UT[h] = GMT[h] = \frac{GHA_{MeanSun}[°]}{15} + 12$$

(If UT is greater than 24 h, subtract 24 hours.)

By definition, the GHA of the mean sun increases by exactly 15° per hour, completing a 360° cycle in 24 hours. The unit for UT (and GMT) is 1 solar day, the time interval between two consecutive meridian transits of the mean sun.

The rate of change of the GHA of the **apparent (observable) sun** varies periodically and is sometimes slightly greater, sometimes slightly smaller than 15° per hour during the course of a year. This behavior is caused by the eccentricity of the earth's orbit and by the obliquity of the ecliptic. The time measured by the hour angle of the apparent sun with respect to the lower branch of the Greenwich meridian is called **Greenwich Apparent Time**, **GAT**. A calibrated sundial located at the Greenwich meridian, for example, would indicate GAT. The difference between GAT and UT (GMT) is called **equation of time, EoT**:

$$EoT = GAT - UT$$

EoT varies periodically between approx. −14 and +17 minutes (*Fig. 3-7*). Predicted values for EoT for each day of the year (at 0:00 and 12:00 UT) are given in the Nautical Almanac (grey background indicates negative EoT). EoT is needed when calculating times of sunrise and sunset, or determining a **noon longitude** (see chapter 6). Formulas for the calculation of EoT are given in chapter 15.

Fig. 3-7
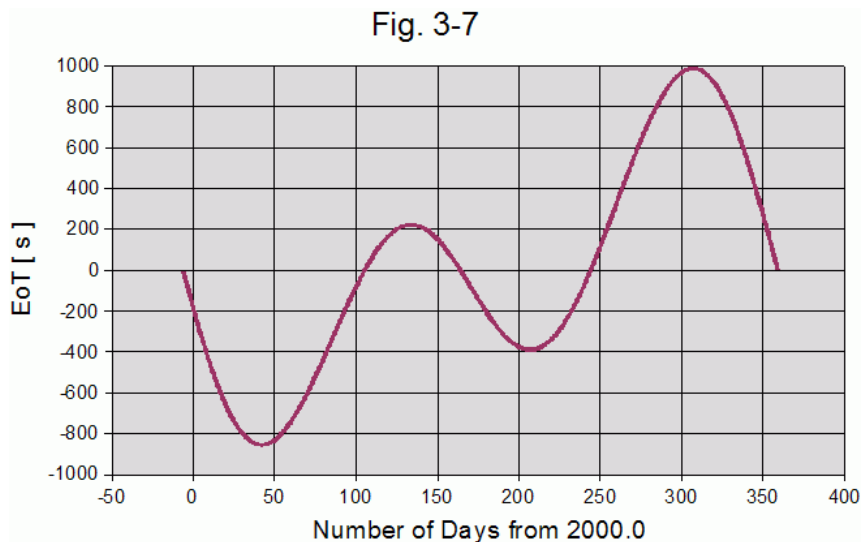
The hour angle of the mean sun with respect to the lower branch of the local meridian (the meridian going through the observer's position) is called **Local Mean Time**, **LMT**. LMT and UT are linked through the following formula:

$$LMT[h] = UT[h] + \frac{Lon[°]}{15}$$

The instant of the mean sun passing through the local meridian (upper branch) is called **Local Mean Noon, LMN**.

A **zone time** is the local mean time with respect to a longitude being a multiple of ±15°. Thus, zone times differ by an integer number of hours. In the US, for example, Eastern Standard Time (UT−5h) is LMT at −75° longitude, Pacific Standard Time (UT−8h) is LMT at −120° longitude. Central European Time (UT+1h) is LMT at +15° longitude.

The hour angle of the apparent sun with respect to the lower branch of the local meridian is called **Local Apparent Time, LAT**:

$$LAT[h] = GAT[h] + \frac{Lon[°]}{15}$$

The instant of the apparent sun crossing the local meridian (upper branch) is called **Local Apparent Noon, LAN**.

Time measurement by the earth's rotation does not necessarily require the sun as the reference point in the sky. **Greenwich Apparent Sidereal Time**, **GAST**, is a time scale based upon the Greenwich hour angle (upper branch) of the true vernal equinox of date, GHA_{Aries} (see Fig. 3-3).

$$GAST[h] = \frac{GHA_{Aries}[\,^\circ\,]}{15}$$

The values for GHA_{Aries} tabulated in the Nautical Almanac refer to the true equinox of date.

GAST is easily measured by the Greenwich meridian transit of stars since GAST and the right ascension of the observed star are numerically equal at the moment of meridian transit.

The Greenwich hour angle of the imaginary mean vernal equinox of date (traveling along the celestial equator at a constant rate) defines **Greenwich Mean Sidereal Time**, **GMST**. The difference to GMST is called **equation of the equinoxes**, **EQ**, or **nutation in right ascension**. EQ can be predicted precisely. It varies within approx. ±1s.

$$EQ = GAST - GMST$$

GMST is of some importance for it is the actual basis for UT. Since time measurement by meridian transits of the sun is not accurate enough for many scientific applications, Universal Time is by definition calculated from GMST. This is possible because there is a close correlation between GMST and mean solar time at Greenwich. The time thus obtained is called **UT0**. Applying a small correction (milliseconds) for the effects of **polar motion** (a quasi-circular movement of the earth's crust with respect to the axis of rotation) yields **UT1**, commonly called **UT**.

Due to the earth's revolution around the sun, a mean sidereal day (the time interval between two concecutive meridian transits of the mean equinox) is slightly shorter than a mean solar day:

$$24\,h \ \text{mean sidereal time} = 23\,h\,56\,m\,4.090524\,s \ \text{mean solar time}$$

In analogy with LMT and LAT, there is a **Local Mean Sidereal Time, LMST**, and a **Local Apparent Sidereal Time, LAST**:

$$LMST[h] = GMST[h] + \frac{Lon[\,^\circ\,]}{15} \qquad\qquad LAST[h] = GAST[h] + \frac{Lon[\,^\circ\,]}{15}$$

Solar time and sidereal time are both linked to the earth's rotation. The earth's rotational speed, however, decreases slowly (tidal friction) and, moreover, fluctuates in an unpredictable manner due to random movements of matter within the earth's body (magma) and on the surface (water, air). Therefore, neither of both time scales is strictly uniform. Many astronomical applications, however, require a linear time scale. One example is the calculation of ephemerides since the motions of celestial bodies in space are independent of the earth's rotation.

International Atomic Time, **TAI**, is the most precise time standard presently available. It is obtained by statistical analysis of data supplied by a great number of atomic clocks all over the world. Among others, two important time scales are derived from TAI:

Today, civil life is mostly determined by **Coordinated Universal Time, UTC**, which is the basis for time signals broadcast by radio stations, e. g., WWV or WWVH. UTC is controlled by TAI. Due to the varying rotational speed of the earth, UT tends to drift away from UTC. This is undesirable since the cycle of day and night is linked to UT. Therefore, UTC is synchronized to UT, if necessary, by inserting (or omitting) leap seconds at certain times (June 30 and December 31) in order to avoid that the difference, ΔUT, exceeds the specified maximum value of ±0.9 s.

$$UT = UTC + \Delta UT$$

$$UTC = TAI - N$$

N is the cumulative number of leap seconds inserted until now (N = 32 in 2004.0). Due to the occasional leap seconds, UTC is **not** a continuous time scale! Predicted values for ΔUT (= UT1-UTC) are published by the **IERS Rapid Service** [15] on a weekly basis (IERS Bulletin A). The IERS also announces the insertion (or omission) of leap seconds in advance (IERS Bulletins A + C).

Terrestrial Time, TT (formerly called **Terrestrial Dynamical Time, TDT**), is another derivative of TAI:

$$TT = TAI + 32.184\,s$$

TT has replaced **Ephemeris Time**, **ET**. The offset of 32.184 s with respect to TAI is necessary to ensure a seamless continuation of ET. TT is used in astronomy (calculation of ephemerides) and space flight. The difference between TT and UT is called ΔT:

$$\Delta T = TT - UT$$

At the beginning of the year 2004, ΔT was +64.6s. ΔT is of some importance since computer almanacs require TT (TDT) as time argument (programs using UT calculate on the basis of extrapolated ΔT values). A precise long-term prediction of ΔT is impossible. Therefore, computer almanacs using only UT as time argument may become less accurate in the long term. ΔT values for the near future can be calculated with the following formula:

$$\Delta T = 32.184\,\mathrm{s} + (TAI - UTC) - (UT1 - UTC)$$

Like UT1-UTC, TAI-UTC (cumulative number of leap seconds) is published in the IERS Bulletin A.

A final remark on GMT:

The term GMT has become ambigous since it is often used as a synonym for UTC now. Moreover, astronomers used to reckon GMT from the upper branch of the Greenwich meridian until 1925 (the time thus obtained is sometimes called Greenwich Mean Astronomical Time, GMAT). Therefore, the term GMT should be avoided in scientific publications, except when used in a historical context.

The Nautical Almanac

Predicted values for GHA and Dec of sun, moon and the navigational planets with reference to UT are tabulated for each whole hour of the year on the daily pages of the **Nautical Almanac, N.A.,** and similar publications [12, 13]. GHA_{Aries} is tabulated in the same manner.

Listing GHA and Dec of all 57 fixed stars used in navigation for each whole hour of the year would require too much space. Since declinations and sidereal hour angles of stars change only slowly, tabulated values for periods of 3 days are accurate enough for navigational purposes.

GHA and Dec for each second of the year are obtained using the **interpolation tables** at the end of the N.A. (printed on tinted paper), as explained in the following directions:

1.

We note the exact time of observation (UT), determined with a chronometer, for each celestial body. If UT is not available, we can use UTC. The resulting error is tolerable in most cases.

2.

We look up the day of observation in the N.A. (two pages cover a period of three days).

3.

We go to the nearest whole hour preceding the time of observation and note GHA and Dec of the observed body. In case of a fixed star, we form the sum of GHA Aries and the SHA of the star, and note the tabulated declination. When observing planets, we note the *v* and *d* factors given at the bottom of the appropriate column. For the moon, we take *v* and *d* for the nearest whole hour preceding the time of observation.

The quantity *v* is necessary to apply an additional correction to the following interpolation of the GHA of moon and planets. It is not required for stars. The sun does not require a *v* factor since the correction has been incorporated in the tabulated values for the sun's GHA.

The quantity *d*, which is negligible for stars, is the rate of change of Dec, measured in arcminutes per hour. It is needed for the interpolation of Dec. The sign of *d* is critical!

We look up the minute of observation in the interpolation tables (1 page for each 2 minutes of the hour), go to the second of observation, and note the **increment** from the respective column.

We enter one of the three columns to the right of the increment columns with the *v* and *d* factors and note the corresponding **corr**(ection) values (*v*-corr and *d*-corr).

The sign of *d*-corr depends on the trend of declination at the time of observation. It is positive if Dec at the whole hour following the observation is greater than Dec at the whole hour preceding the observation. Otherwise it is negative.

v -corr is negative for Venus. Otherwise, it is always positive.

We form the sum of Dec and *d*-corr (if applicable).

We form the sum of GHA (or GHA Aries and SHA of star), increment, and *v*-corr (if applicable).
SHA values tabulated in the Nautical Almanac refer to the true vernal equinox of date.

Interactive Computer Ephemeris

Interactive Computer Ephemeris, **ICE**, is a computer almanac developed by the U.S. Naval Observatory (successor of the **Floppy Almanac**).

ICE is FREEWARE (no longer supported by USNO), compact, easy to use, and provides a vast quantity of accurate astronomical data for a time span of almost 250 (!) years. In spite of the old design (DOS program), ICE is an extremely useful tool for navigators and astronomers.

Among many other features, ICE calculates GHA and Dec for a given body and time as well as altitude and azimuth of the body for an **assumed position** (see chapter 4) and sextant altitude corrections. Since the navigation data are as accurate as those tabulated in the **Nautical Almanac** (approx. 0.1'), the program makes an adequate alternative, although a printed almanac (and sight reduction tables) should be kept as a backup in case of a computer failure. The following instructions refer to the final version (0.51). Only program features relevant to navigation are explained.

1. Installation

Copy the program files to a chosen directory on the hard drive or to a floppy disk.

2. Getting Started

Go to the program directory (or floppy disk) and enter "ice". The main menu appears.
Use the function keys F1 to F10 to navigate through the submenus. The program is more or less self-explanatory.
Go to the submenu INITIAL VALUES (F1). Follow the directions on the screen to enter date and time of observation (F1), assumed latitude (F2), assumed longitude (F3), and your local time zone (F6). Assumed latitude and longitude define your assumed position.

Use the correct data format, as shown on the screen (decimal format for latitude and longitude). After entering the above data, press F7 to accept the values displayed. To change the default values permanently, edit the file ice.dft with a text editor (after making a backup copy) and make the desired changes. Do not change the data format. The numbers have to be in columns 21-40. An output file can be created to store calculated data. Go to the submenu FILE OUTPUT (F2) and enter a chosen file name, e.g., OUTPUT.TXT.

3. Calculation of Navigational Data

From the main menu, go to the submenu NAVIGATION (F7). Enter the name of the body. The program displays GHA and Dec of the body, GHA and Dec of the sun (if visible), and GHA of the vernal equinox for the date and time (UT) stored in INITIAL VALUES.

Hc (computed altitude) and Zn (azimuth) mark the apparent position of the body as observed from the assumed position. Approximate altitude corrections (refraction, SD, PA), based upon Hc, are also displayed (for lower limb of body). The semidiameter of the moon includes augmentation. The coordinates calculated for Venus and Mars do **not** include phase correction. Therefore, the upper or lower limb (if visible) should be observed. ΔT is TDT-UT, the predicted difference between **terrestrial dynamical time** and UT for the given date. The ΔT value for 2004.0 predicted by ICE is 66.1s, the actual value is 64.6s (see below).

Horizontal parallax and semidiameter of a body can be extracted, if required, from the submenu POSITIONS (F3). Choose APPARENT GEOCENTRIC POSITIONS (F1) and enter the name of the body (sun, moon, planets). The last column shows the distance of the center of the body from the center of the earth, measured in astronomical units (1 AU = 149.6 · 10^6 km). HP and SD are calculated as follows:

$$ HP \ = \ \arcsin \frac{r_E\left[km\right]}{distance\left[km\right]} \qquad\qquad SD \ = \ \arcsin \frac{r_B\left[km\right]}{distance\left[km\right]} $$

r_E is the equatorial radius of the earth (6378 km). r_B is the radius of the body (Sun: 696260 km, Moon: 1378 km, Venus: 6052 km, Mars: 3397 km, Jupiter: 71398 km, Saturn: 60268 km).

The apparent geocentric positions refer to TDT, but the difference between TDT and UT has no significant effect on HP and SD.

To calculate times of rising and setting of a body, go to the submenu RISE & SET TIMES (F6) and enter the name of the body. The columns on the right display the time of rising, meridian transit, and setting for the assumed location (UT+xh, according to the time zone specified).

The increasing error of ΔT values predicted by ICE may lead to reduced precision when calculating navigation data for the future. The coordinates of the moon are particularly sensitive to errors of ΔT. Unfortunately, ICE has no option for editing and modifying ΔT. The high-precision part of ICE, however, is not affected since TDT is the time argument. To extract GHA and Dec, the following procedure is recommended:

1.   Compute GAST using SIDEREAL TIME (F5). The time argument is UT.

2.   Edit date and time at INITIAL VALUES (F1). The new time argument is UT+ΔT. Compute RA and Dec using POSITIONS (F3) and APPARENT GEOCENTRIC POSITIONS (F1).

3.   Use the following formula to calculate GHA from GAST and RA (RA refers to the true vernal equinox of date):

$$ GHA\left[°\right] = 15 \cdot \left(GAST\left[h\right] + 24h - RA\left[h\right]\right) $$

Add or subtract 360° if necessary.

High-precision GHA and Dec values thus obtained can be used as an internal standard to check medium-precision data obtained through NAVIGATION (F7).