Package 'conicfit'

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Type Package

Title Algorithms for Fitting Circles, Ellipses and Conics Based on the Work by Prof. Nikolai Chernov

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Description Geometric circle fitting with Levenberg-Marquardt (a, b, R), Levenberg-Marquardt reduced (a, b), Landau, Spath and Chernov-Lesort. Algebraic circle fitting with Taubin, Kasa, Pratt and Fitzgibbon-Pilu-Fisher. Geometric ellipse fitting with ellipse LMG (geometric parameters) and conic LMA (algebraic parameters). Algebraic ellipse fitting with Fitzgibbon-Pilu-Fisher and Taubin.

License GPL (>= 3)

Depends R (>= 2.7.0), pracma, geigen

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R topics documented:

`AtoG` *Conversion of algebraic parameters to geometric parameters*

Description

`AtoG` converts algebraic parameters (A, B, C, D, E, F) to geometric parameters (Center(1:2), Axes(1:2), Angle).

Usage

```
AtoG(ParA)
```

Arguments

`ParA`	vector or array with geometric parameters (A, B, C, D, E, F)

Format

code is: -1 - degenerate cases 0 - imaginary ellipse 4 - imaginary parell lines 1 - ellipse 2 - hyperbola 3 - parabola

Value

`list(ParG, exitCode`
list with algebraic parameters (Center(1:2), Axes(1:2), Angle) and exit code

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
AtoG(c(0.0551,-0.0908,0.1588,0.0489,-0.9669,0.1620))
```

---

calculateCircle *Generate points from a circle*

Description

`calculateCircle` generates points from a circle with many options, equally spaced, randomly spaced, with noise added to the radius or limited to a segment of angle alpha.

Usage

```
calculateCircle(x, y, r, steps=50,sector=c(0,360),randomDist=FALSE,
randomFun=runif, noiseFun = NA, ...)
```

Arguments

`x`	center point x
`y`	center point y
`r`	radius
`steps`	number of points
`sector`	limited circular sector
`randomDist`	logical, TRUE = randomly spaced
`randomFun`	random function for the position of the points in the circle
`noiseFun`	random function for the noise
`...`	optional parameters to pass to randomFun

Value

`points`	array n x 2 of point coordinates.

Author(s)

Jose Gama

Examples

```
## Not run:
# 100 points from a circle at c(0,0) with radius=200
a<-calculateCircle(0,0,200,100)
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250))
par(new=T)
# 12 points from a circle at c(0,0) with radius=190, points between 0 and 90
#degrees
a<-calculateCircle(0,0,190,12,c(0,90))
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250),col='red')
par(new=T)
# 12 points from a circle at c(0,0) with radius=180, points between 0 and 180
#degrees, uniform random distribution
a<-calculateCircle(0,0,180,12,c(0,180),TRUE)
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250),col='green')
par(new=T)
# 12 points from a circle at c(0,0) with radius=170, points between 0 and 180
#degrees, normal random distribution
a<-calculateCircle(0,0,170,12,c(0,180),TRUE,rnorm)
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250),col='blue')
par(new=T)
# 12 points from a circle at c(0,0) with radius=200, points between 0 and 180
#degrees, positioned by uniform random distribution, noise=normal random
#distribution with sd=10
a<-calculateCircle(0,0,200,12,c(180,360),TRUE,noiseFun=function(x)
(x+rnorm(1,mean=0,sd=10)))
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250),col='orange')

## End(Not run)
```

---

calculateEllipse *Generate points from a ellipse*

Description

`calculateEllipse` generates points from a ellipse with many options, equally spaced, randomly spaced, with noise added to the radius or limited to a segment of angle alpha.

Usage

```
calculateEllipse(x, y, a, b, angle = 0, steps = 50, sector = c(0, 360),
randomDist = FALSE, randomFun = runif, noiseFun = NA, ...)
```

Arguments

`x`	center point x
`y`	center point y
`a`	axis a
`b`	axis b
`angle`	tilt angle
`steps`	number of points
`sector`	limited circular sector
`randomDist`	logical, TRUE = randomly spaced
`randomFun`	random function for the position of the points in the ellipse
`noiseFun`	random function for the noise
`...`	optional parameters to pass to randomFun

Value

`points`	array n x 2 of point coordinates.

Author(s)

Jose Gama

Examples

```
## Not run:
# 50 points from an ellipse at c(0,0) with axis (200, 100), angle 45 degrees
a<-calculateEllipse(0,0,200,100,45,50)
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250))
par(new=T)
# 10 points from an ellipse at c(0,0) with axis (200, 100), angle 45 degrees,
#points between 0 and 180 # degrees, normal random distribution
b<-calculateEllipse(0,0,200,100,45,10,c(0,90))
plot(b[,1],b[,2],xlim=c(-250,250),ylim=c(-250,250),col='red')
par(new=T)
# 50 points from an ellipse at c(0,0) with axis (200, 100), angle 45 degrees
a<-calculateEllipse(0,0,200,100,45,50, randomDist=TRUE,noiseFun=function(x)
(x+rnorm(1,mean=0,sd=10)))
plot(a[,1],a[,2],xlim=c(-250,250),ylim=c(-250,250),col='cyan')

## End(Not run)
```

CircleFitByKasa *Algebraic circle fit (Kasa method)*

Description

CircleFitByKasa applies the simple algebraic circle fit (Kasa method)

Usage

```
CircleFitByKasa(XY)
```

Arguments

XY array of sample data

Value

vector(a, b, R)
vector with the values for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c3 <- CircleFitByKasa(xy)
xyc3<-calculateCircle(c3[1],c3[2],c3[3])
plot(xyc3[,1],xyc3[,2],xlim=c(-250,250),ylim=c(-250,250),col='green',type='l');par(new=TRUE)
```

CircleFitByLandau — *Geometric circle fit (minimizing orthogonal distances) by Landau algorithm*

Description

`CircleFitByLandau` applies the Geometric circle fit (minimizing orthogonal distances) by Landau algorithm

Usage

```
CircleFitByLandau(XY,ParIni = NA, epsilon = 1e-06, IterMAX = 50)
```

Arguments

`XY`	array of sample data
`ParIni`	initial guess (a, b, R)
`epsilon`	tolerance (small threshold)
`IterMAX`	maximal number of iterations, with a bad initial guess it may take >100 iterations

Value

`vector(a, b, R)`
vector with the values for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Geometric circle fit (minimizing orthogonal distances) by Landau algorithm M. Landau, "Estimation of a circular arc center and its radius", Computer Vision, Graphics and Image Processing, Vol. 38, pages 317-326, (1987)

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Geometric circle fit (minimizing orthogonal distances) by Landau algorithm M. Landau, "Estimation of a circular arc center and its radius", Computer Vision, Graphics and Image Processing, Vol. 38, pages 317-326, (1987)

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c6 <- CircleFitByLandau(xy)
xyc6<-calculateCircle(c6[1],c6[2],c6[3])
plot(xyc6[,1],xyc6[,2],xlim=c(-250,250),ylim=c(-250,250),col='purple',type='l');par(new=TRUE)
```

CircleFitByPratt *Algebraic circle fit by Pratt*

Description

`CircleFitByPratt` applies the Algebraic circle fit by Pratt

Usage

```
CircleFitByPratt(XY)
```

Arguments

`XY`	array of sample data

Value

`vector(a, b, R)`
: vector with the values for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c2 <- CircleFitByPratt(xy)
xyc2<-calculateCircle(c2[1],c2[2],c2[3])
plot(xyc2[,1],xyc2[,2],xlim=c(-250,250),ylim=c(-250,250),col='blue',type='l');par(new=TRUE)
```

`CircleFitBySpath` *Geometric circle fit by Spath*

Description

`CircleFitBySpath` applies the Geometric circle fit by Spath

Usage

```
CircleFitBySpath(XY, ParIni = NA, epsilon = 1e-06, IterMAX = 50)
```

Arguments

`XY`	array of sample data
`ParIni`	initial guess (a, b, R)
`epsilon`	tolerance (small threshold)
`IterMAX`	maximal number of iterations, with a bad initial guess it may take >100 iterations

Value

`vector(a, b, R)`
vector with the values for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c5 <- CircleFitBySpath(xy)
xyc5<-calculateCircle(c5[1],c5[2],c5[3])
plot(xyc5[,1],xyc5[,2],xlim=c(-250,250),ylim=c(-250,250),col='magenta',type='l');par(new=TRUE)
```

`CircleFitByTaubin` *Algebraic circle fit (Taubin method)*

Description

`CircleFitByTaubin` applies the simple algebraic circle fit (Taubin method)

Usage

```
CircleFitByTaubin(XY)
```

Arguments

`XY` array of sample data

Value

`vector(a, b, R)`
vector with the values for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c1 <- CircleFitByTaubin(xy)
xyc1<-calculateCircle(c1[1],c1[2],c1[3])
plot(xyc1[,1],xyc1[,2],xlim=c(-250,250),ylim=c(-250,250),col='red',type='l');par(new=TRUE)
```

EllipseDirectFit *Algebraic ellipse fit method by Fitzgibbon-Pilu-Fisher*

Description

`EllipseDirectFit` applies the algebraic ellipse fit method by Fitzgibbon-Pilu-Fisher

Usage

```
EllipseDirectFit(XY)
```

Arguments

`XY`	array of sample data

Value

`vector(A,B,C,D,E,F)`
: vector of algebraic parameters of the fitting ellipse: ax^2 + bxy + cy^2 +dx + ey + f = 0

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

A. W. Fitzgibbon, M. Pilu, R. B. Fisher, 1999 Direct Least Squares Fitting of Ellipses IEEE Trans. PAMI, Vol. 21, pages 476-480

A. W. Fitzgibbon, M. Pilu, R. B. Fisher, "Direct Least Squares Fitting of Ellipses", IEEE Trans. PAMI, Vol. 21, pages 476-480 (1999) Halir R, Flusser J (1998) Proceedings of the 6th International Conference in Central Europe on Computer Graphics and Visualization, Numerically stable direct least squares fitting of ellipses (WSCG, Plzen, Czech Republic), pp 125–132.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

A. W. Fitzgibbon, M. Pilu, R. B. Fisher, 1999 Direct Least Squares Fitting of Ellipses IEEE Trans. PAMI, Vol. 21, pages 476-480

A. W. Fitzgibbon, M. Pilu, R. B. Fisher, "Direct Least Squares Fitting of Ellipses", IEEE Trans. PAMI, Vol. 21, pages 476-480 (1999) Halir R, Flusser J (1998) Proceedings of the 6th International Conference in Central Europe on Computer Graphics and Visualization, Numerically stable direct least squares fitting of ellipses (WSCG, Plzen, Czech Republic), pp 125–132.

Examples

```
xy<-calculateEllipse(0,0,200,100,45,50, randomDist=TRUE,noiseFun=function(x)
(x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250),col='magenta');par(new=TRUE)

ellipDirect <- EllipseDirectFit(xy)
ellipDirectG <- AtoG(ellipDirect)$ParG
xyDirect<-calculateEllipse(ellipDirectG[1], ellipDirectG[2], ellipDirectG[3],
ellipDirectG[4], 180/pi*ellipDirectG[5])
plot(xyDirect[,1],xyDirect[,2],xlim=c(-250,250),ylim=c(-250,250),type='l',
col='cyan');par(new=TRUE)
```

`EllipseFitByTaubin` *Algebraic ellipse fit by Taubin*

Description

`EllipseFitByTaubin` applies the Algebraic ellipse fit by Taubin

Usage

```
EllipseFitByTaubin(XY)
```

Arguments

`XY`	array of sample data

Value

`vector(A,B,C,D,E,F)`
: vector with the values for the ellipse

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateEllipse(0,0,200,100,45,50, randomDist=TRUE,noiseFun=function(x)
(x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250),col='magenta');par(new=TRUE)

ellipTaubin <- EllipseFitByTaubin(xy)
ellipTaubinG <- AtoG(ellipTaubin)$ParG
xyTaubin<-calculateEllipse(ellipTaubinG[1], ellipTaubinG[2], ellipTaubinG[3],
ellipTaubinG[4], 180/pi*ellipTaubinG[5])
plot(xyTaubin[,1],xyTaubin[,2],xlim=c(-250,250),ylim=c(-250,250),type='l',
col='red');par(new=TRUE)
```

`ellipticity` *Formulas for the ellipse*

Description

`ellipticity` ellipticity = flattening factor `ellipseEccentricity` eccentricity of the ellipse `ellipseFocus` focus of the ellipse `ellipseRa` radius at apoapsis (the farthest distance) `ellipseRp` radius at periapsis (the closest distance) `ellipse.l` semi-latus rectum l

Usage

```
ellipticity(minorAxis,majorAxis)
```

Arguments

`minorAxis`	minor ellipse axis
`majorAxis`	major ellipse axis

Value

scalar result

Author(s)

Jose Gama

Source

Wikipedia Ellipse http://en.wikipedia.org/wiki/Ellipse#Mathematical_definitions_and_properties

References

Wikipedia Ellipse http://en.wikipedia.org/wiki/Ellipse#Mathematical_definitions_and_properties

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estimateInitialGuessCircle

Estimate Initial Guess Circle values

Description

estimateInitialGuessCircle estimates initial guess values for the center and radius of the circle

Usage

```
estimateInitialGuessCircle(XY)
```

Arguments

XY	array of sample data

Value

vector(a, b, R)
: vector with the estimates for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
estimateInitialGuessCircle(xy)
```

fit.conicLMA *Fitting a conic to a given set of points (Implicit method)*

Description

fit.conicLMA fits a conic to a given set of points (Implicit method) using algebraic parameters. Conic: Ax^2 + Bxy + Cy^2 +Dx + Ey + F = 0

Usage

```
fit.conicLMA(XY, ParAini, LambdaIni, epsilonP = 1e-10, epsilonF = 1e-13,
IterMAX = 2e+06)
```

Arguments

XY	array of sample data
ParAini	initial parameter vector c(A,B,C,D,E,F)
LambdaIni	initial value of the control parameter Lambda
epsilonP	tolerance (small threshold)
epsilonF	tolerance (small threshold)
IterMAX	maximum number of (main) iterations, usually 10-20 will suffice

Value

list(ParA, RSS, iters
list with algebraic parameters (Center(1:2), Axes(1:2), Angle), Residual Sum of Squares and number of iterations

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

N. Chernov and H. Ma, 2011 Least squares fitting of quadratic curves and surfaces In: Computer Vision, Editor S. R. Yoshida, Nova Science Publishers; pp. 285-302.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

N. Chernov and H. Ma, 2011 Least squares fitting of quadratic curves and surfaces In: Computer Vision, Editor S. R. Yoshida, Nova Science Publishers; pp. 285-302.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParAini <- matrix(c(0.2500,0, 1.0000, 0, 0, -1.0000),ncol=1)
LambdaIni=0.1
fit.conicLMA(XY,ParAini,LambdaIni)
```

`fit.ellipseLMG` *Fitting an ellipse using Implicit method*

Description

`fit.ellipseLMG` Fits an ellipse to a given set of points (Implicit method) using geometric parameters. Conic:

Usage

```
fit.ellipseLMG(XY,ParGini,LambdaIni = 1, epsilon = 1e-06, IterMAX = 200,
L = 200)
```

Arguments

`XY`	array of sample data
`ParGini`	initial parameter vector c(Center(1:2), Axes(1:2), Angle)
`LambdaIni`	initial value of the control parameter Lambda
`epsilon`	tolerance (small threshold)
`IterMAX`	maximum number of (main) iterations, usually 10-20 will suffice
`L`	boundary for major/minor axis

Value

`list(ParG,RSS,iters,TF)`
list with geometric parameters (A,B,C,D,E,F), Residual Sum of Squares, number of iterations and TF==TRUE if the method diverges

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

N. Chernov and H. Ma, 2011 Least squares fitting of quadratic curves and surfaces In: Computer Vision, Editor S. R. Yoshida, Nova Science Publishers; pp. 285-302.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

N. Chernov and H. Ma, 2011 Least squares fitting of quadratic curves and surfaces In: Computer Vision, Editor S. R. Yoshida, Nova Science Publishers; pp. 285-302.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParGini <- matrix(c(0,0,2,1,0),ncol=1)
LambdaIni=0.1
fit.ellipseLMG(XY,ParGini,LambdaIni)
```

`fitbookstein` *Linear ellipse fit using bookstein constraint*

Description

`fitbookstein` Linear ellipse fit using bookstein constraint

`conic2parametric` Diagonalise A - find Q, D such at A = Q' * D * Q

`fitggk` Linear least squares with the Euclidean-invariant constraint Trace(A) = 1

Usage

```
fitbookstein(x)
```

Arguments

`x`	array of sample data

Value

`list(z, a, b, alpha)`
list with fitted ellipse parameters

Author(s)

Jose Gama

Source

Richard Brown, May 28, 2007 http://www.mathworks.com/matlabcentral/fileexchange/15125-fitellipse-m/content/demo/html/ellipsedemo.html

References

Richard Brown, May 28, 2007 http://www.mathworks.com/matlabcentral/fileexchange/15125-fitellipse-m/content/demo/html/ellipsedemo.html

W. Gander, G. H. Golub, R. Strebel, 1994 Least-Squares Fitting of Circles and Ellipses BIT Numerical Mathematics, Springer

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GtoA *Conversion of geometric parameters to algebraic parameters*

Description

`GtoA` converts geometric parameters (A, B, C, D, E, F) to algebraic parameters (Center(1:2), Axes(1:2), Angle).

Usage

```
GtoA(ParG)
```

Arguments

`ParG`	list with geometric parameters (A, B, C, D, E, F)

Value

`ParA`	vector or array with algebraic parameters (Center(1:2), Axes(1:2), Angle)

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

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Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
GtoA(c(0,0,20,60,45))
```

JmatrixLMA *Compute the Jacobian matrix using algebraic parameters*

Description

`JmatrixLMA` Computes the Jacobian matrix(Implicit method) using algebraic parameters

Usage

```
JmatrixLMA(XY,ParA,XYproj)
```

Arguments

`XY`	array of sample data
`ParA`	initial parameter vector c(Center(1:2), Axes(1:2), Angle)
`XYproj`	corresponding n projection points on the conic

Value

`list(Res,J)`	list with the Residual Sum of Squares and the Jacobian matrix

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

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Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParA <- matrix(c(0.250000000000000,0,1,0,0,-1),ncol=1)
XYproj=matrix(c(0.394467220216675,0.980356518335872,0.833315950425981,
0.909063326557293,1.40466123643977,0.711850899213363,
1.70601340510202,0.521899957274429,1.89925244997324,0.313384799914835,
1.06482258038841,0.846485805004280,1.95308457257492,
0.215325713960169,1.91319150256275,0.291418202297698),8,2,byrow=TRUE)
JmatrixLMA(XY,ParA,XYproj)
```

`JmatrixLMG` *Compute the Jacobian matrix using geometric parameters*

Description

`JmatrixLMG` Computes the Jacobian matrix (Implicit method) using geometric parameters

Usage

```
JmatrixLMG(XY,A,XYproj)
```

Arguments

`XY`	array of sample data
`A`	initial parameter vector c(Xc,Yc,a,b,alpha)
`XYproj`	corresponding n projection points on the conic

Value

`list(Res,J)`	list with the Residual Sum of Squares and the Jacobian matrix

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

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Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
A <- matrix(c(0,0,2,1,0),ncol=1)
XYproj=matrix(c(0.394467220216675,0.980356518335872,0.833315950425981,
0.909063326557293,1.40466123643977,0.711850899213363,
1.70601340510202,0.521899957274429,1.89925244997324,0.313384799914835,
1.06482258038841,0.846485805004280,1.95308457257492,
0.215325713960169,1.91319150256275,0.291418202297698),8,2,byrow=TRUE)
JmatrixLMG(XY,A,XYproj)
```

`LMcircleFit` *Geometric circle fit (minimizing orthogonal distances) based on the Levenberg-Marquardt method*

Description

`LMcircleFit` applies a Geometric circle fit (minimizing orthogonal distances) based on the standard Levenberg-Marquardt scheme

Usage

```
LMcircleFit(XY, ParIni, LambdaIni = 1, epsilon = 1e-06, IterMAX = 50)
```

Arguments

`XY`	array of sample data
`ParIni`	initial guess (a, b, R)
`LambdaIni`	initial value for the correction factor lambda
`epsilon`	tolerance (small threshold)
`IterMAX`	maximum number of (main) iterations, usually 10-20 will suffice

Value

`vector(a, b, R)`
vector with the estimates for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c4 <- LMcircleFit(xy)
xyc4<-calculateCircle(c4[1],c4[2],c4[3])
plot(xyc4[,1],xyc4[,2],xlim=c(-250,250),ylim=c(-250,250),col='cyan',type='l')
```

LMreducedCircleFit — *Geometric circle fit (minimizing orthogonal distances) based on the Levenberg-Marquardt method*

Description

`LMreducedCircleFit` applies a Geometric circle fit (minimizing orthogonal distances) based on the standard Levenberg-Marquardt scheme in the "reduced" (a,b) parameter space

Usage

```
LMreducedCircleFit(XY, ParIni, LambdaIni = 1, epsilon = 1e-06,
IterMAX = 50)
```

Arguments

`XY`	array of sample data
`ParIni`	initial guess (a, b)
`LambdaIni`	initial value for the correction factor lambda
`epsilon`	tolerance (small threshold)
`IterMAX`	maximum number of (main) iterations, usually 10-20 will suffice

Value

`vector(a, b, R)`
vector with the estimates for the circle: center (a,b) and radius R

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

Nikolai Chernov, 2010 Circular and linear regression: Fitting circles and lines by least squares Chapman & Hall/CRC, Monographs on Statistics and Applied Probability, Volume 117

Examples

```
xy<-calculateCircle(0,0,200,50,randomDist=TRUE,noiseFun=function(x) (x+rnorm(1,mean=0,sd=50)))
plot(xy[,1],xy[,2],xlim=c(-250,250),ylim=c(-250,250));par(new=TRUE)
c7 <- LMreducedCircleFit(xy)
xyc7<-calculateCircle(c7[1],c7[2],c7[3])
plot(xyc7[,1],xyc7[,2],xlim=c(-250,250),ylim=c(-250,250),col='pink',type='l')
```

`Residuals.ellipse` *Projecting a given set of points onto an ellipse*

Description

`Residuals.ellipse` projects a given set of points onto an ellipse and computing the distances from the points to the ellipse

Usage

```
Residuals.ellipse(XY,ParG)
```

Arguments

`XY`	array of sample data
`ParG`	vector 5x1 of the ellipse parameters (Center(1:2), Axes(1:2), Angle)

Value

`list(Res,J)`	list with the Residual Sum of Squares and the Jacobian matrix

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParG <- matrix(c(0,0,2,1,0),ncol=1)
Residuals.ellipse(XY,ParG)
```

Residuals.hyperbola *Projecting a given set of points onto an hyperbola*

Description

`Residuals.hyperbola` projects a given set of points onto an hyperbola and computing the distances from the points to the hyperbola

Usage

```
Residuals.hyperbola(XY,ParG)
```

Arguments

XY	array of sample data
ParG	vector 5x1 of the hyperbola parameters (Center(1:2), Axes(1:2), Angle)

Value

`list(RSS, XYproj)`
list with the Residual Sum of Squares and the array of coordinates of projections

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParG <- matrix(c(0,0,2,1,0),ncol=1)
Residuals.hyperbola(XY,ParG)
```

`Residuals.parabola` *Projecting a given set of points onto an parabola*

Description

`Residuals.parabola` projects a given set of points onto an parabola and computing the distances from the points to the parabola

Usage

```
Residuals.parabola(XY,ParG)
```

Arguments

`XY`	array of sample data
`ParG`	vector 4x1 of the parabola parameters (Vertex(1:2), p, Angle)

Value

`list(RSS, XYproj)`
list with the Residual Sum of Squares and the array of coordinates of projections

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

References

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParG <- matrix(c(0,0,2,1,0),ncol=1)
Residuals.parabola(XY,ParG)
```

ResidualsG *Projecting a given set of points onto an ellipse*

Description

`ResidualsG` projects a given set of points onto an ellipse and computing the distances from the points to the ellipse

Usage

```
ResidualsG(XY,ParG)
```

Arguments

`XY`	array of sample data
`ParG`	vector 5x1 of the ellipse parameters (Center(1:2), Axes(1:2), Angle)

Value

`list(RSS, XYproj)`
list with the Residual Sum of Squares and the array of coordinates of projections

Author(s)

Jose Gama

Source

Nikolai Chernov, 2014 Fitting ellipses, circles, and lines by least squares http://people.cas.uab.edu/~mosya/cl/

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N. Chernov, Q. Huang, and H. Ma, 2014 Fitting quadratic curves to data points British Journal of Mathematics & Computer Science, 4, 33-60.

Examples

```
XY <- matrix(c(1,7,2,6,5,8,7,7,9,5,3,7,6,2,8,4),8,2,byrow=TRUE)
ParG <- matrix(c(0,0,2,1,0),ncol=1)
ResidualsG(XY,ParG)
```

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