Supplement: Generators and Relations for Real Stabilizer Operators

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This supplement to "Generators and Relations for Real Stabilizer Operators" establishes that the reduced relations of Definition 5.6 imply the typed relations of Definition 5.1 (as listed in the 8 figures, from Figure 3 to Figure 10 in Appendix B).

1 Proof of Proposition 5.7

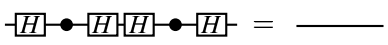
We show that the typed relations, viewed as relations between un-typed circuits, follow from the reduced relations. That is, in this appendix, we *forget* the typing.

Once typing is forgotten, the relations in Figure 6 and Figure 7 coincide so that it is sufficient to derive those in Figure 6. Similarly, of the first 8 rules in Figure 4, the left column is identical to the right column as un-typed rules, so we only prove the left column. Lastly, the next 12 rules of Figure 6 are identical to the 12 rules following them as un-typed rules, so we only derive the first 12 of them.

We start by proving a few useful lemmas before deriving the typed relations. For each one of Figures 3–10, we number the relations left-to-right and row-by-row in order to conveniently refer to them.

1.1 Lemmas

Lemma 1.1. *The following is a consequence of R2 and R3.*



(L1)

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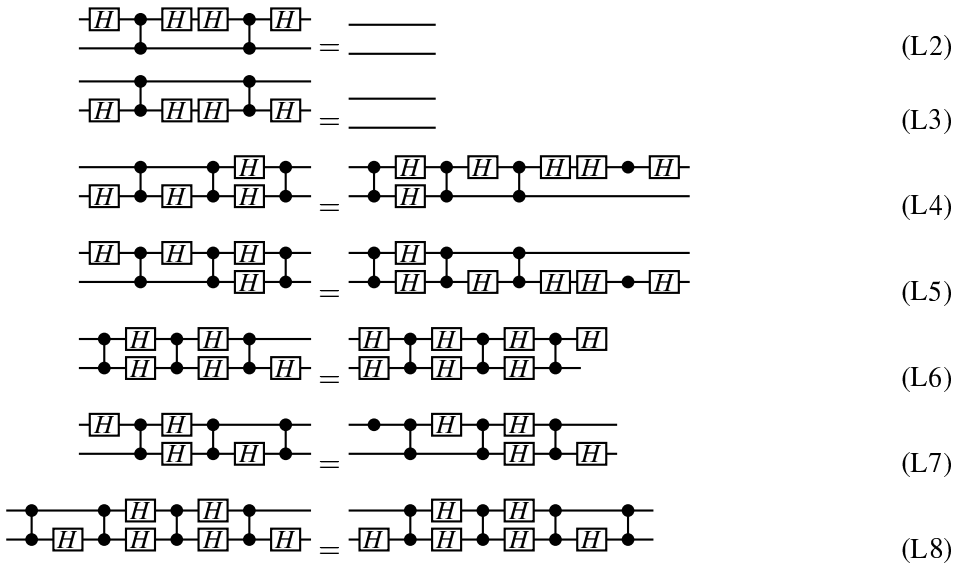
Lemma 1.2. *The following rules are a consequence of* $R1 - R18$



(L2)

(L3)

(L4)

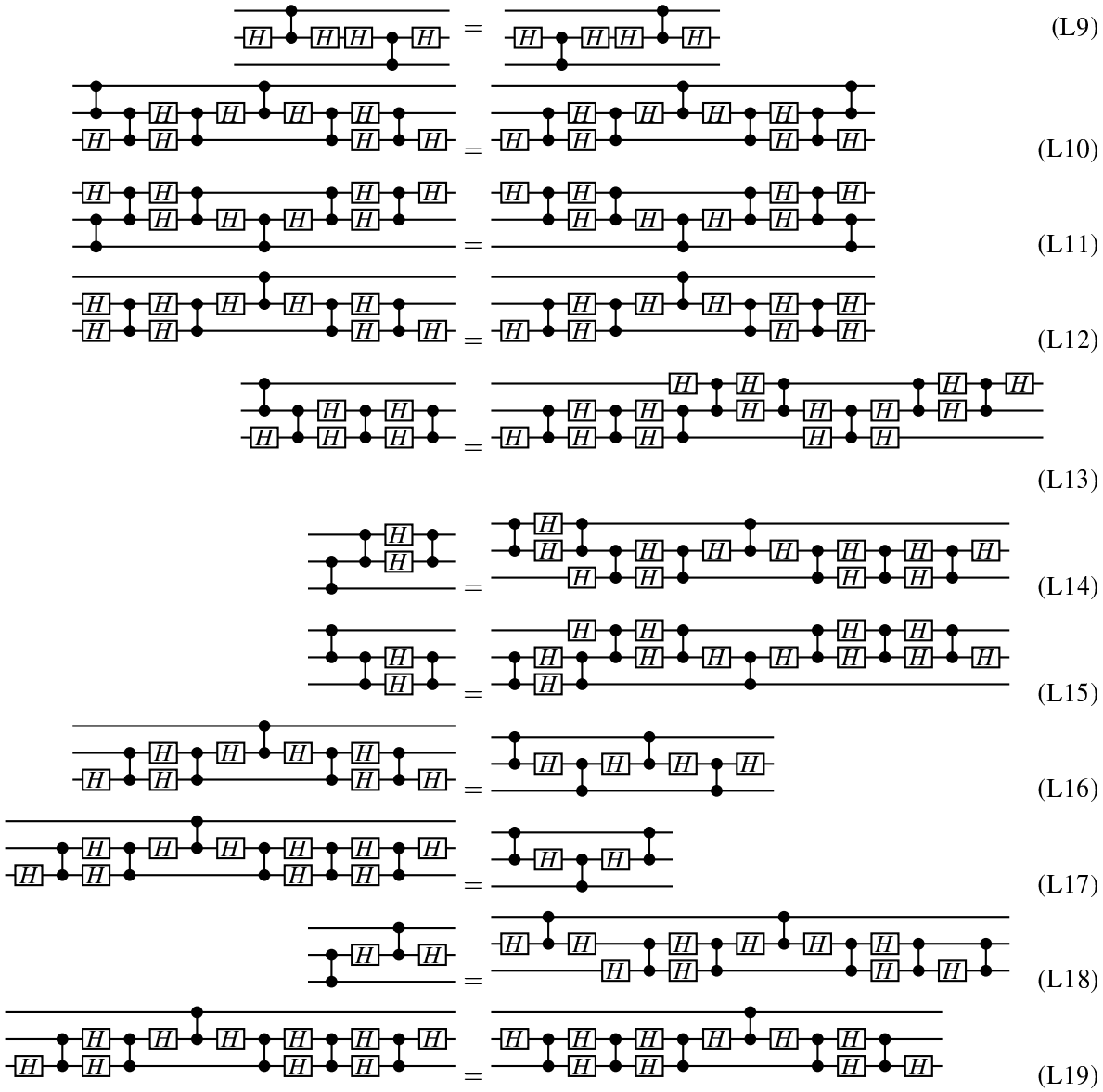
(L5)

(L6)

(L7)

(L8)

Lemma 1.3. *The following rules are a consequence of* $R1 - R18$



(L9)

(L10)

(L11)

(L12)

(L13)

(L14)

(L15)

(L16)

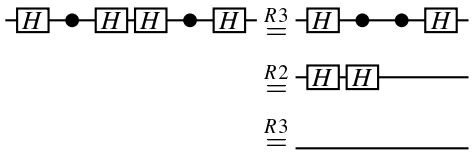
(L17)

(L18)

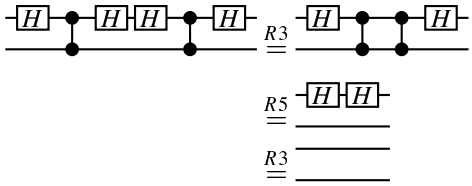
(L19)

1.2 Proofs of Lemmas

L1:



L2:



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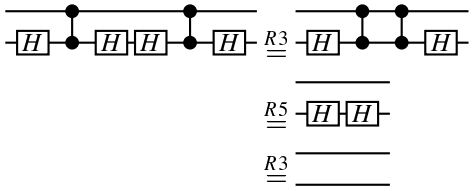

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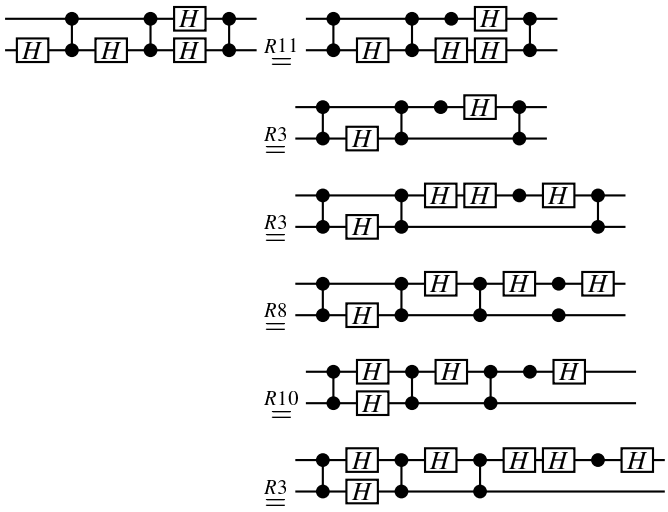

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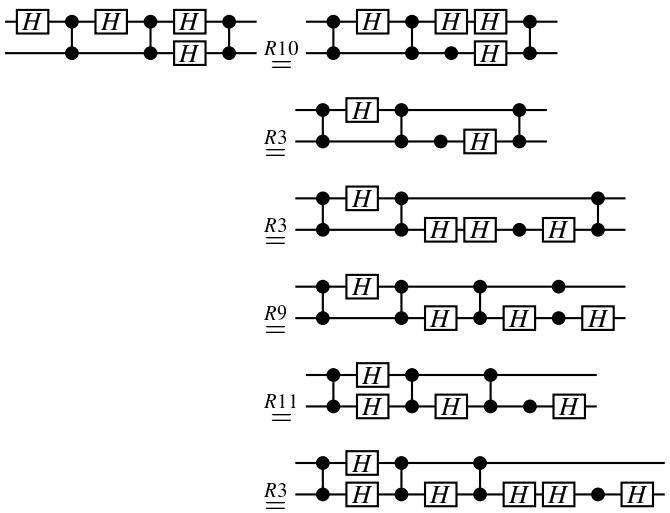

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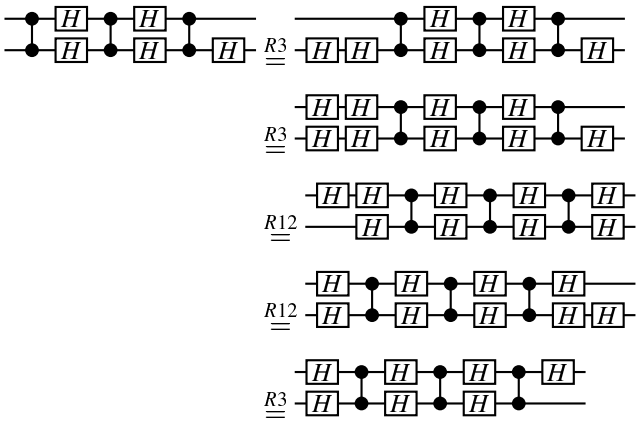

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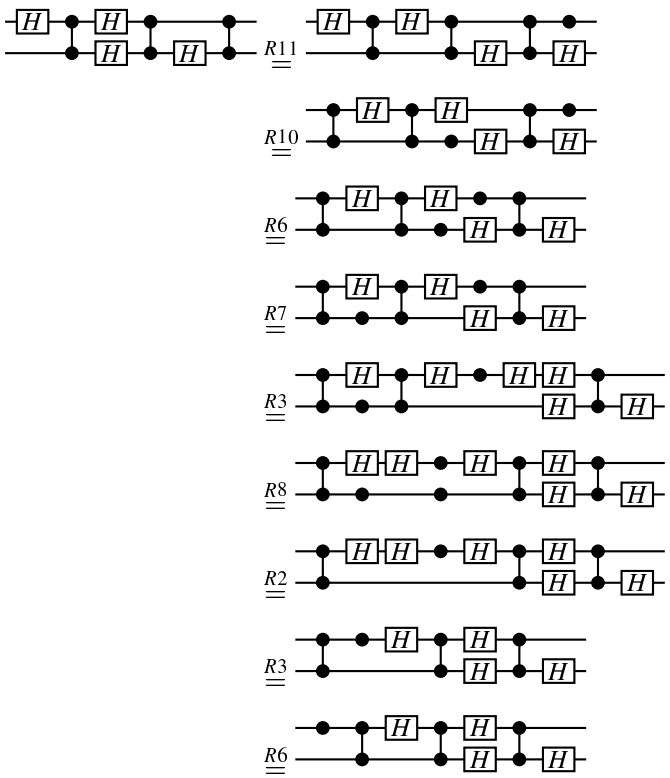

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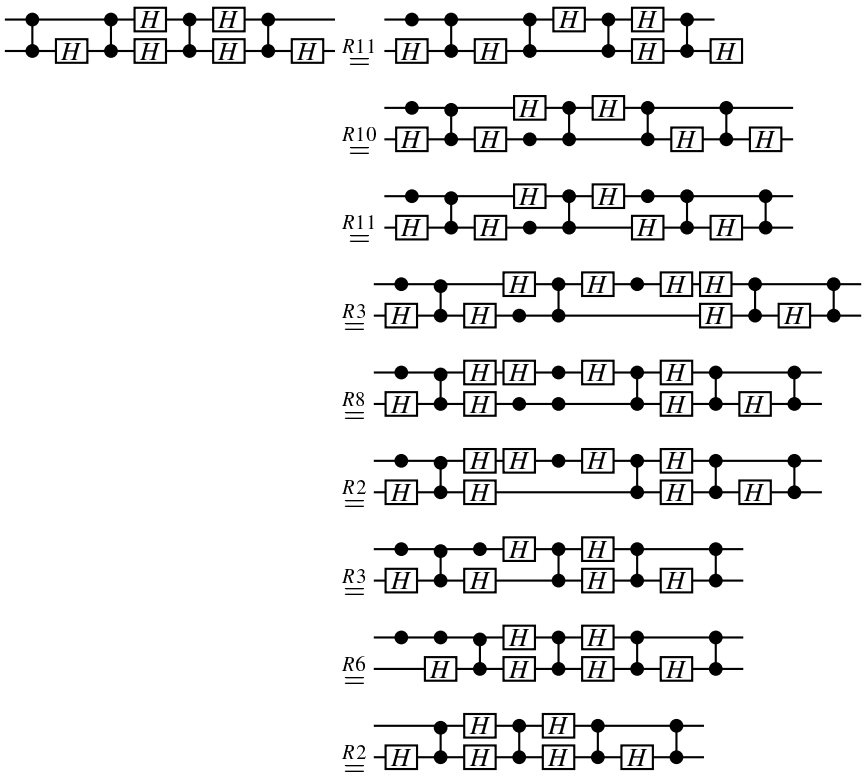

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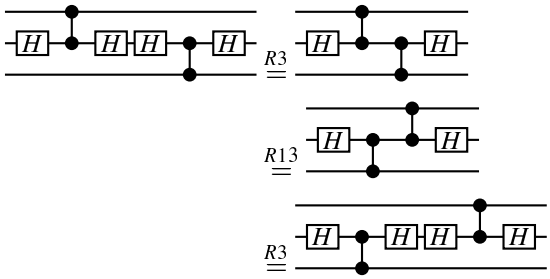

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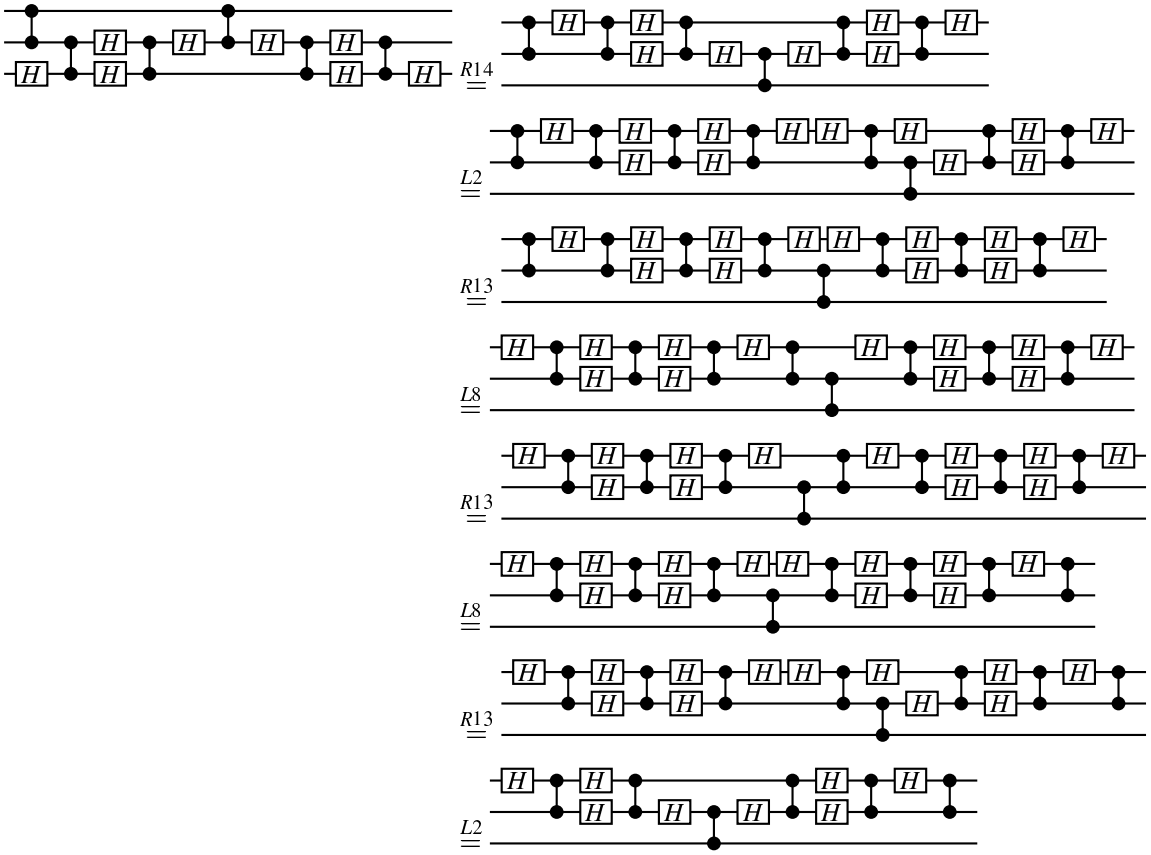

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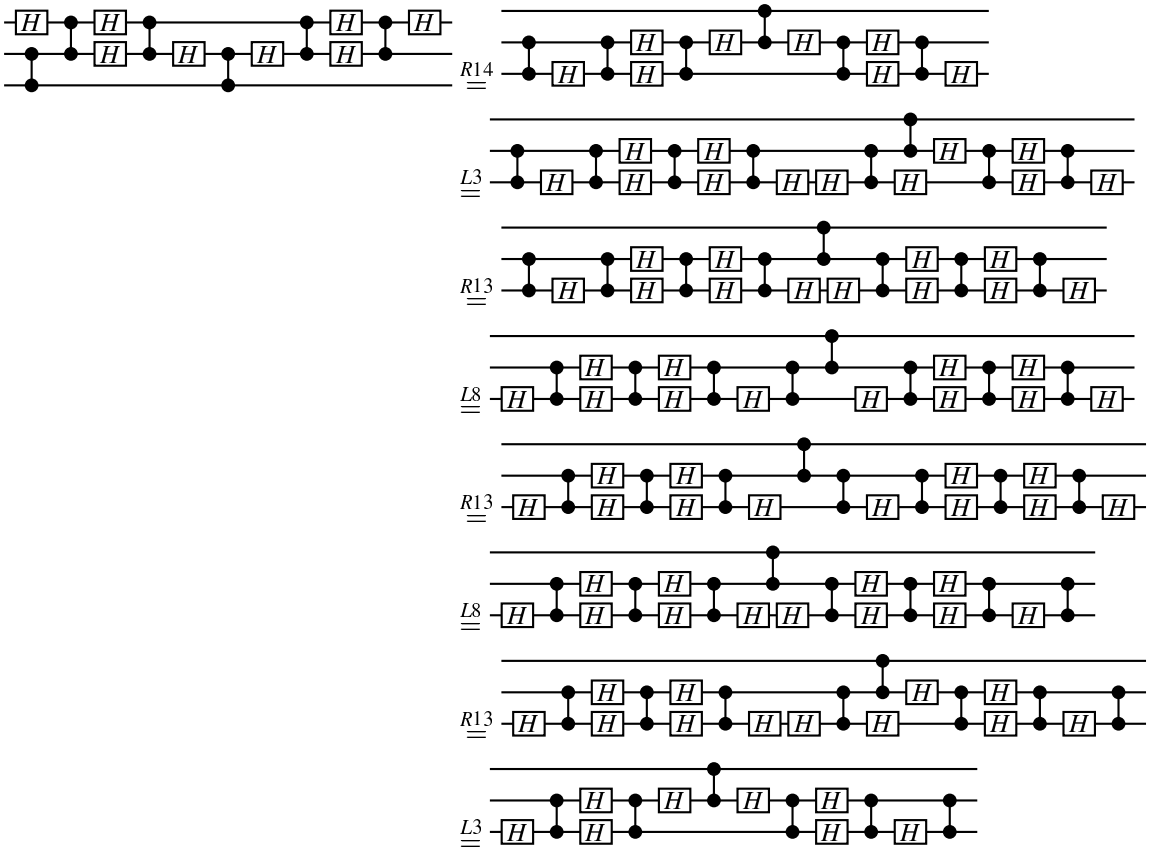

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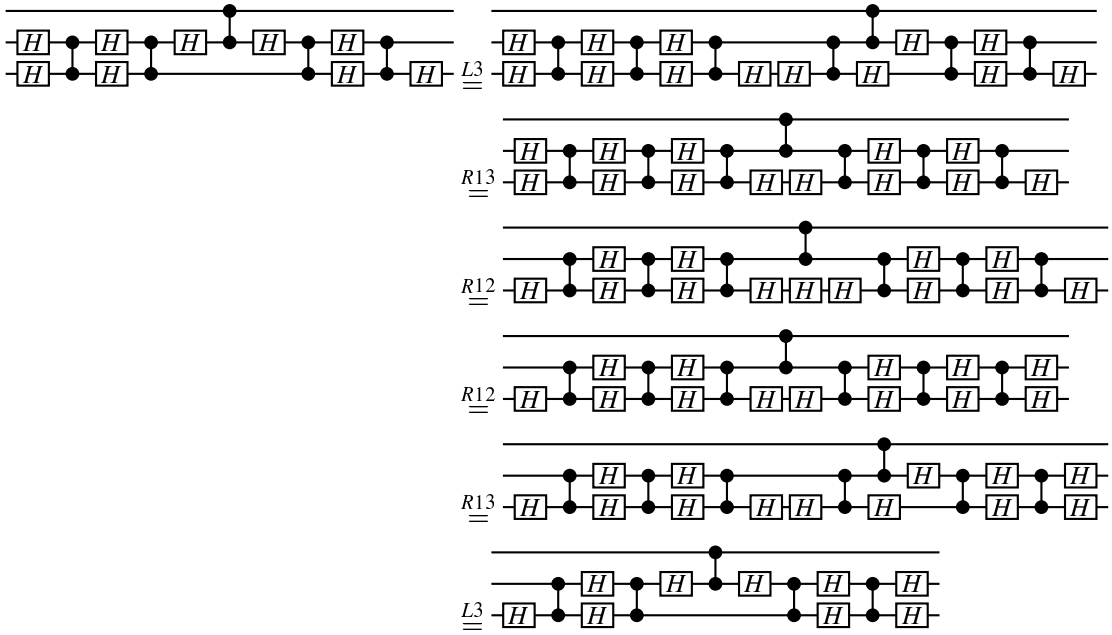

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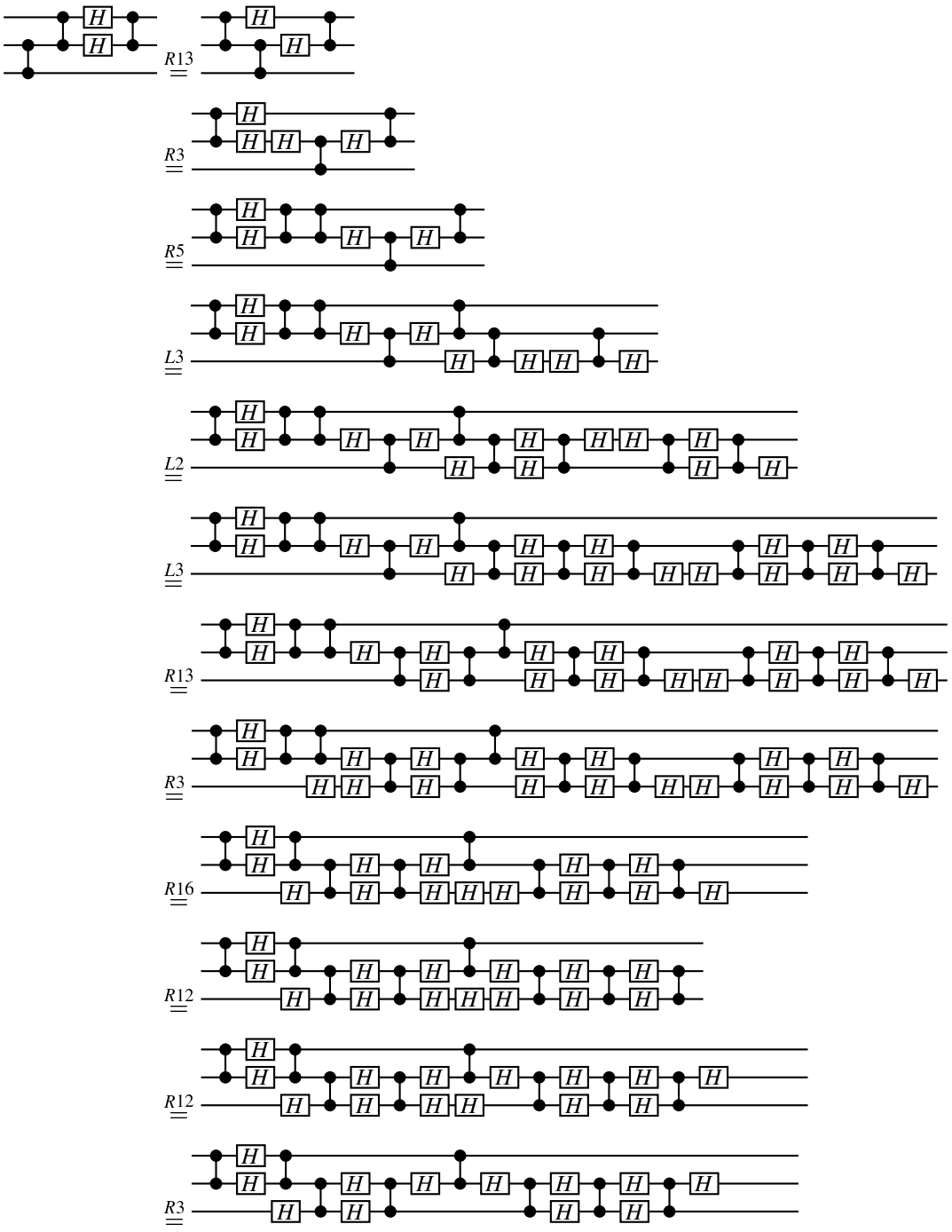

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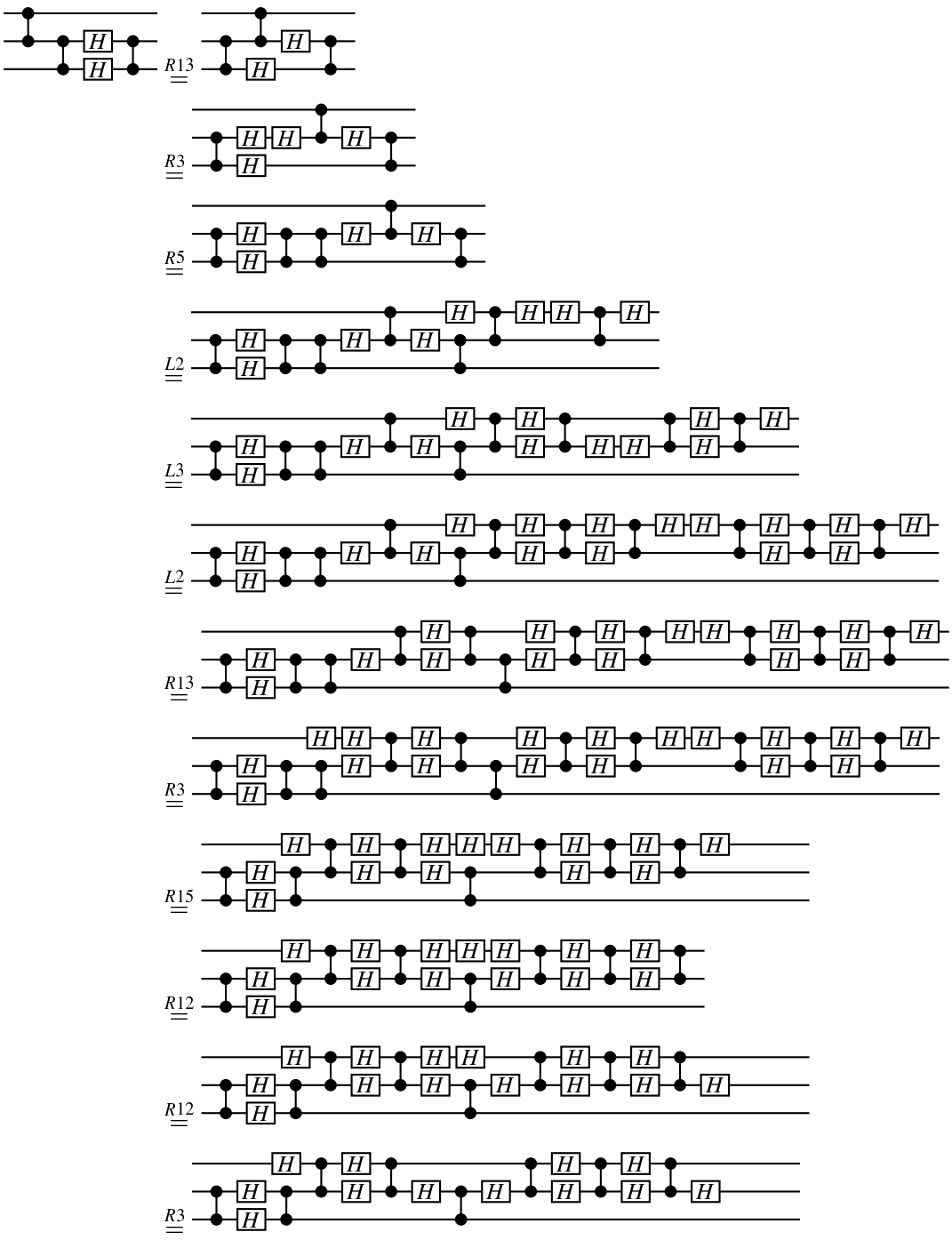

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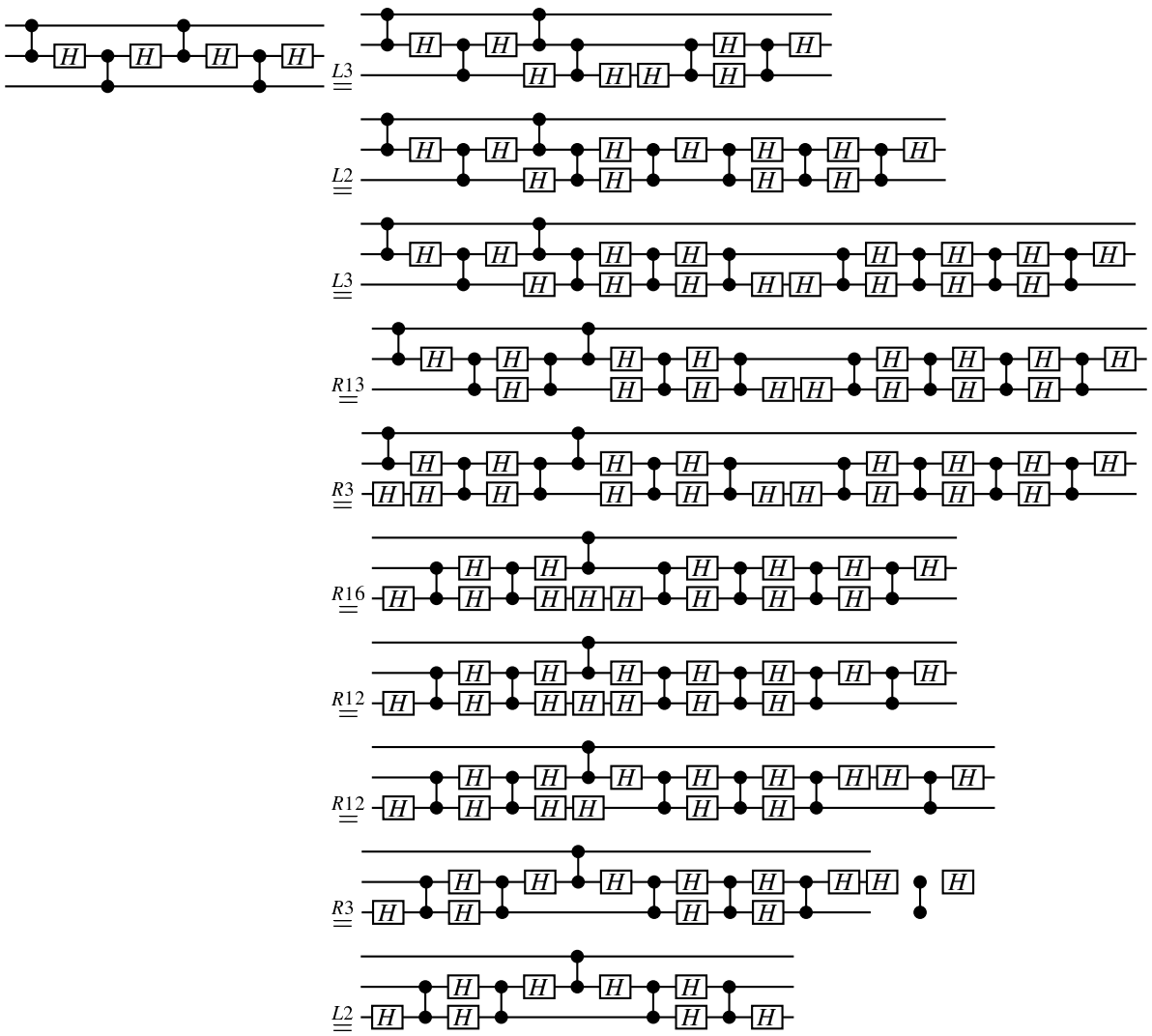

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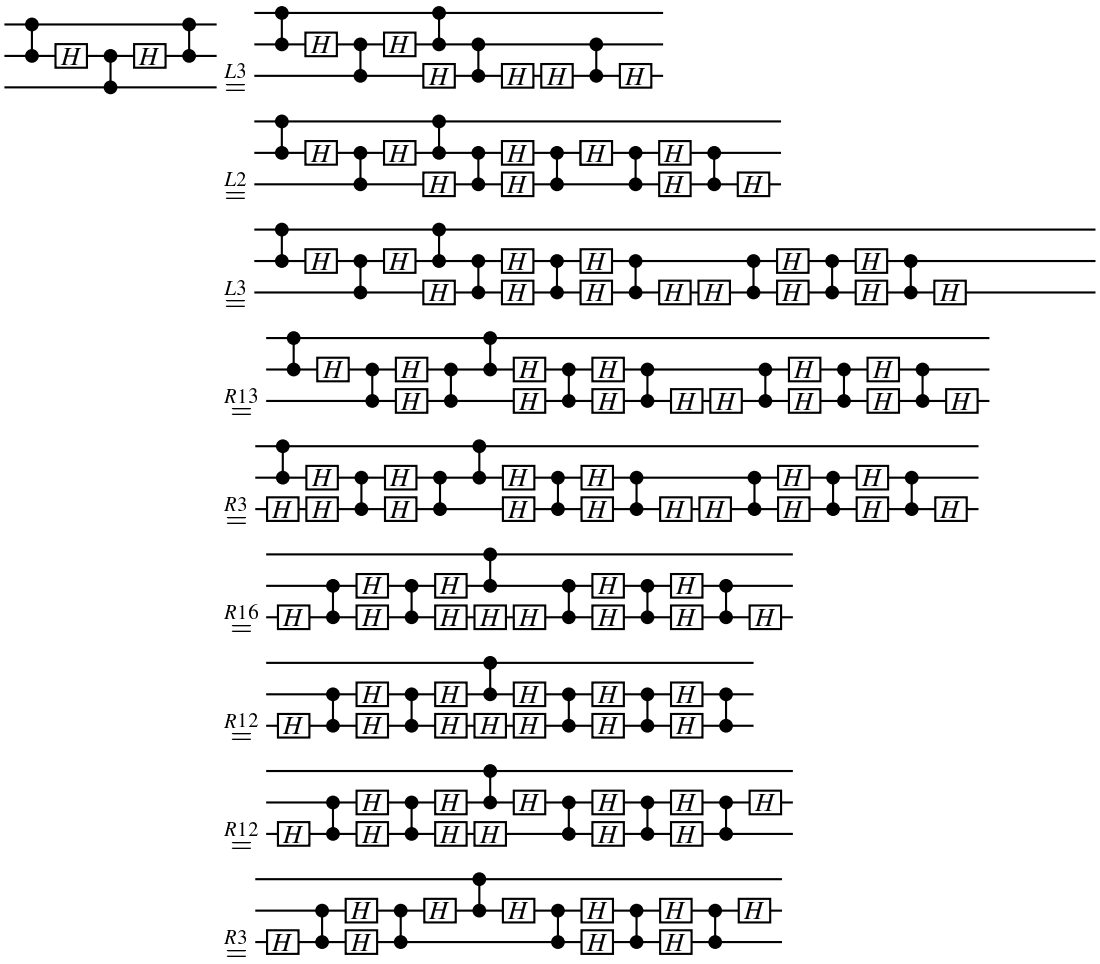

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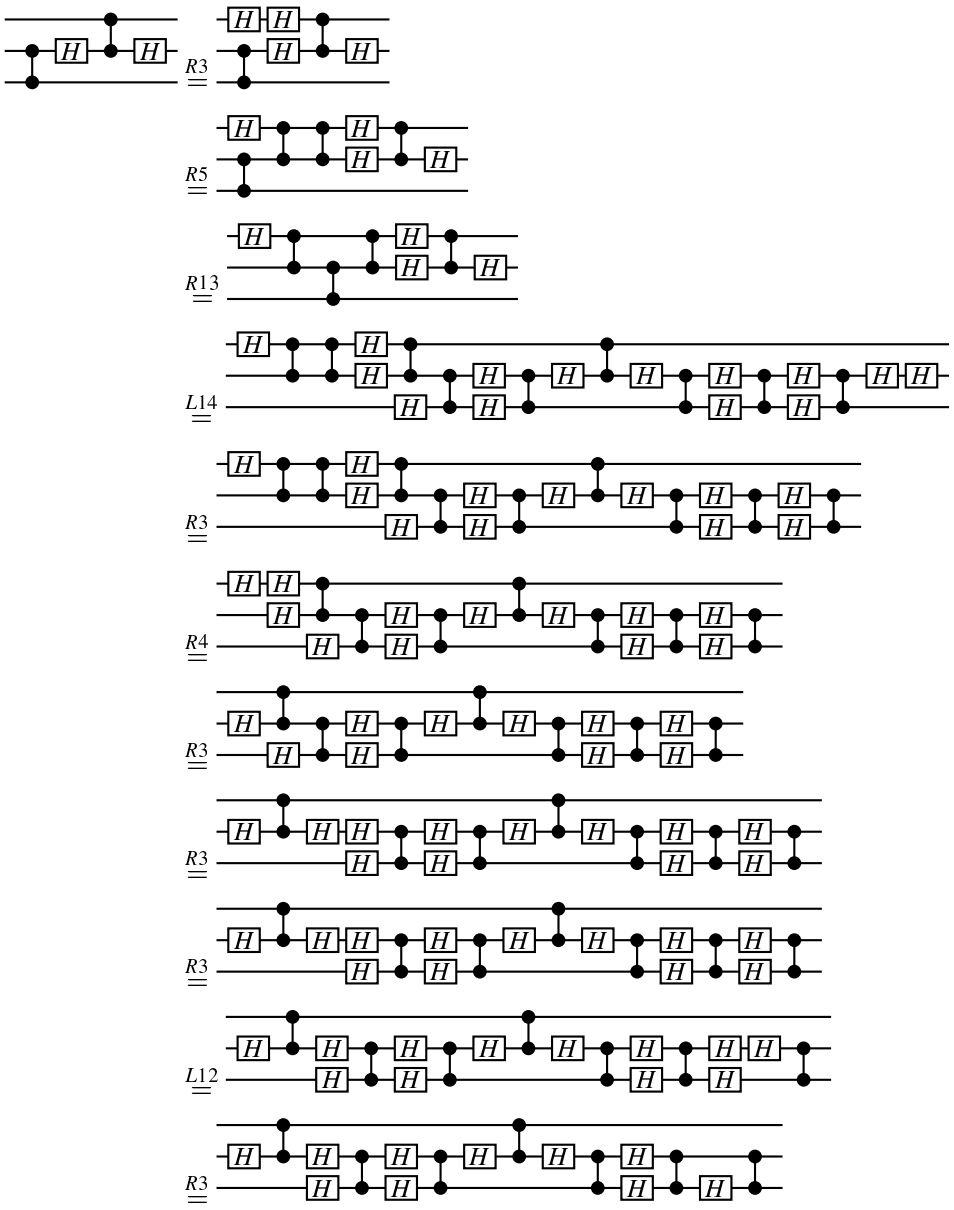

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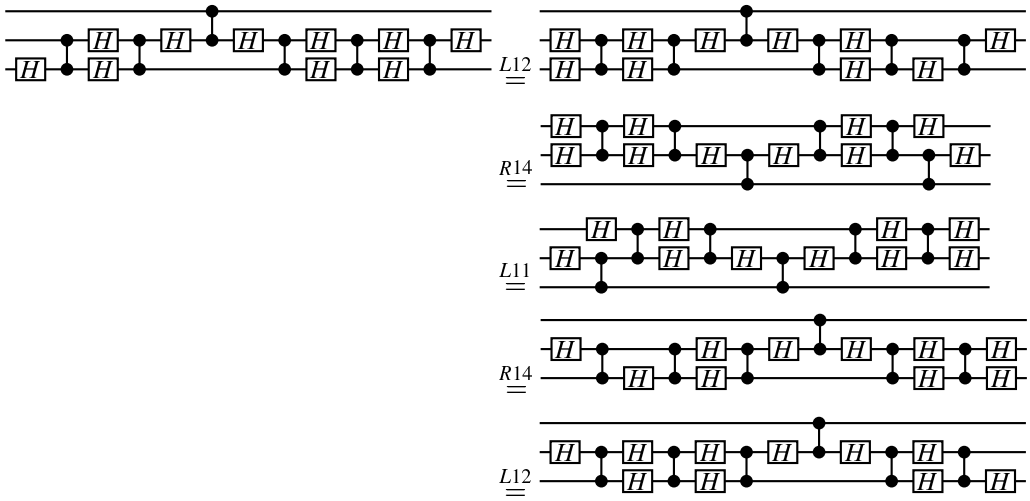

1.3 Proofs of Rewrite Rules

Figure 3:

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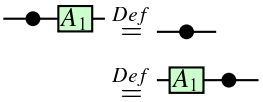

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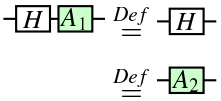

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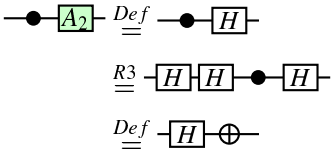

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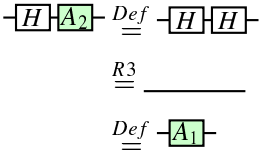

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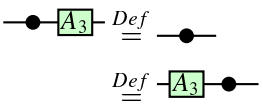
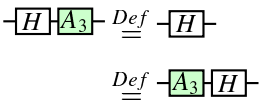
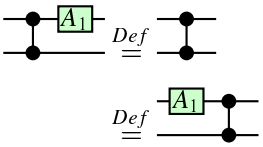

A_3
Def
Def
A_3

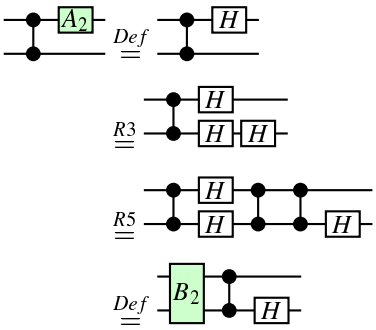
3.6


H
A_3
Def
H
Def
A_3
H

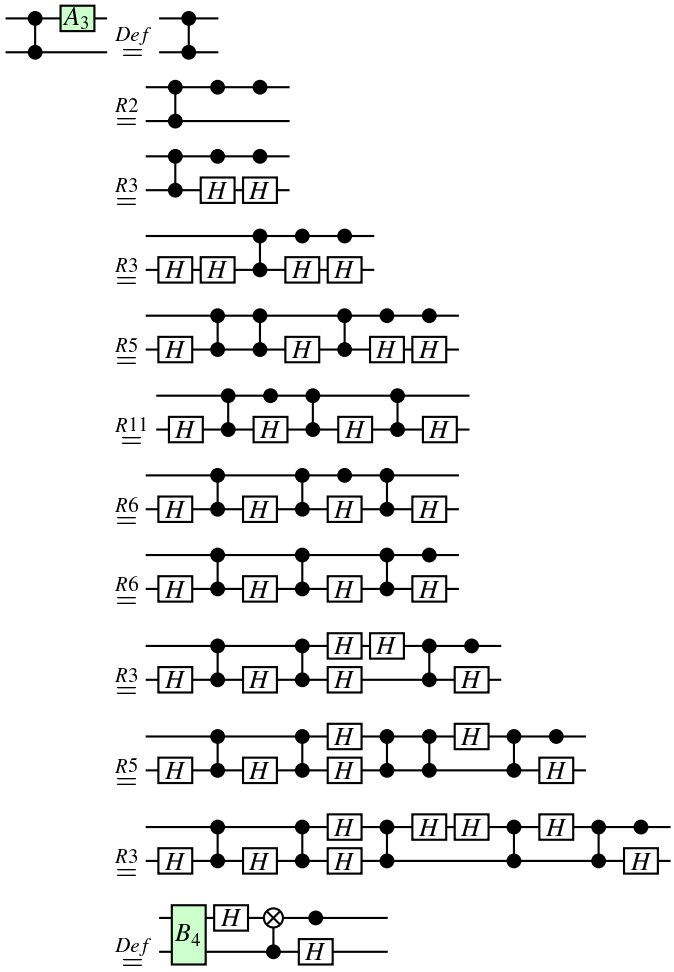
3.7


A_1
Def
Def
A_1

3.8


A_2
Def
H
H
R3
H
H
H
R5
H
H
Def
B_2
H

3.9



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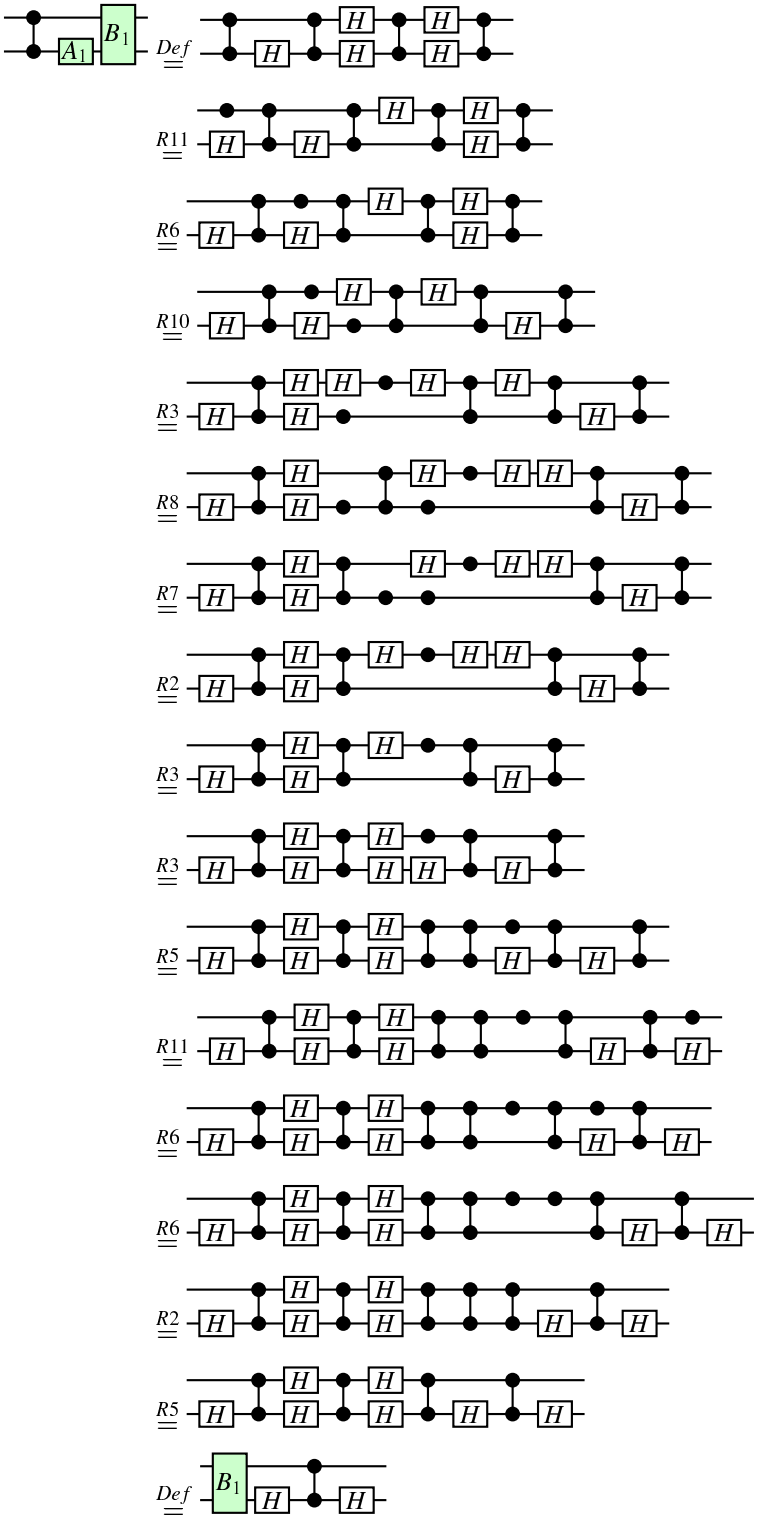

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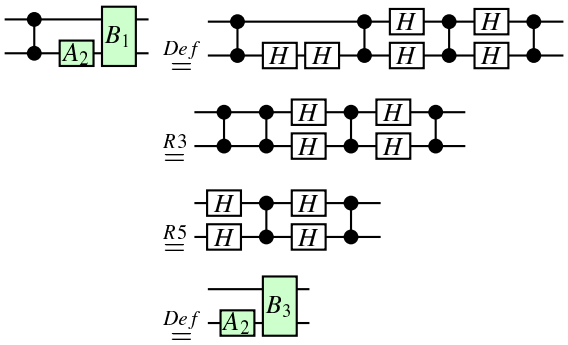
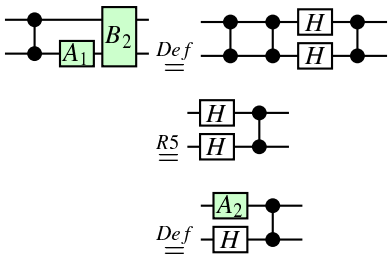
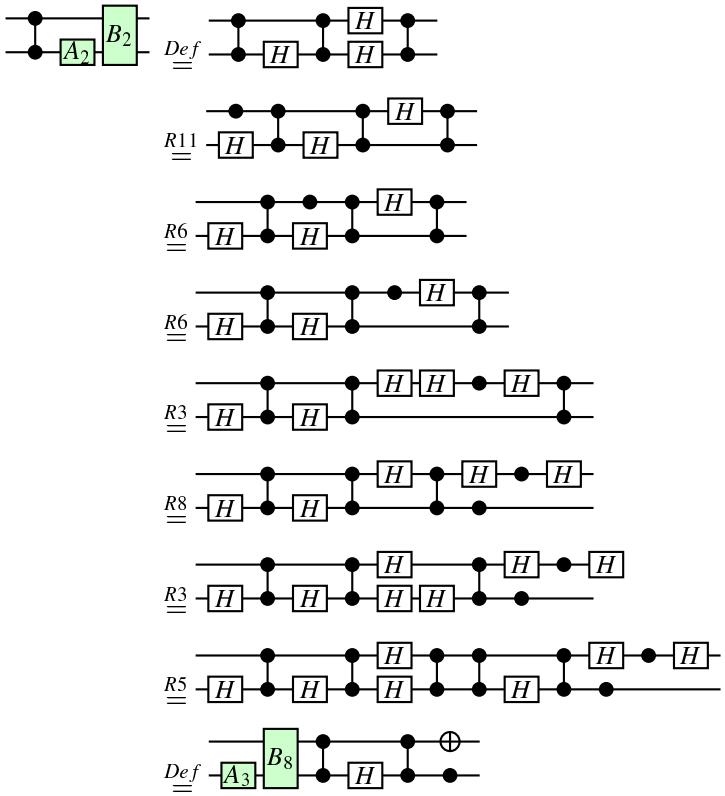

B_1
A_2
Def
H
$R3$
$R5$
Def
A_2
B_3

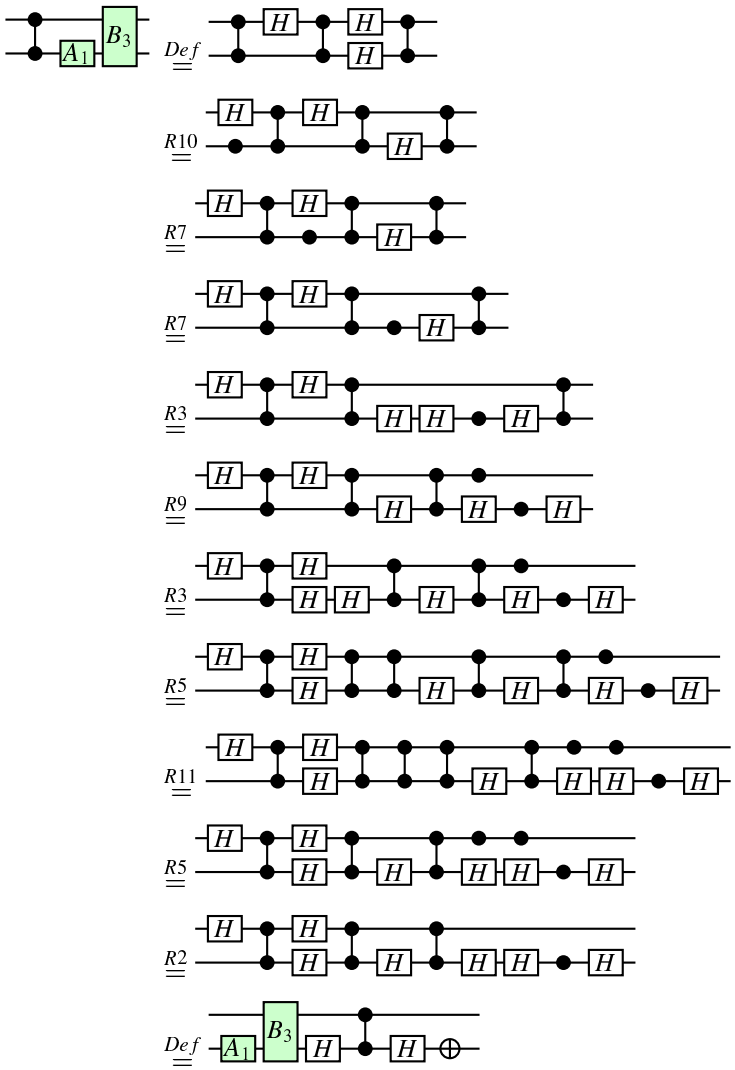
3.12


B_2
A_1
Def
H
$R5$
A_2
Def

3.13


B_2
A_2
Def
$R11$
$R6$
$R6$
$R3$
$R8$
$R3$
$R5$
B_8
A_3
Def

3.14



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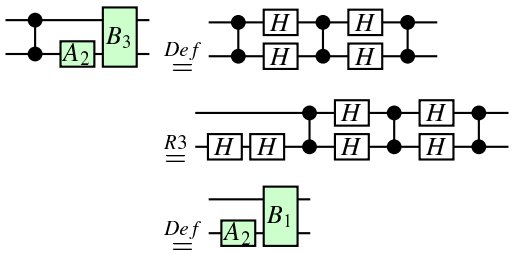

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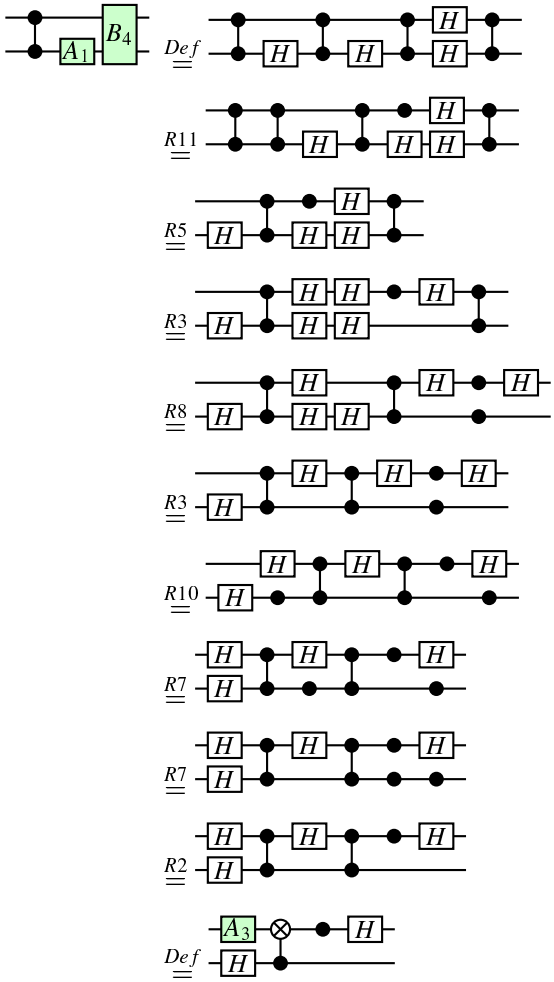

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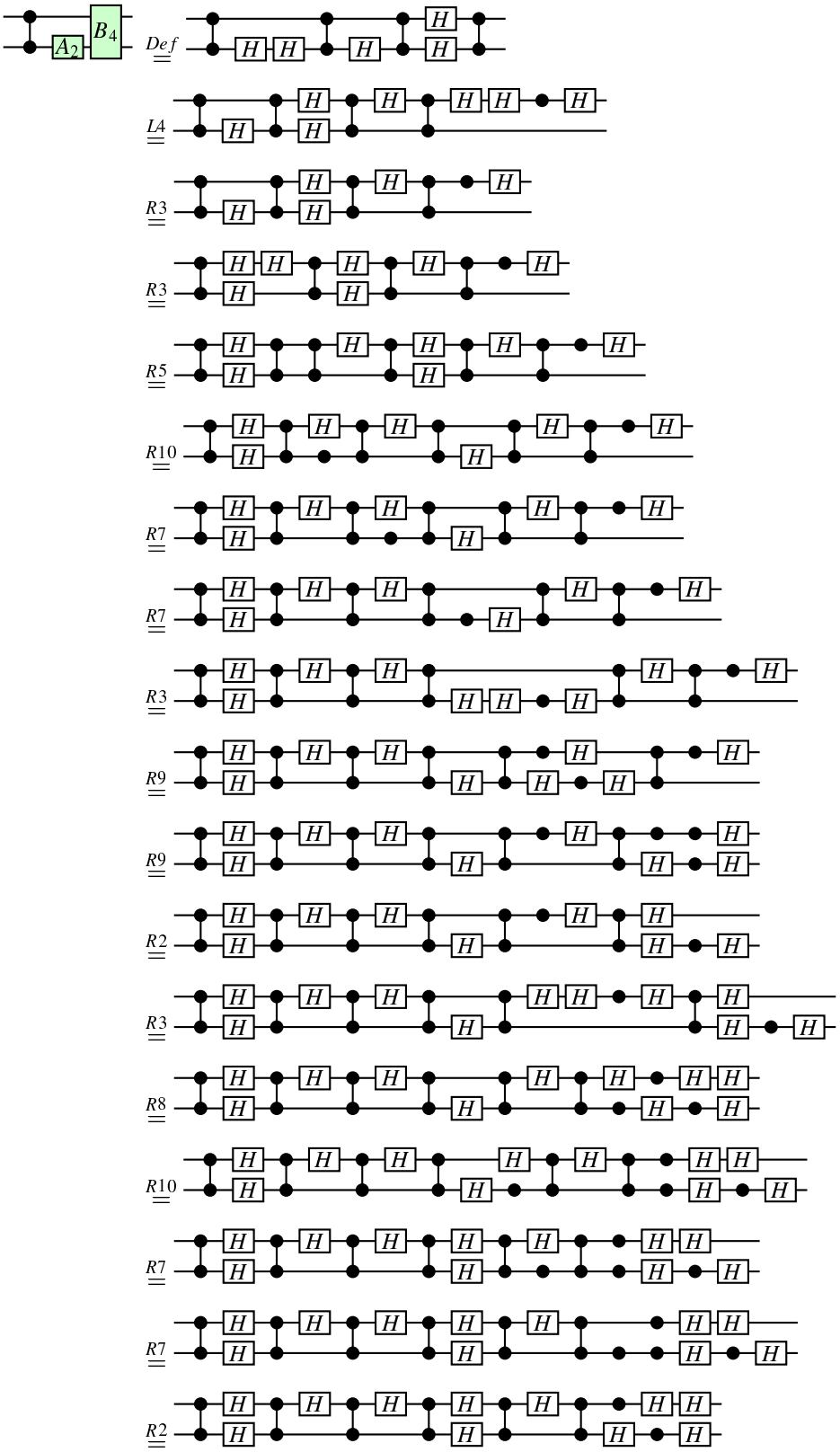
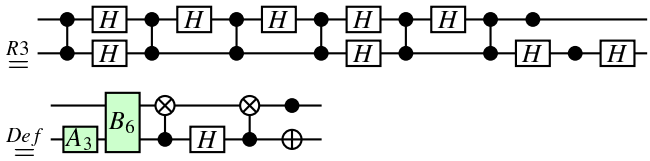
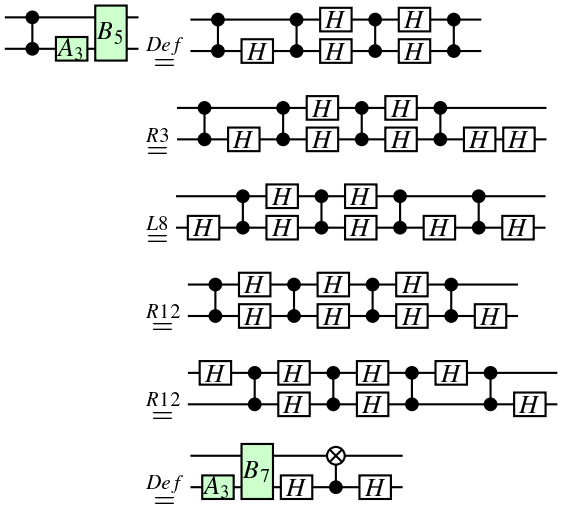
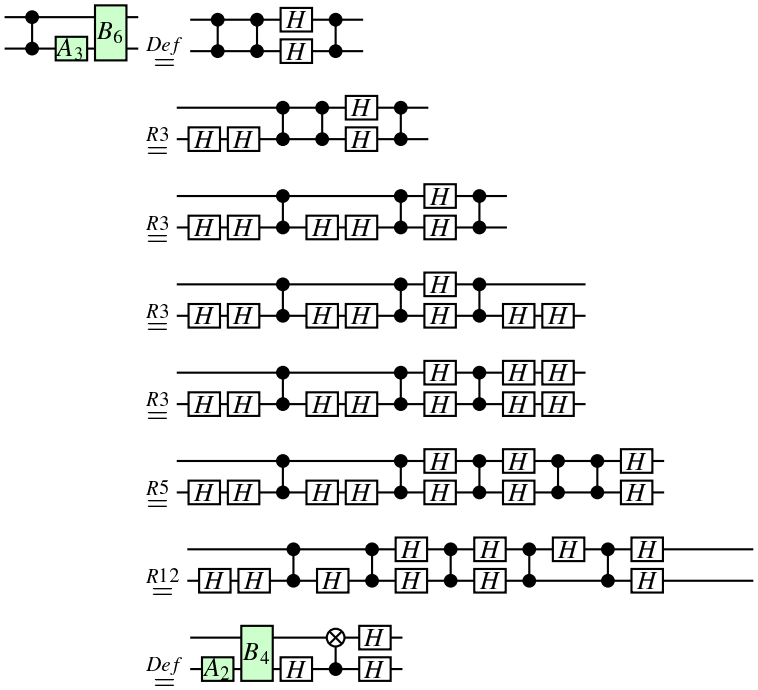



R3
Def
A3
B6
H

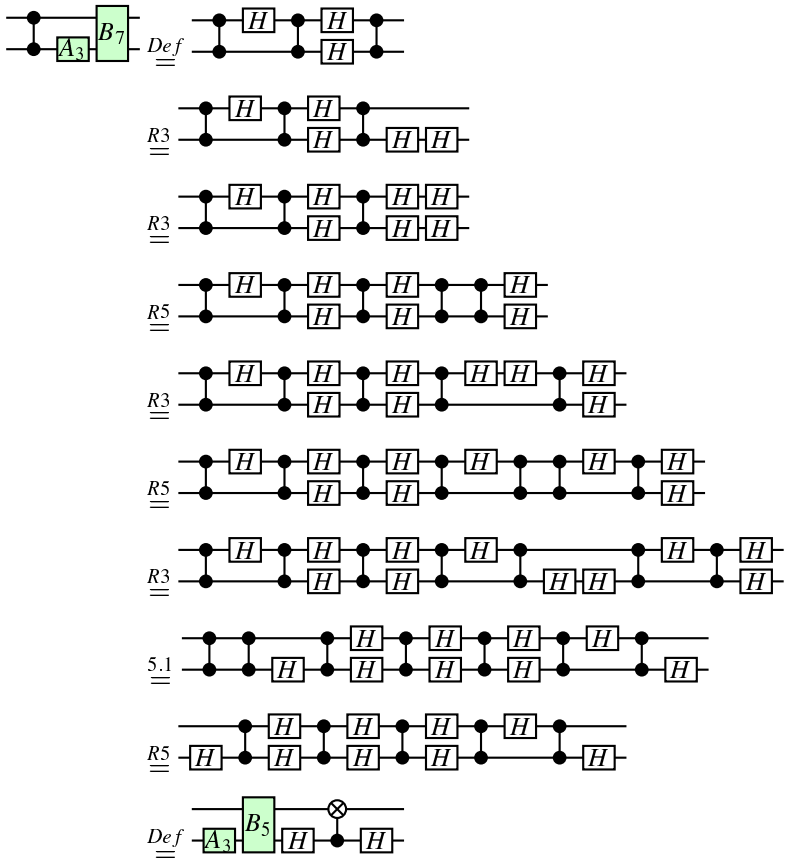
3.18


A3
B5
Def
R3
L8
R12
R12
Def
A3
B7
H

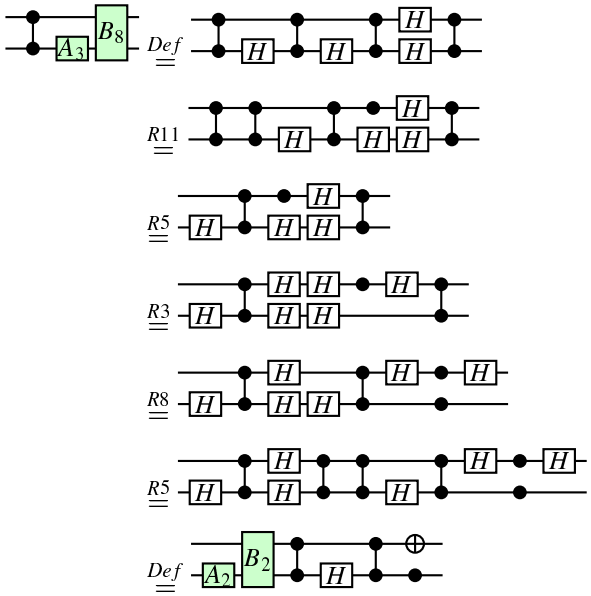
3.19


A3
B6
Def
R3
R3
R3
R3
R5
R12
Def
A2
B4
H

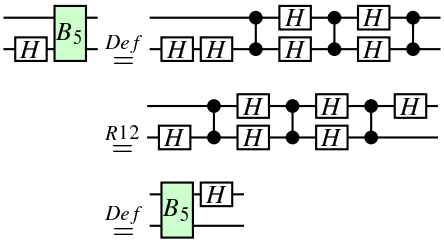
3.20


A_3 B_7 Def
R3
R3
R5
R3
R5
R3
5.1
R5
Def A_3 B_5

3.21


A_3 B_8 Def
R11
R5
R3
R8
R5
Def A_2 B_2

3.22



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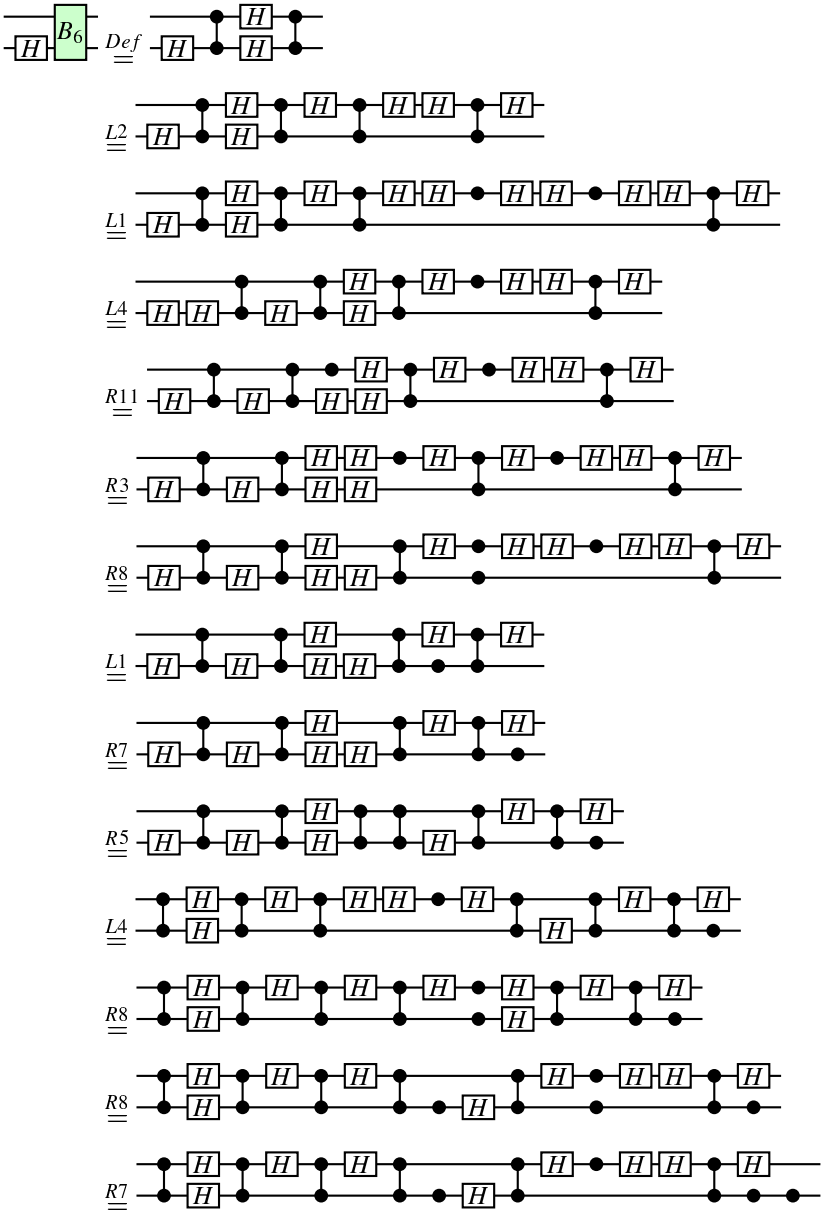
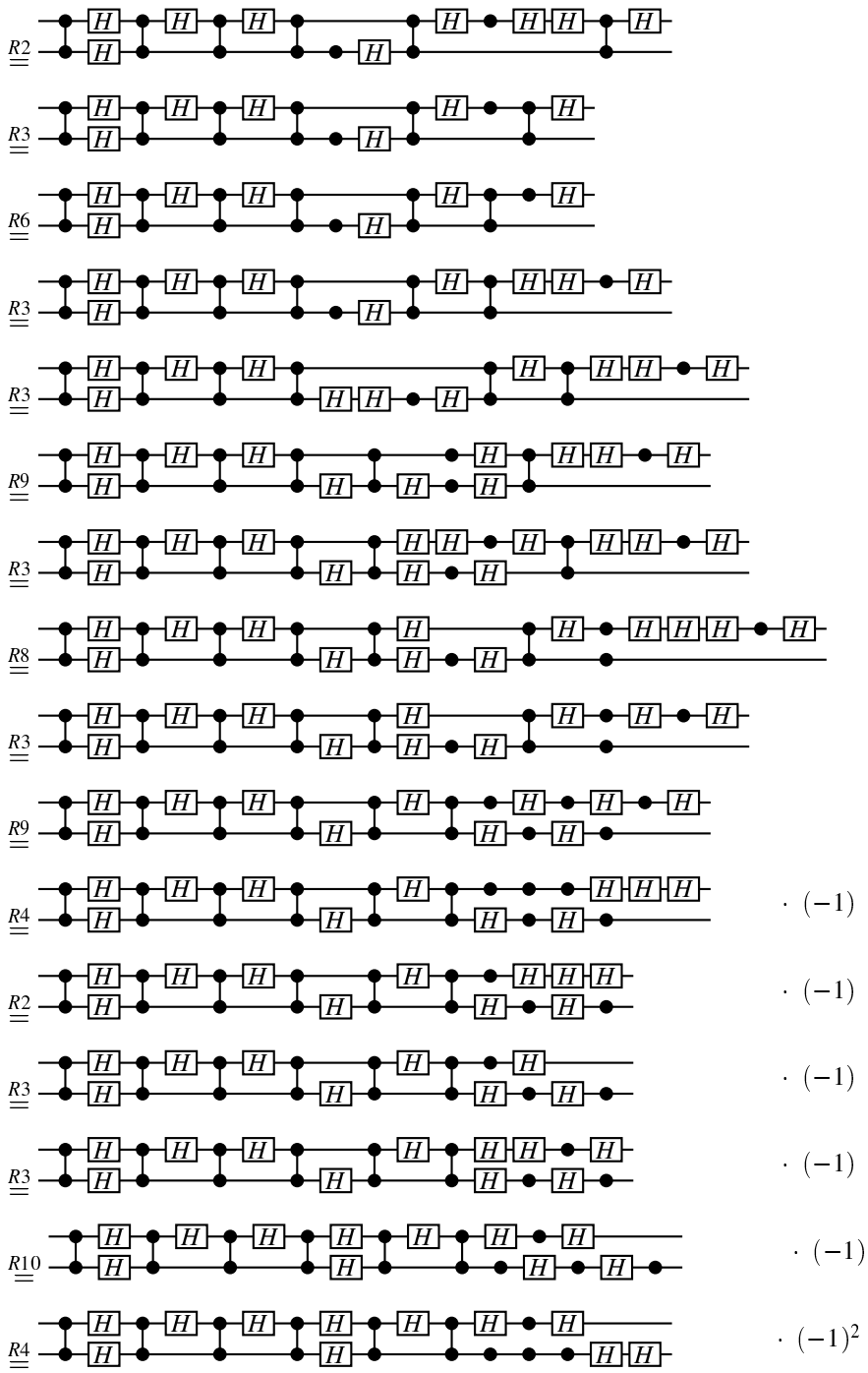
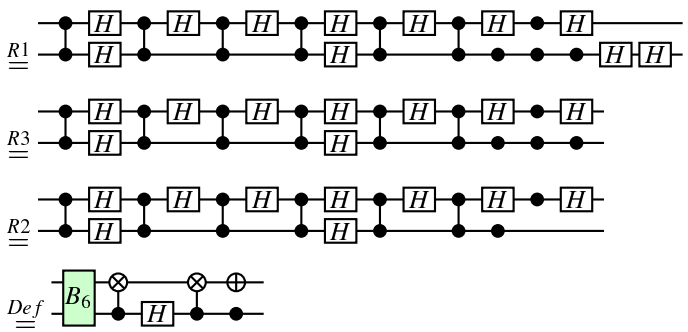



R2
R3
R6
R3
R3
R9
R3
R8
R3
R9
R4
R2
R3
R3
R10
R4
· (−1)
· (−1)
· (−1)
· (−1)
· (−1)
· (−1)²



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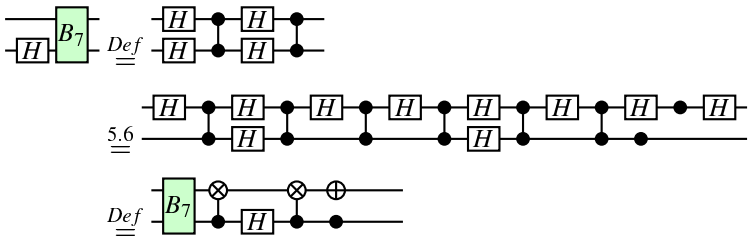

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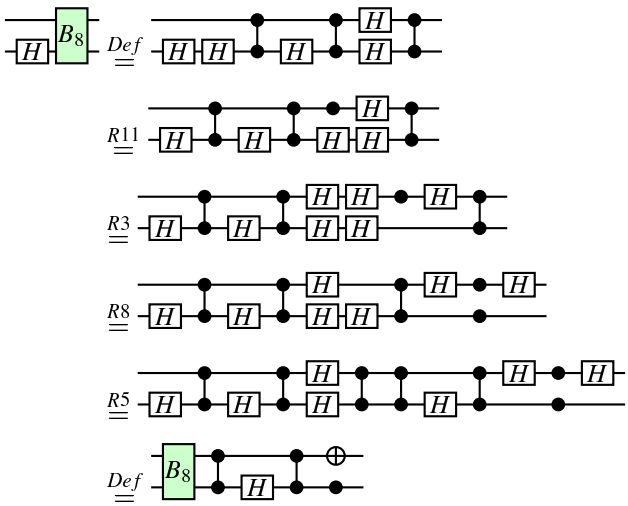
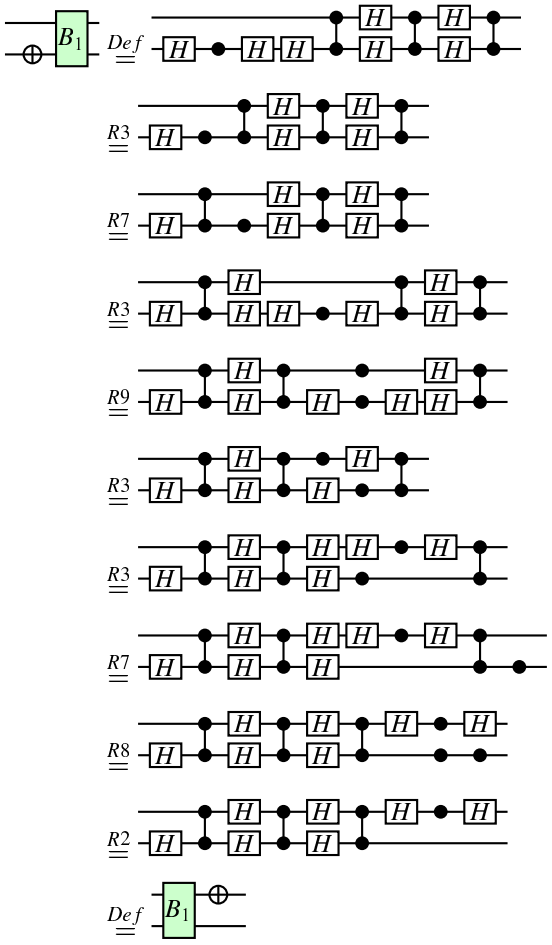
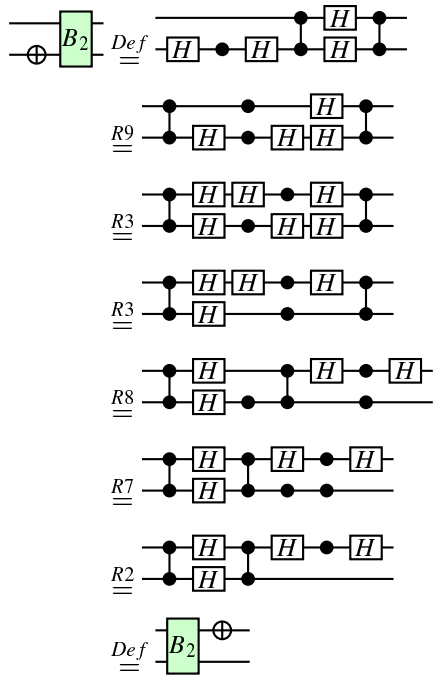


Figure 4:

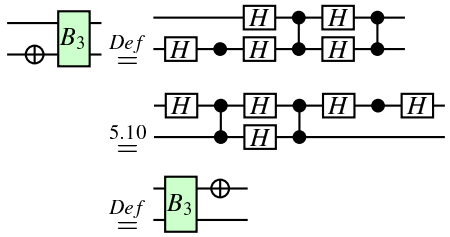
4.1



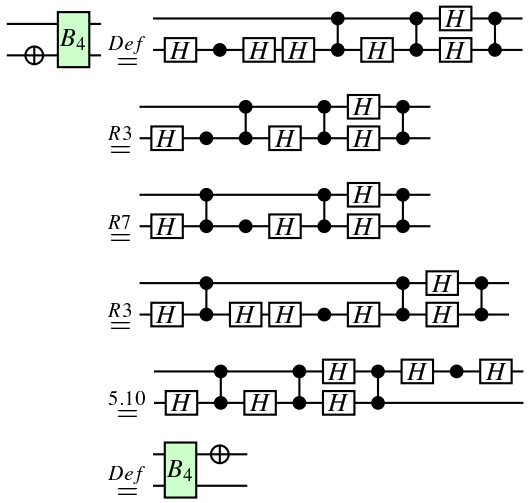
4.2


B_2
Def
H
$R9$
$R3$
$R3$
$R8$
$R7$
$R2$
Def
B_2

4.3

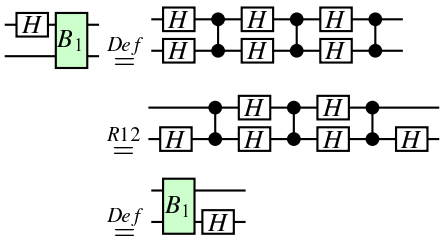

B_3
Def
H
5.10
Def
B_3

4.4

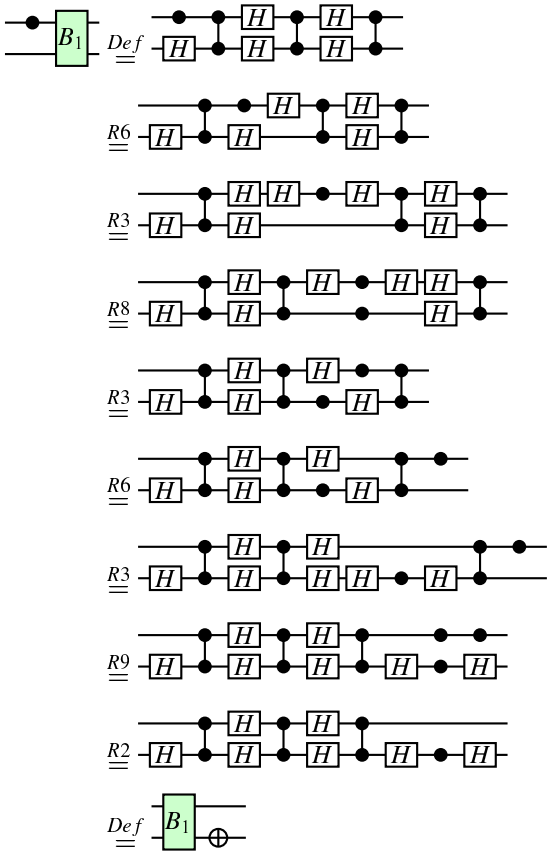

B_4
Def
H
$R3$
$R7$
$R3$
5.10
Def
B_4

The proofs of 4.5–4.8 are the same as those of 4.1–4.4.

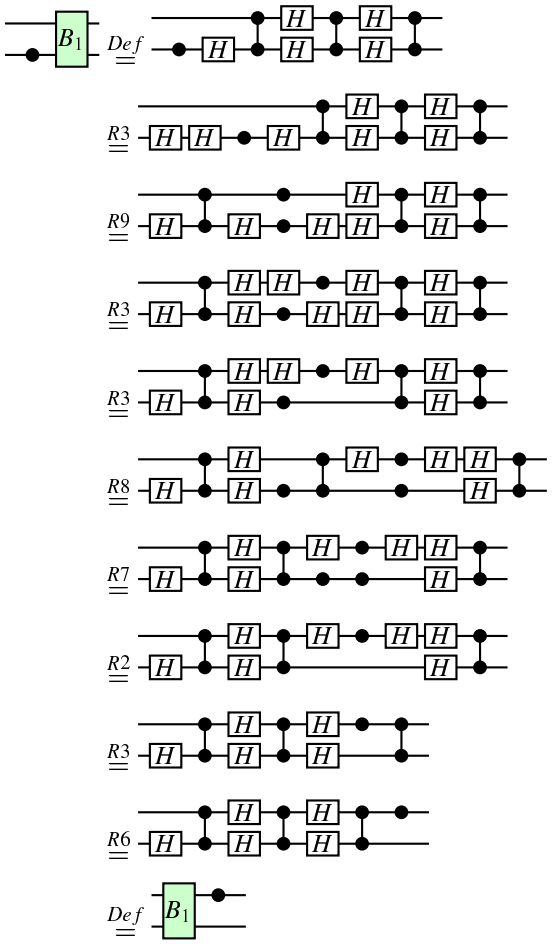
4.9



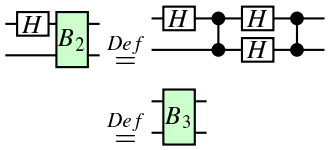
4.10



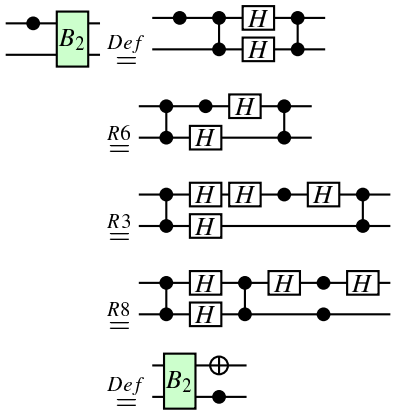
4.11



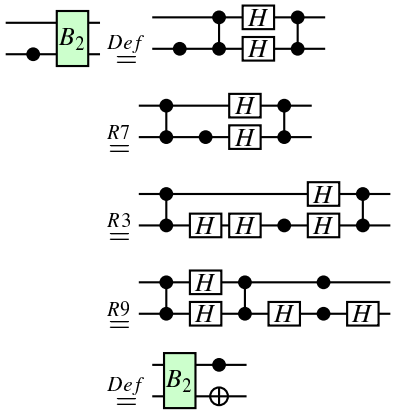
4.12



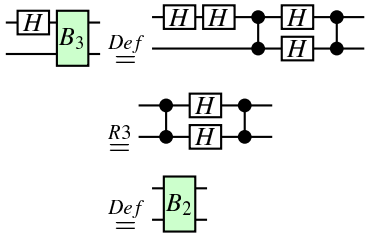
4.13


B2
Def
H
H
R6
H
H
R3
H
H
H
H
R8
H
H
H
H
Def
B2

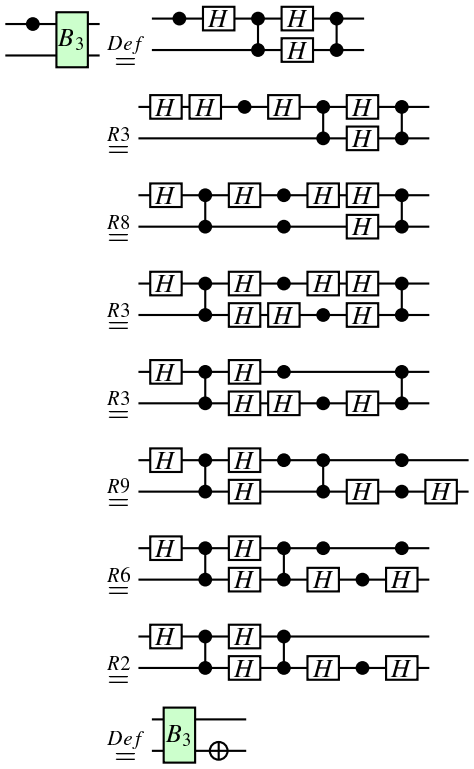
4.14


B2
Def
H
H
R7
H
H
R3
H
H
H
H
R9
H
H
H
H
Def
B2

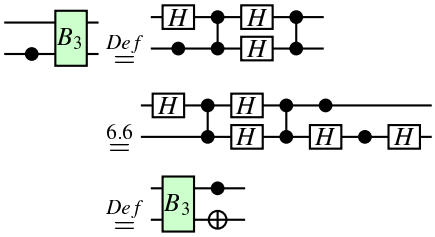
4.15


H
B3
Def
H
H
H
H
R3
H
H
Def
B2

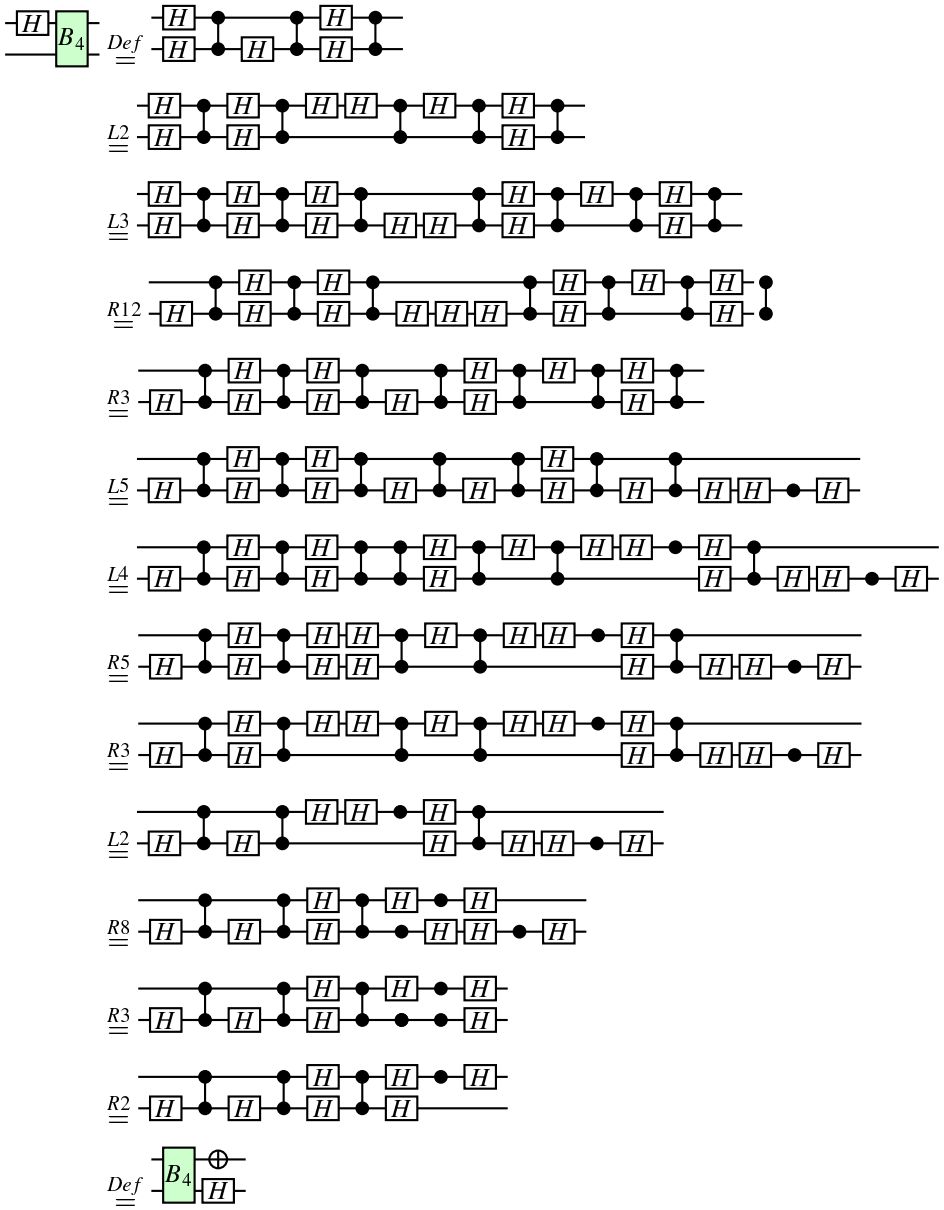
4.16



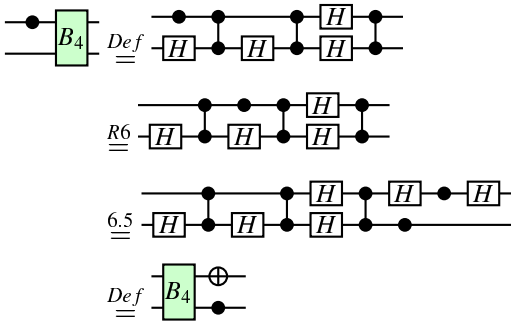
4.17



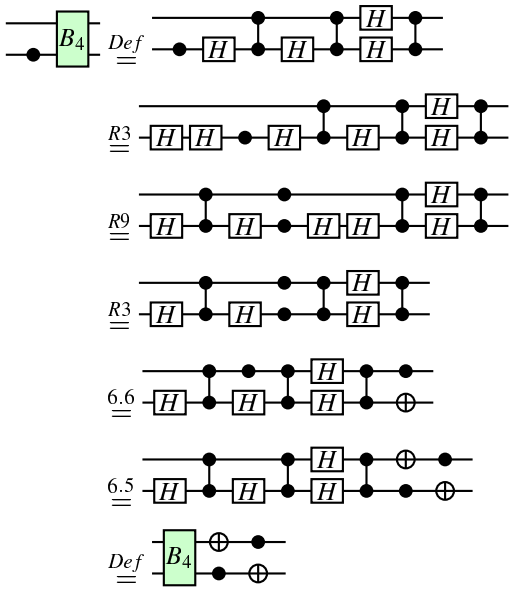
4.18



4.19

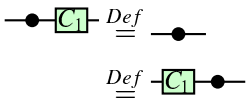


4.20

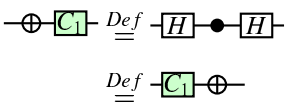


The proofs of 4.21–4.32 are the same as those of 4.9–4.20.

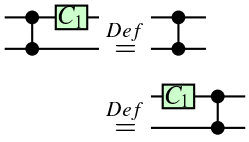
4.33



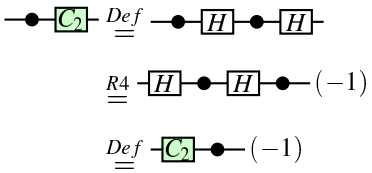
4.34



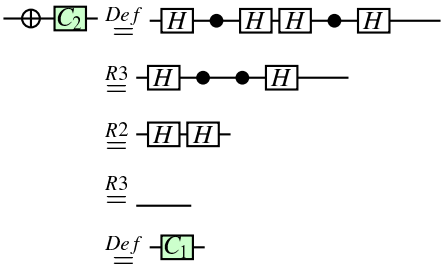
4.35



4.36



4.37



4.38

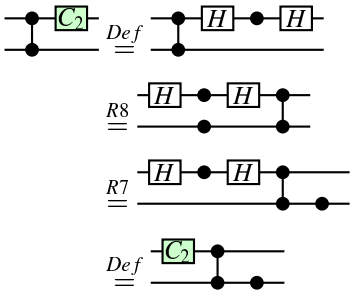
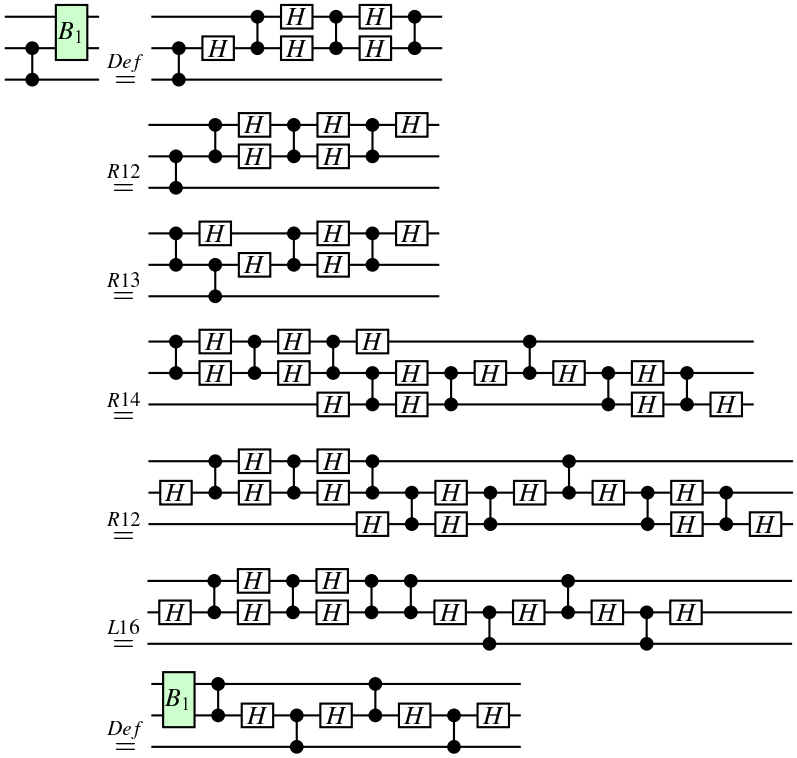
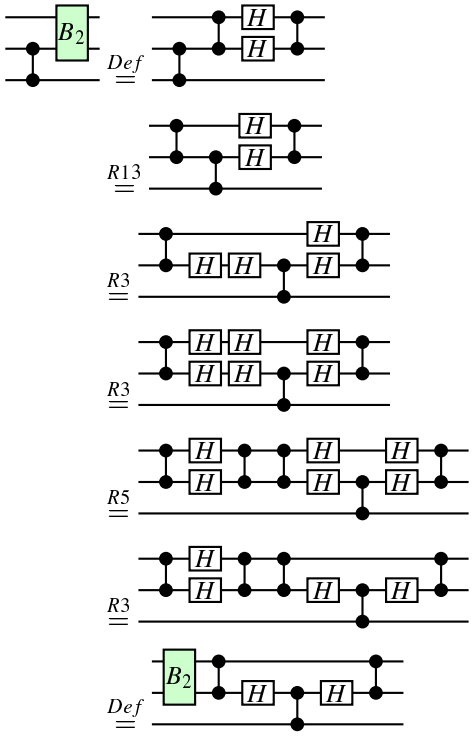


Figure 5:

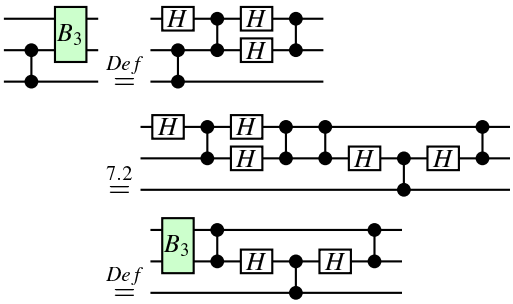
5.1



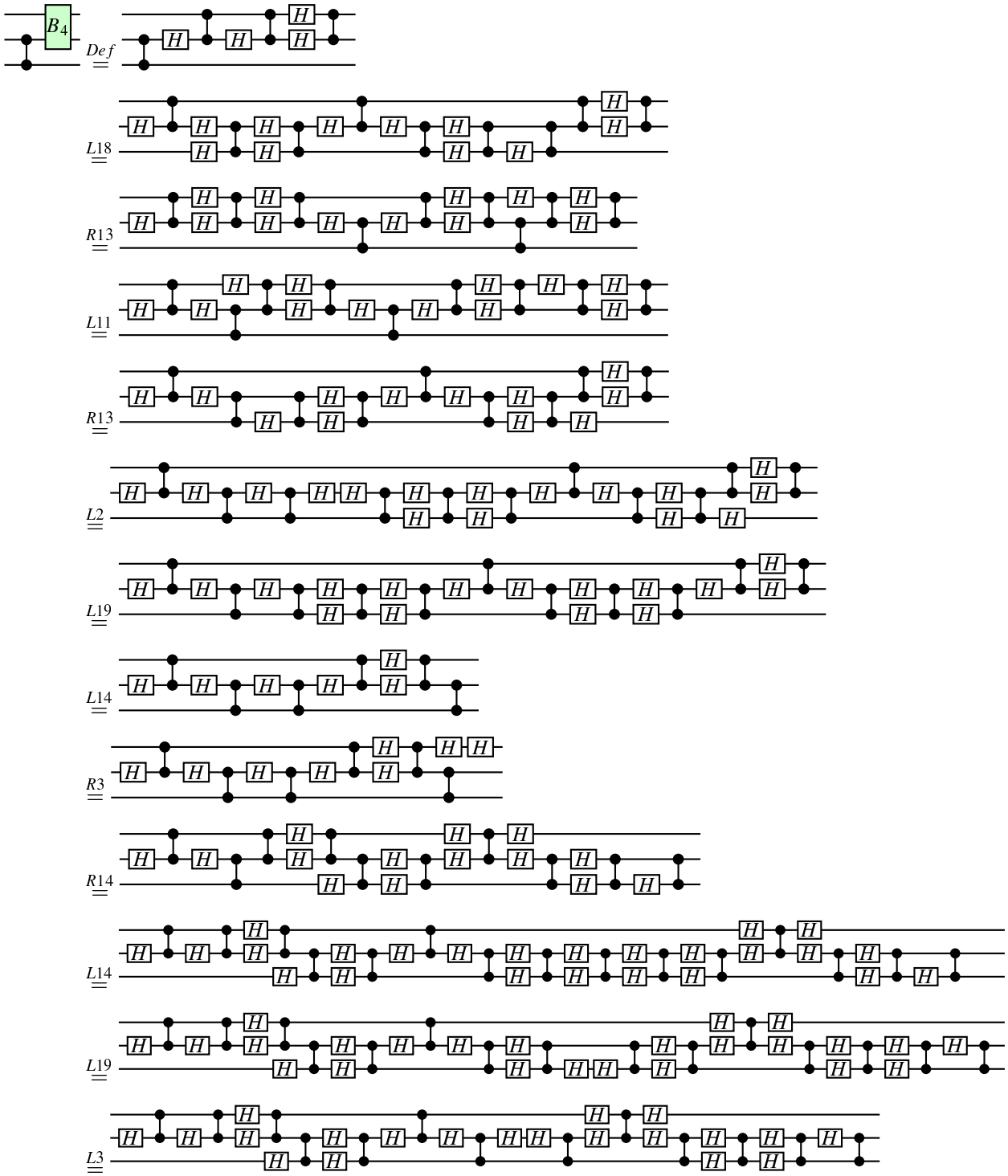
5.2

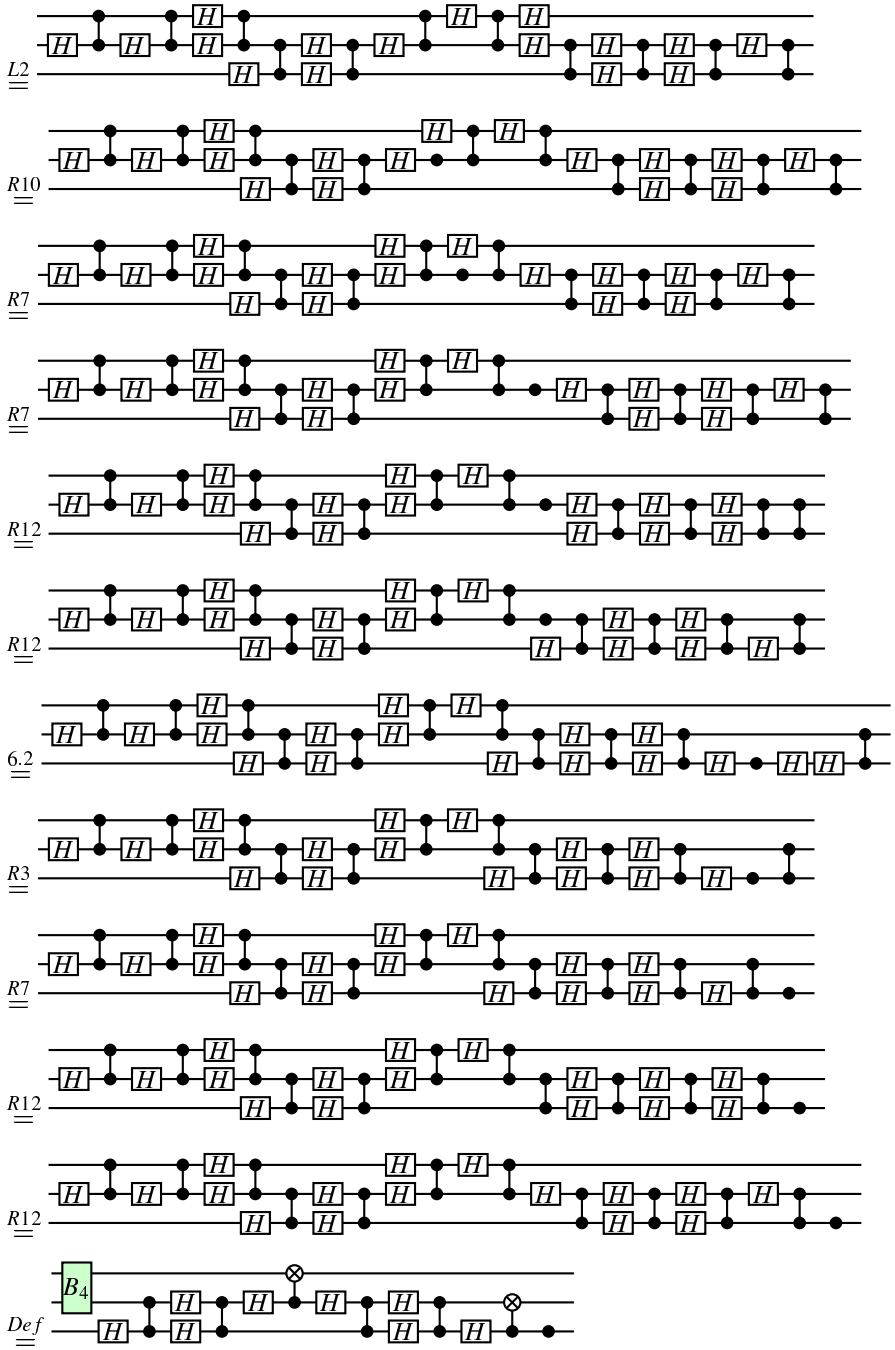


5.3

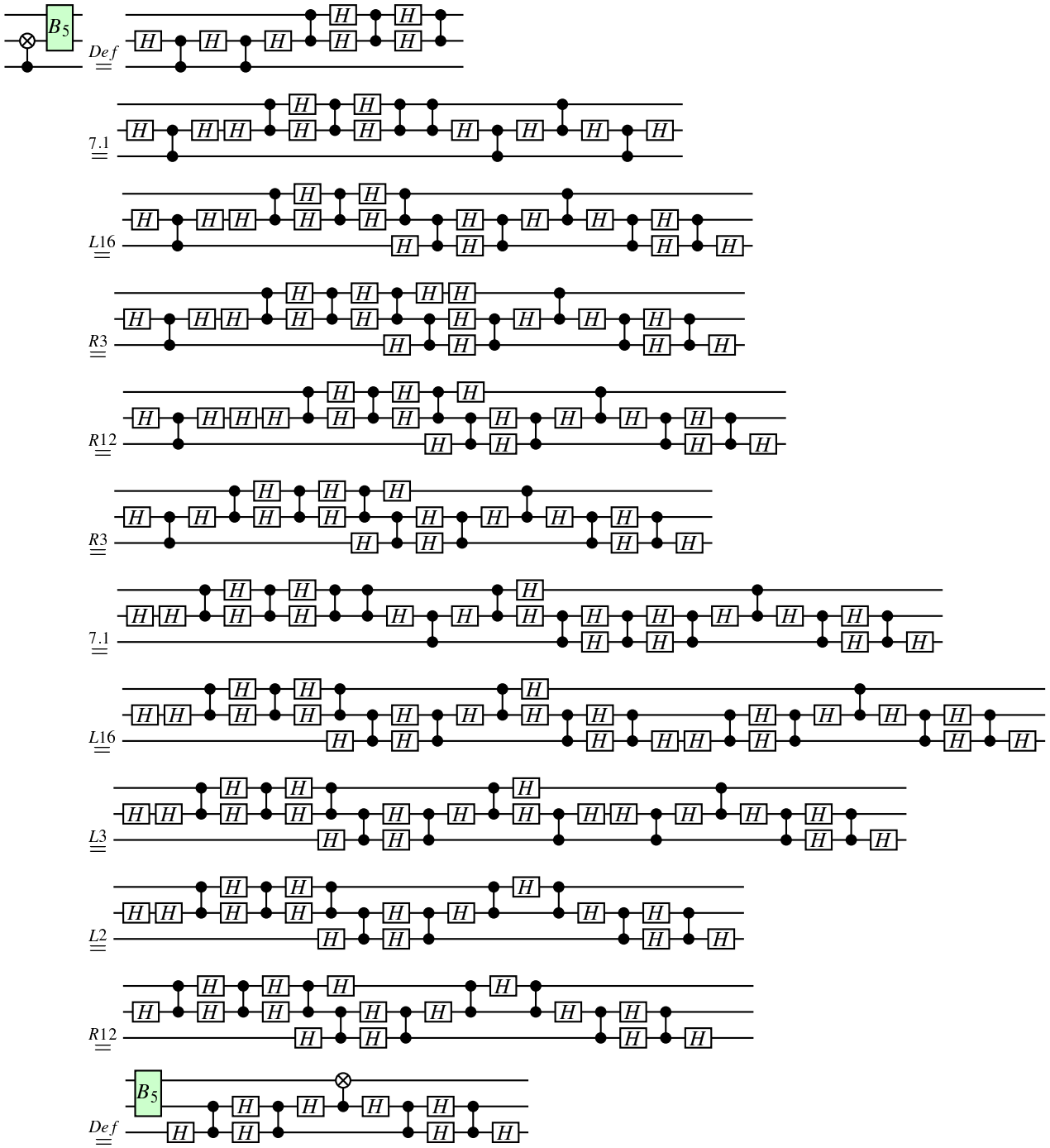


5.4

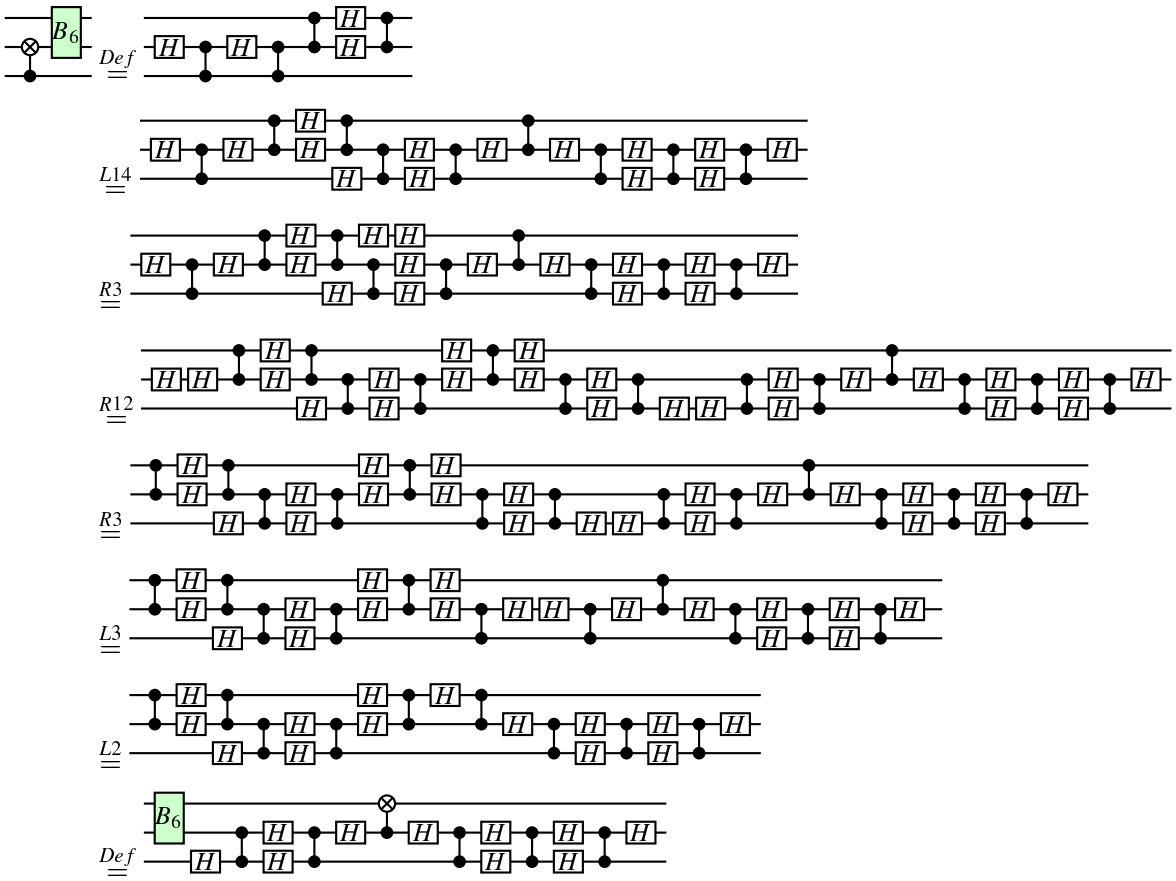



L2
R10
R7
R7
R12
R12
6.2
R3
R7
R12
R12
B_4
Def

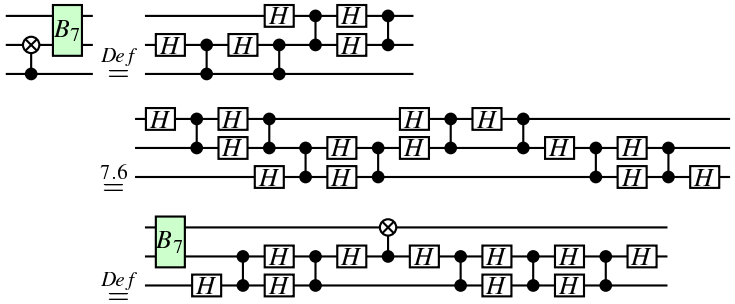
5.5



5.6



5.7



5.8

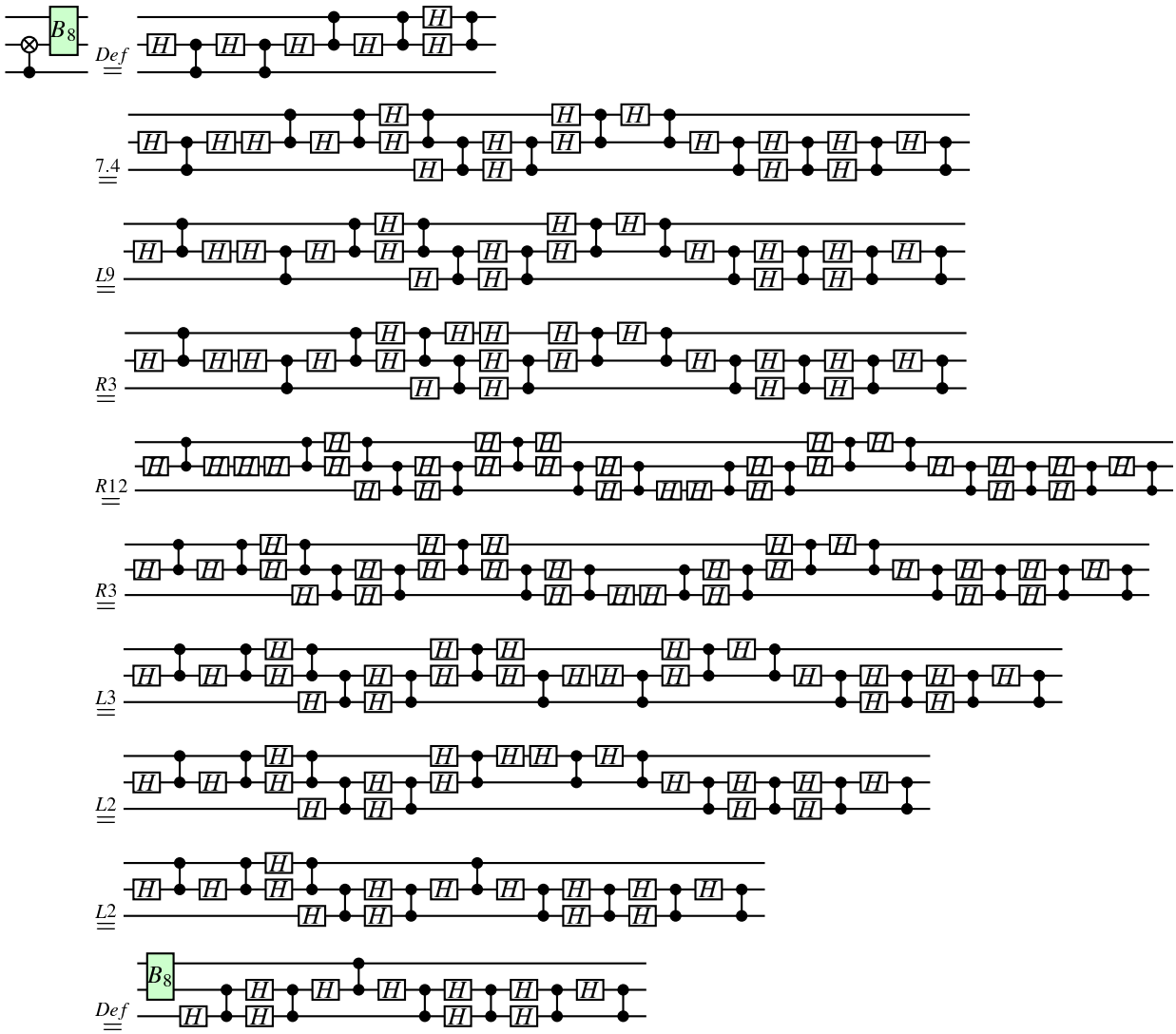
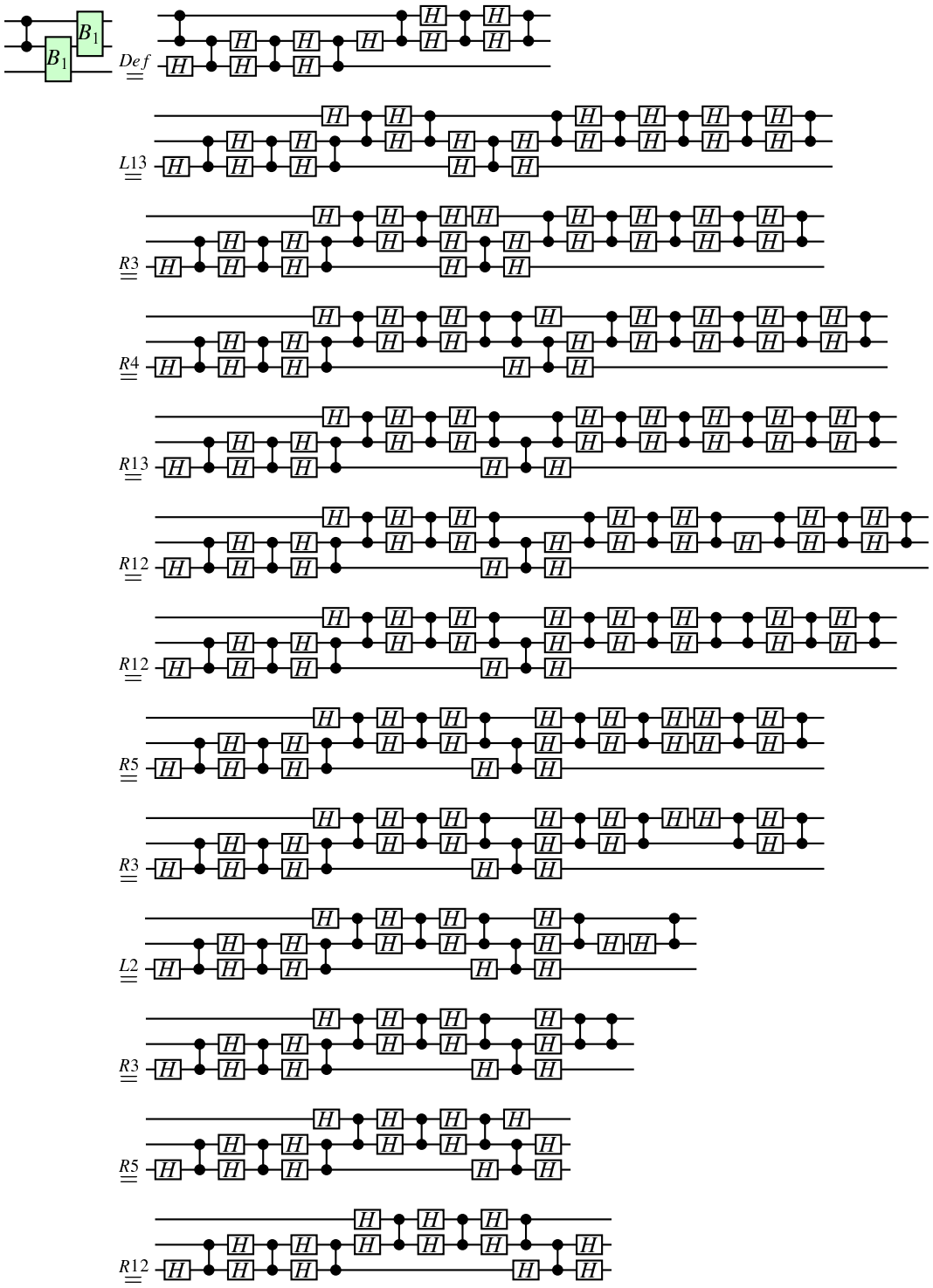
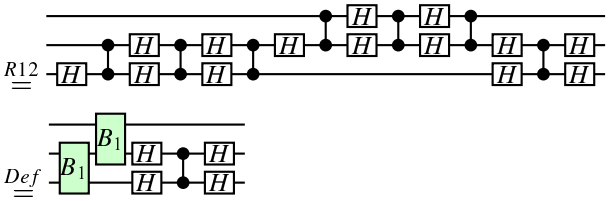


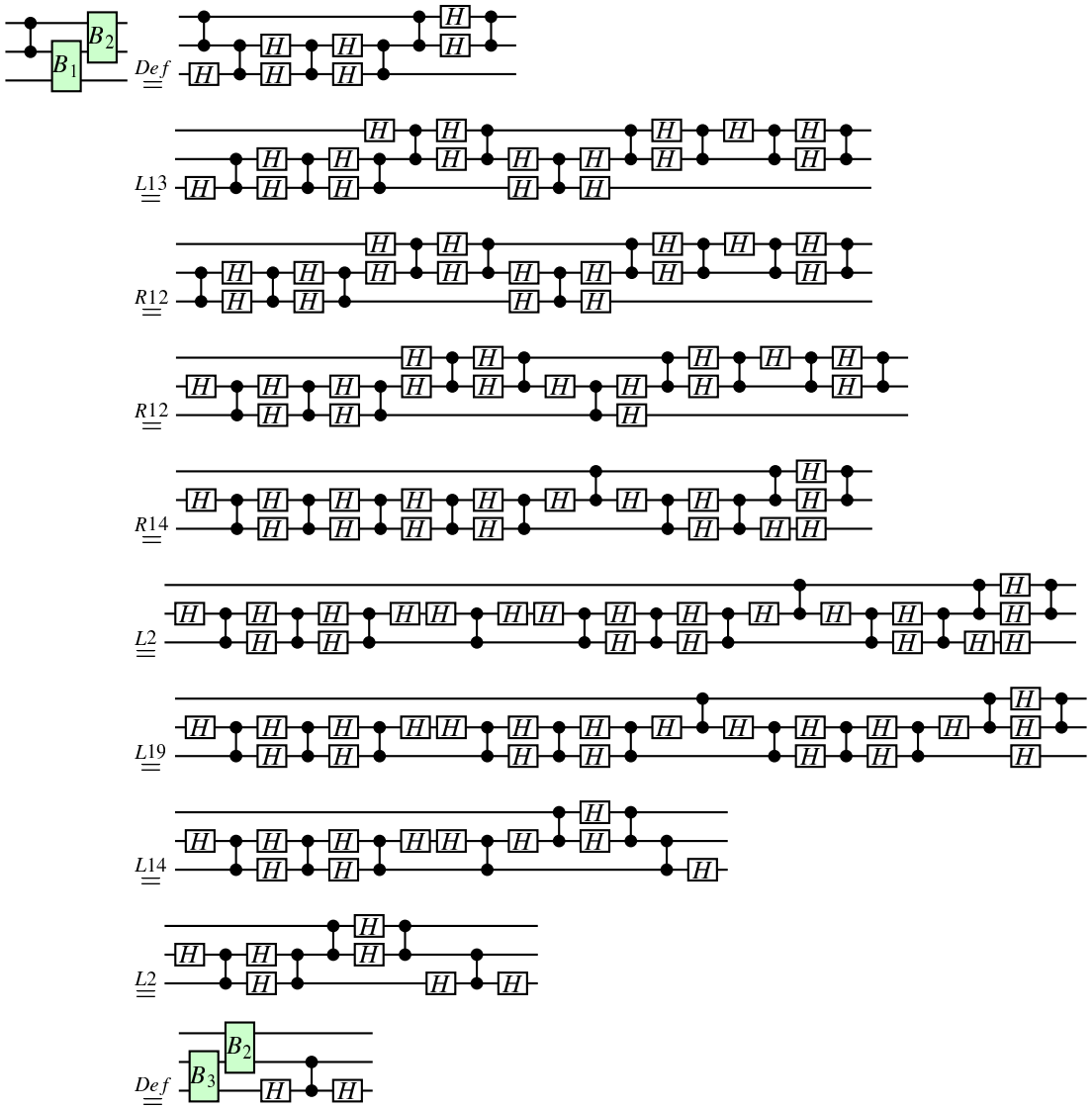
Figure 6:

6.1

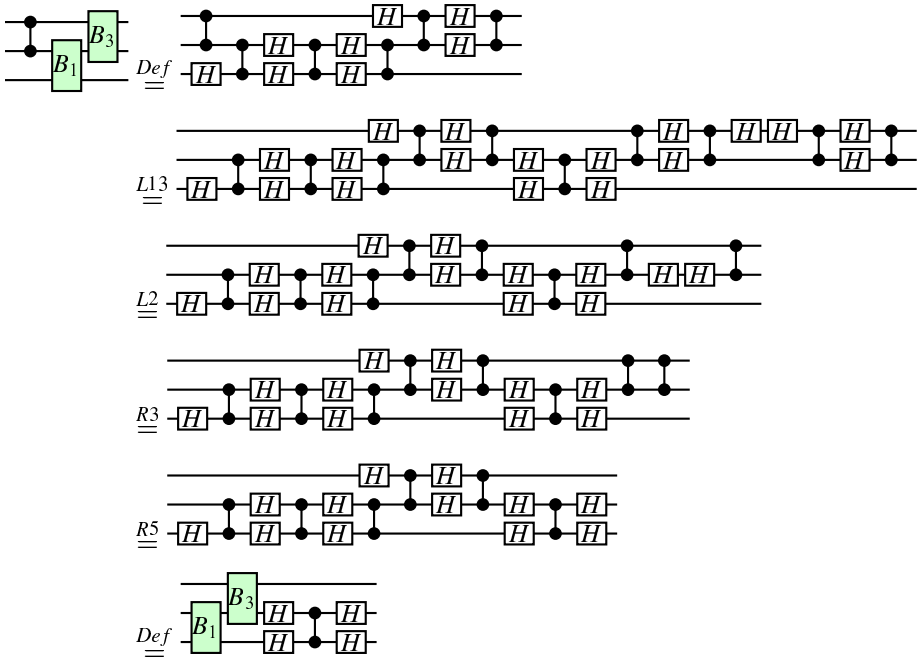




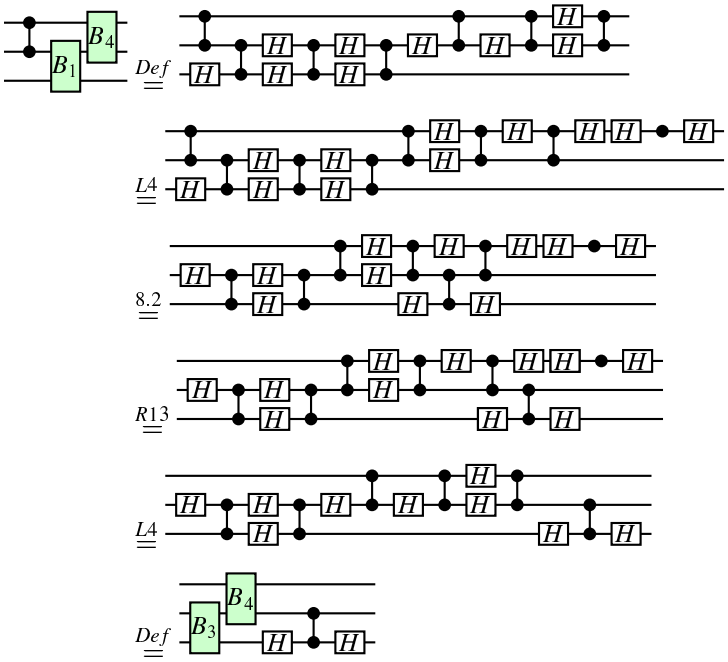
6.2



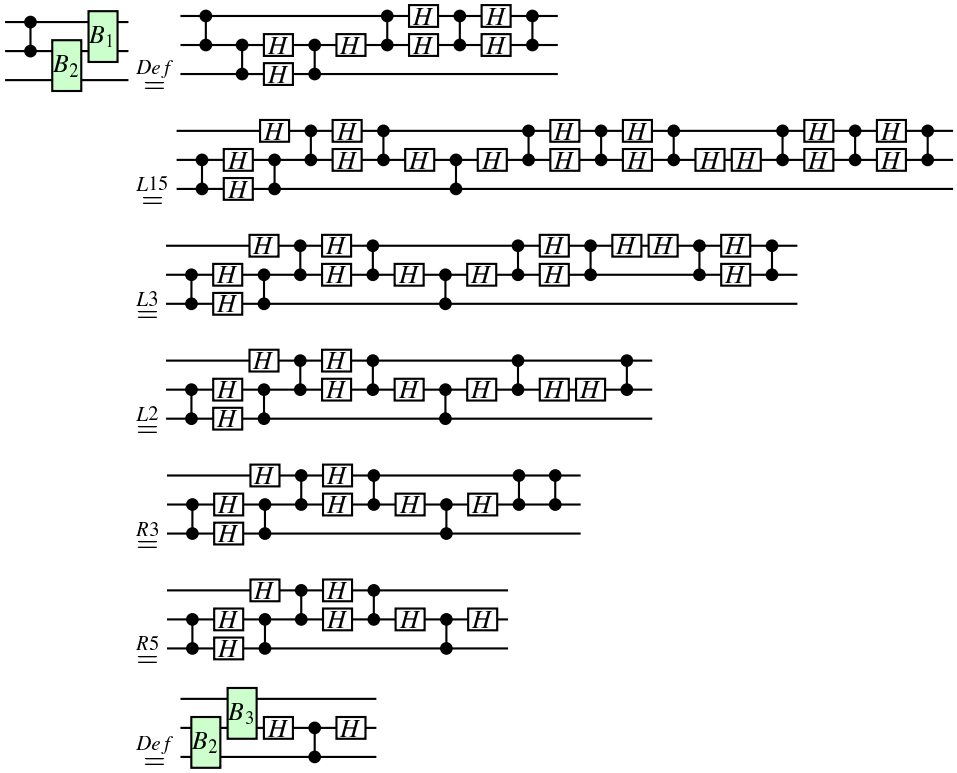
6.3



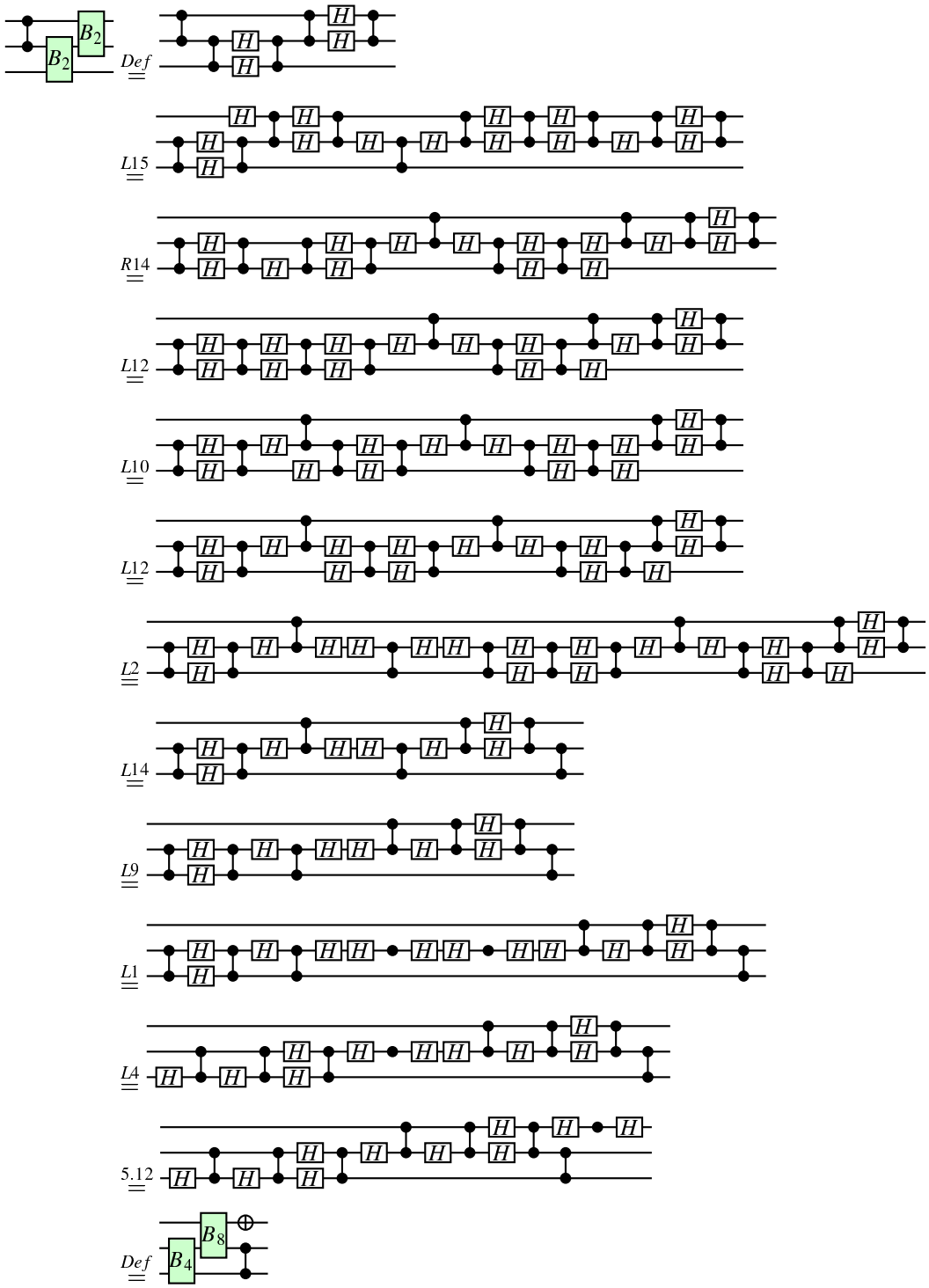
6.4



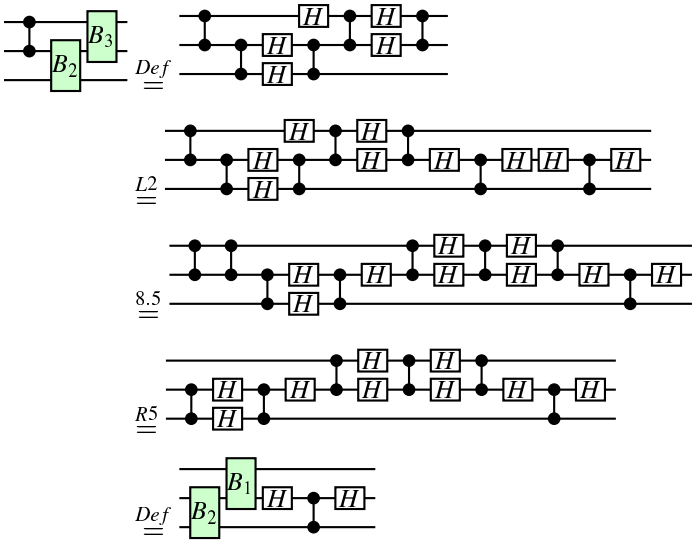
6.5



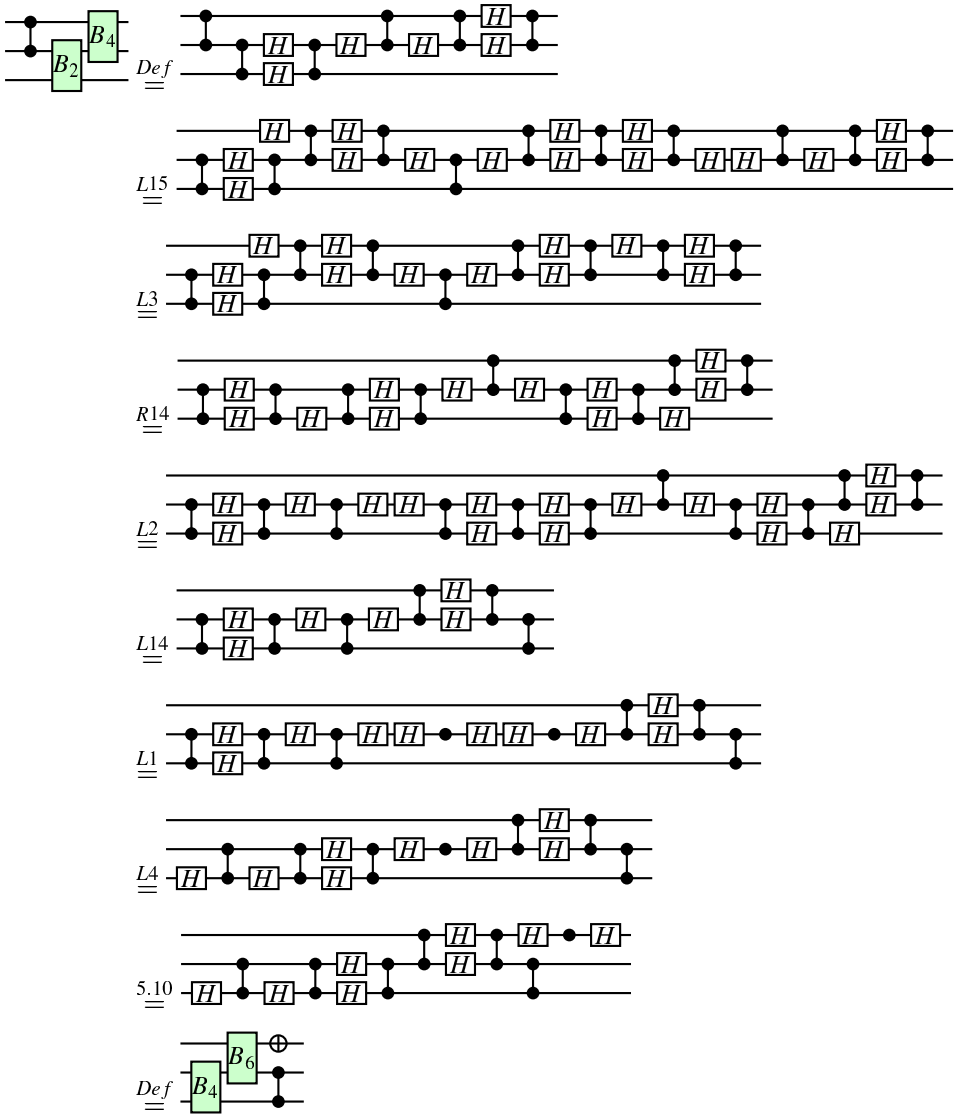
6.6



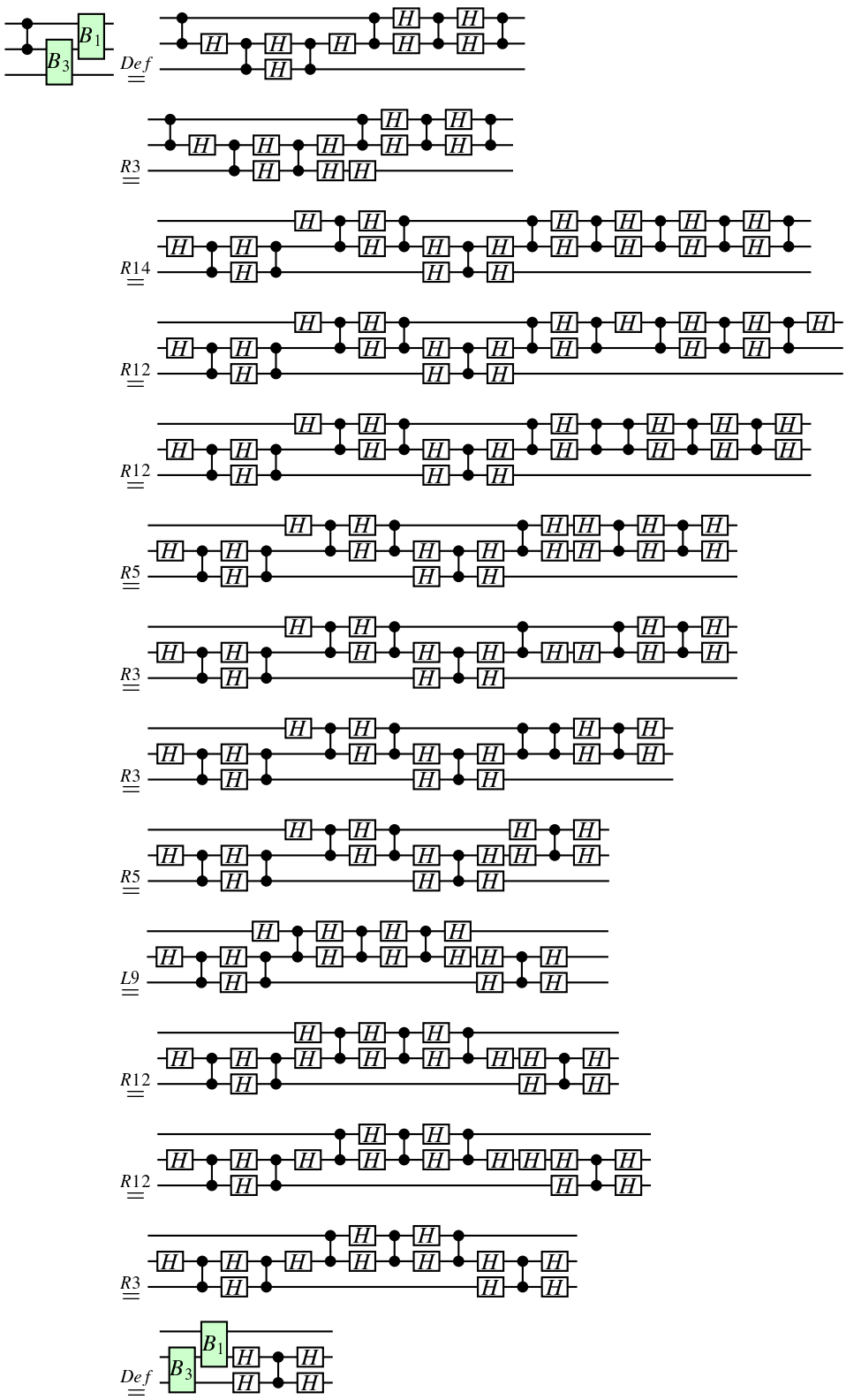
6.7



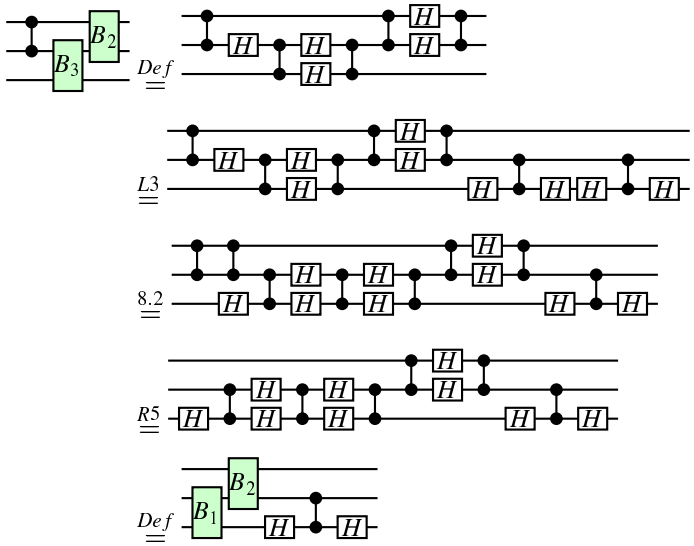
6.8



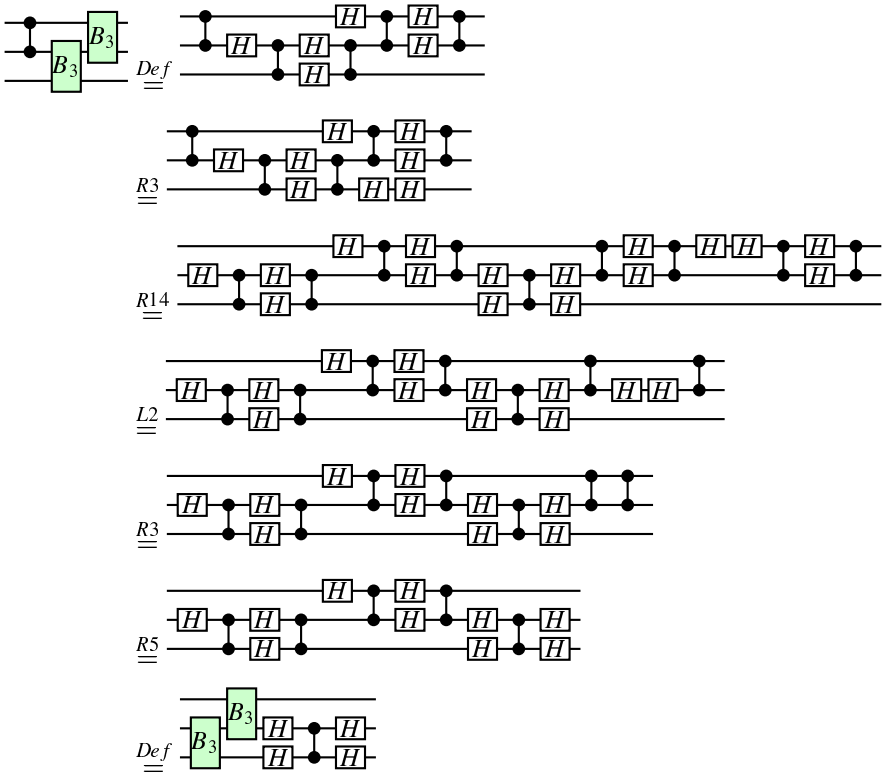
6.9



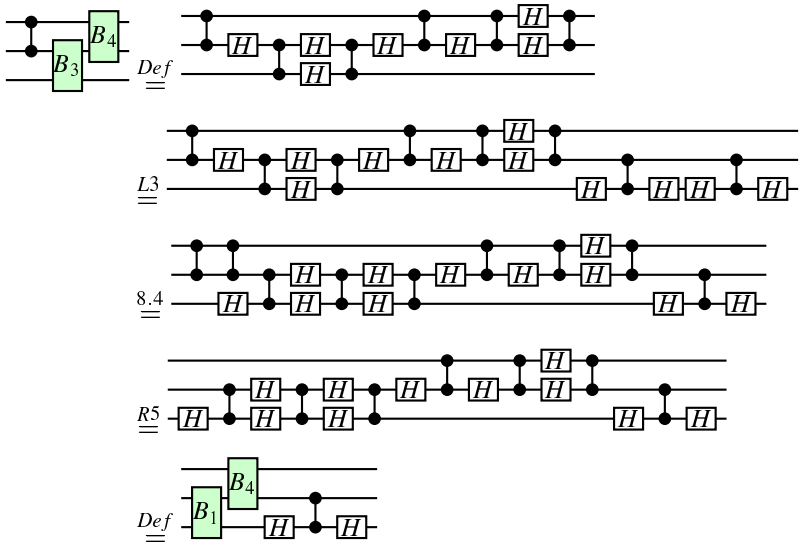
6.10



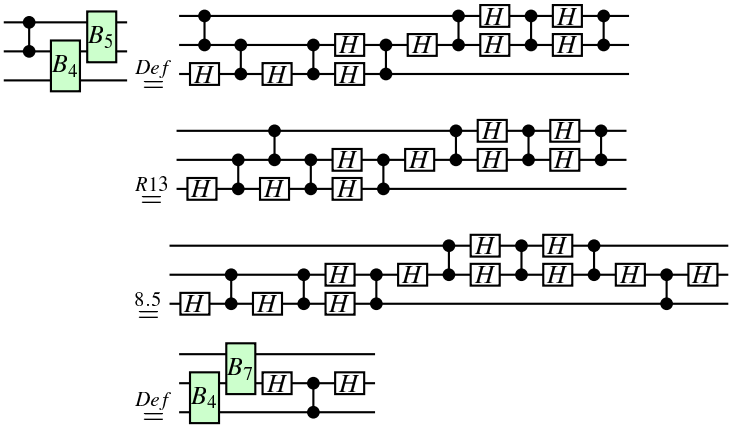
6.11



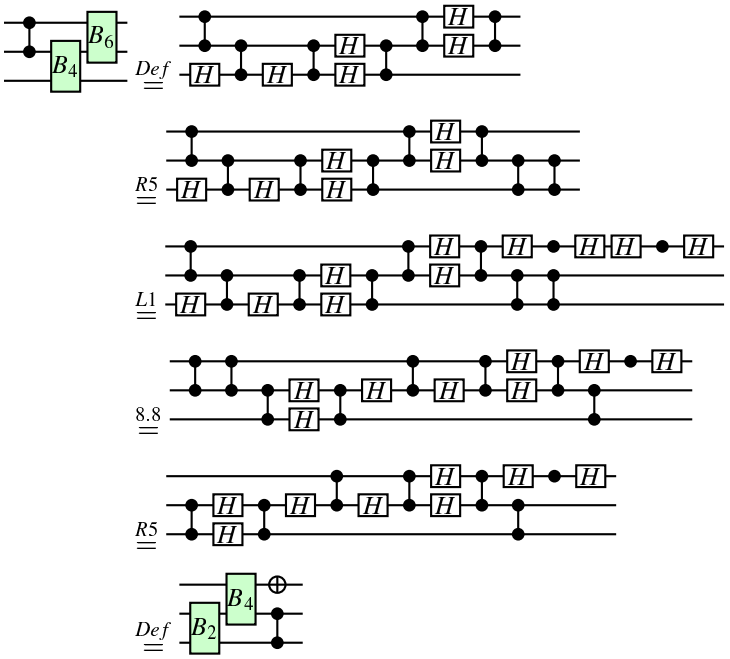
6.12



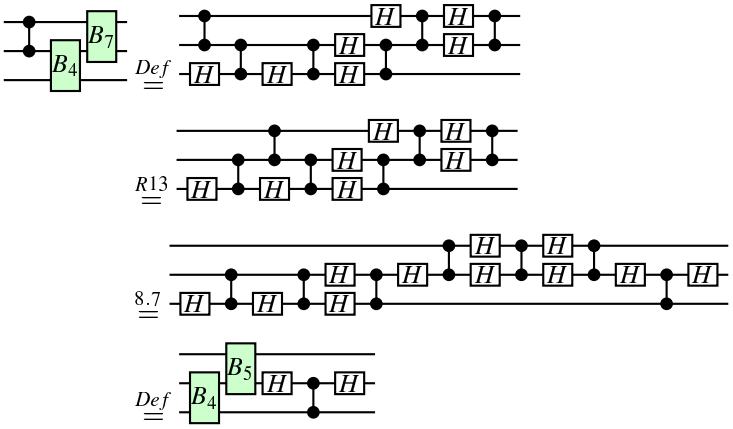
6.13



6.14



6.15



6.16

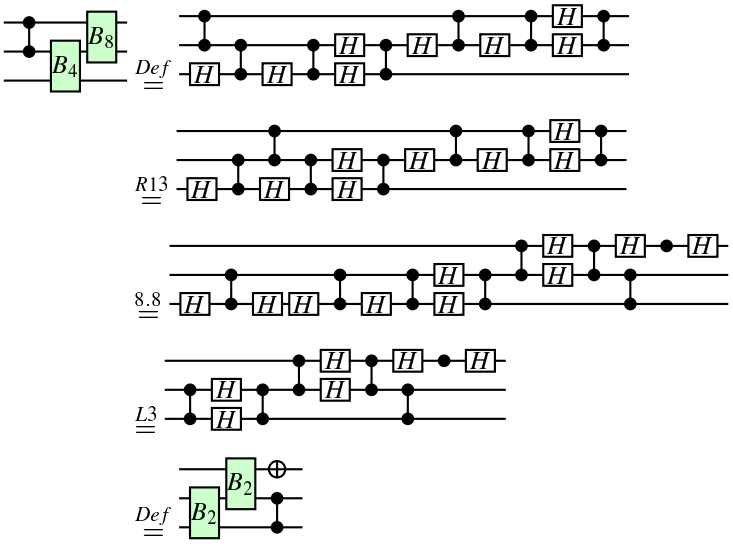
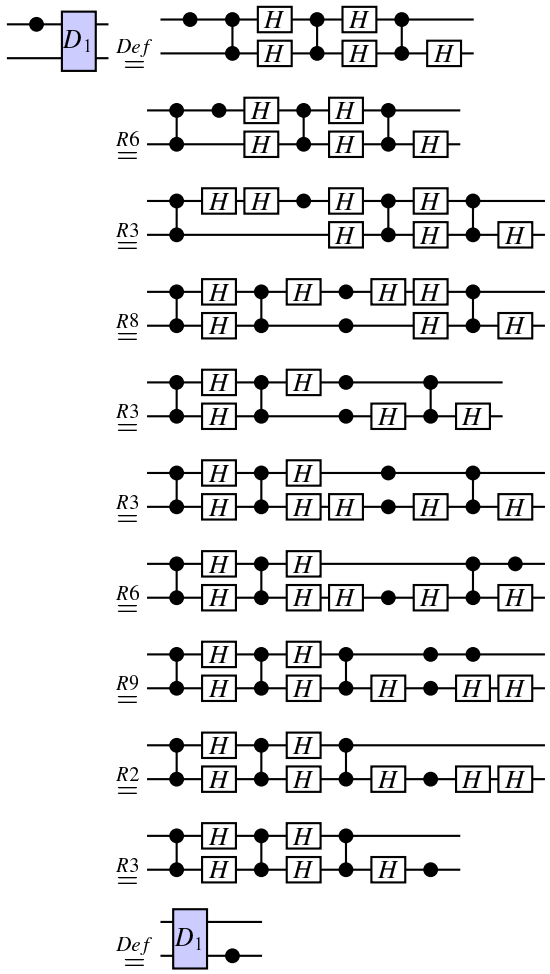


Figure 7:

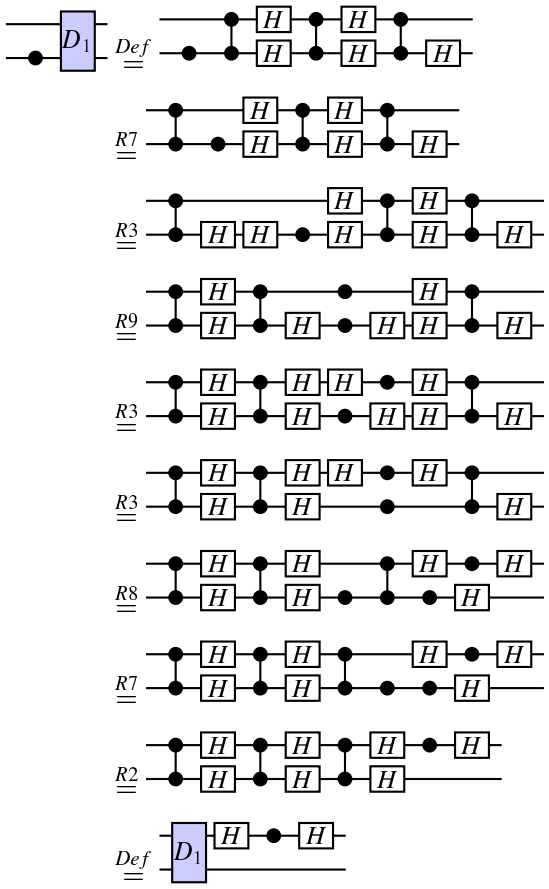
The proofs of 7.1–7.16 are the same as those of 6.1–6.16.

Figure 8:

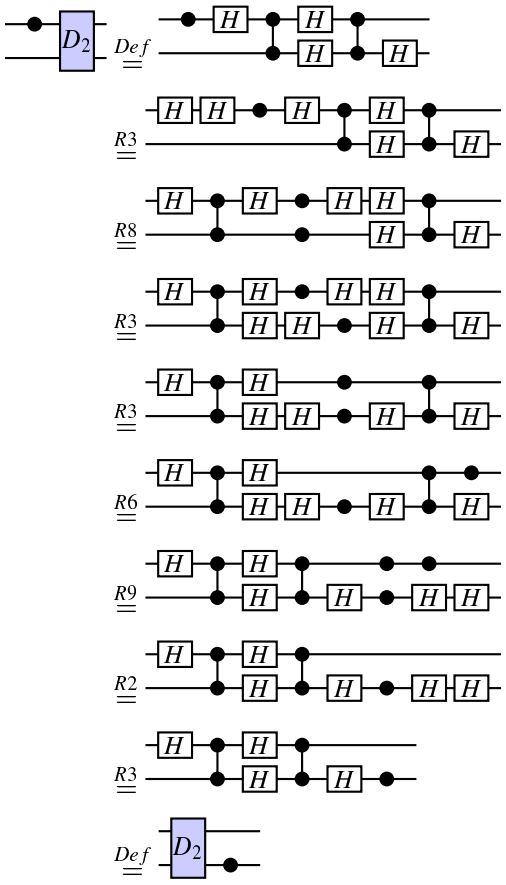
8.1



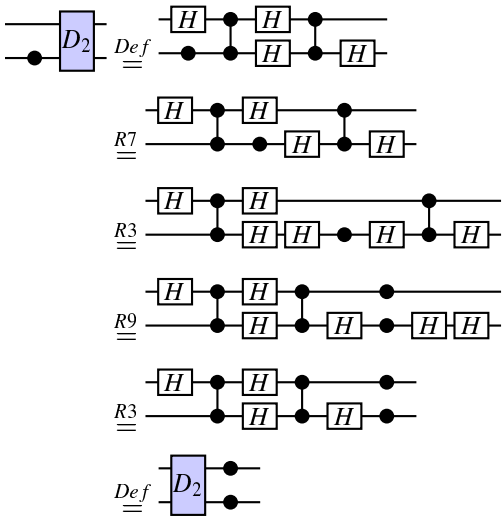
8.2



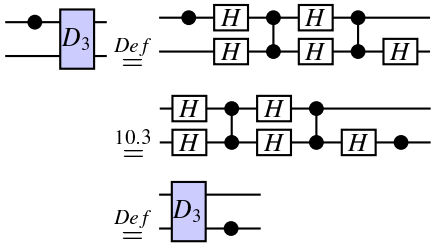
8.3



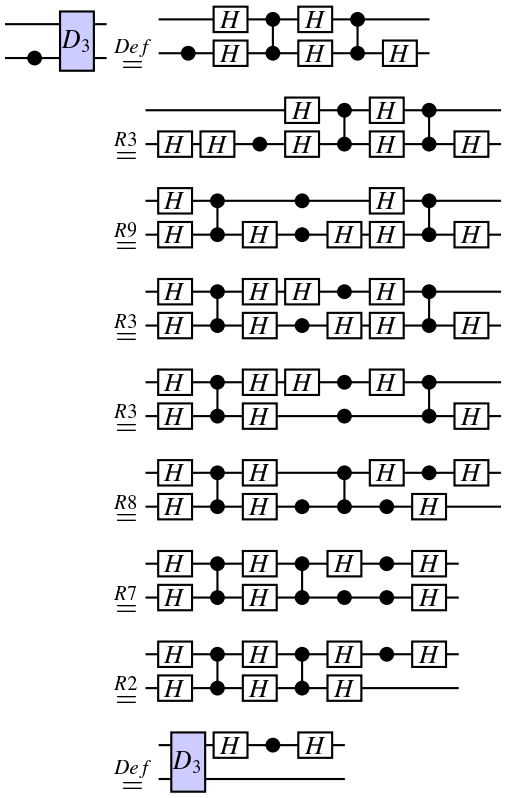
8.4



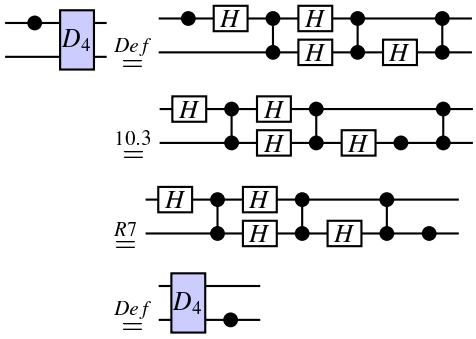
8.5



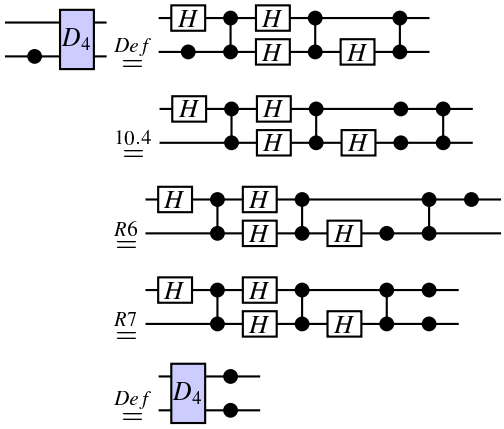
8.6



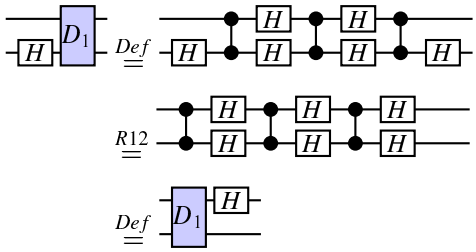
8.7



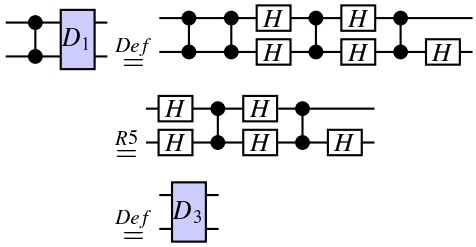
8.8


D4
Def
10.4
R6
R7
Def
D4

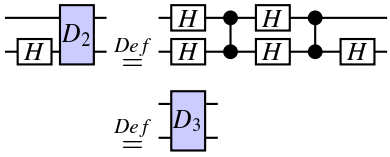
8.9


D1
Def
R12
Def
D1

8.10


D1
Def
R5
Def
D3

8.11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Def
Def
D3

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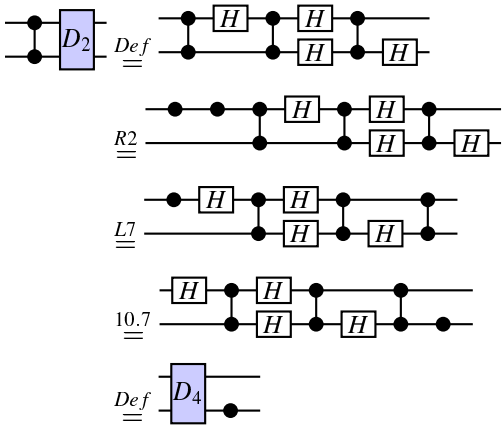

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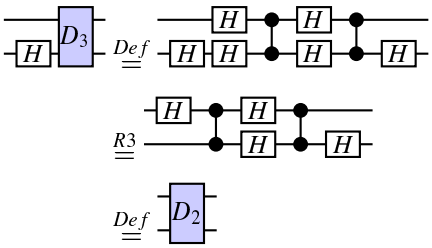

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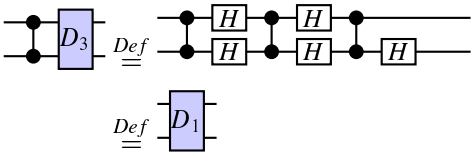

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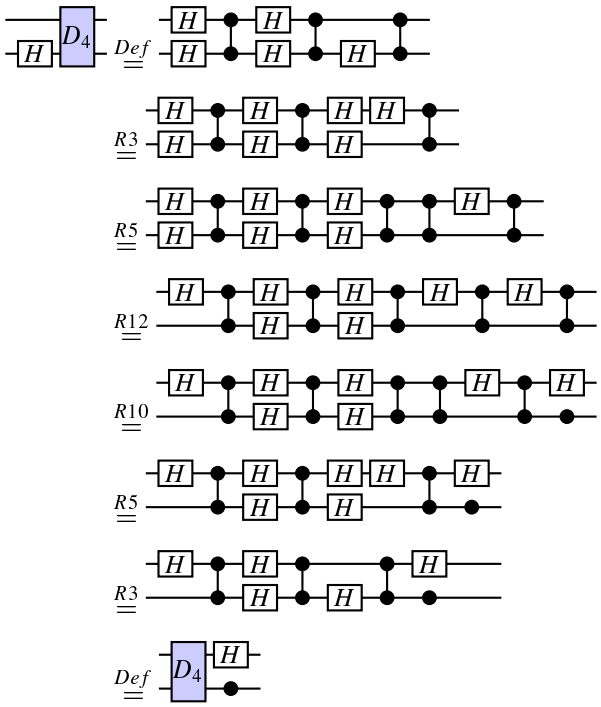

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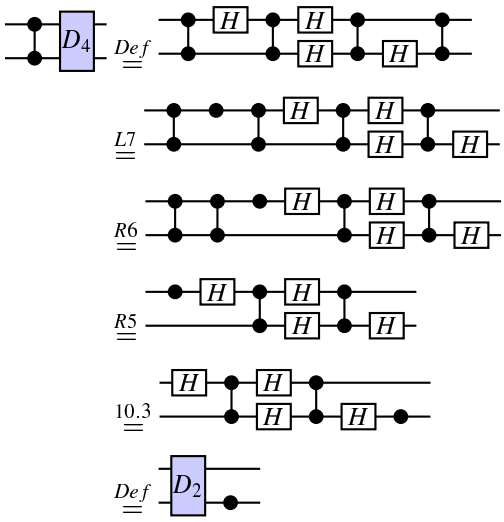

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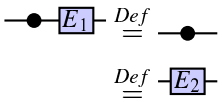

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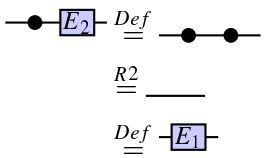
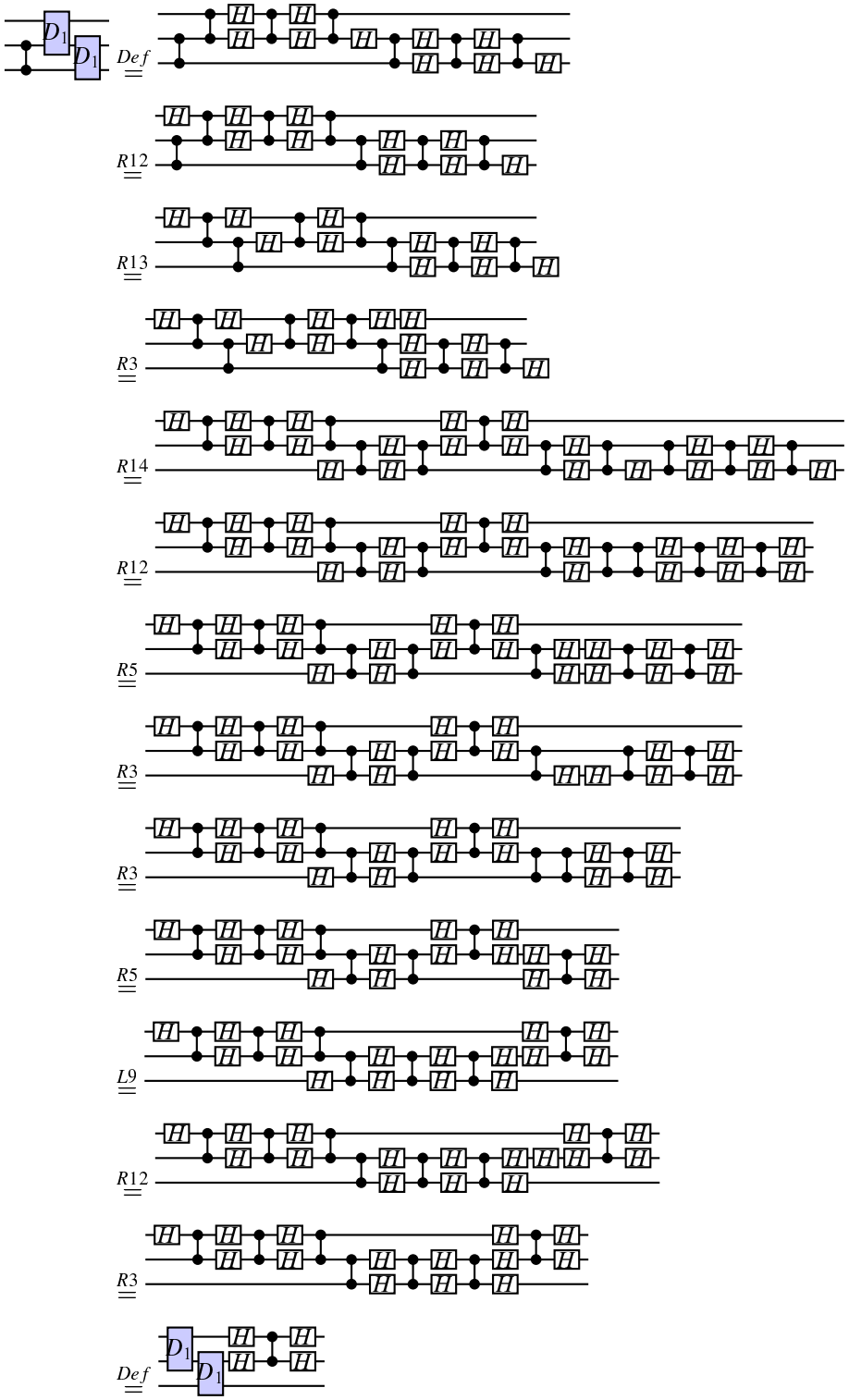


Figure 9:

9.1



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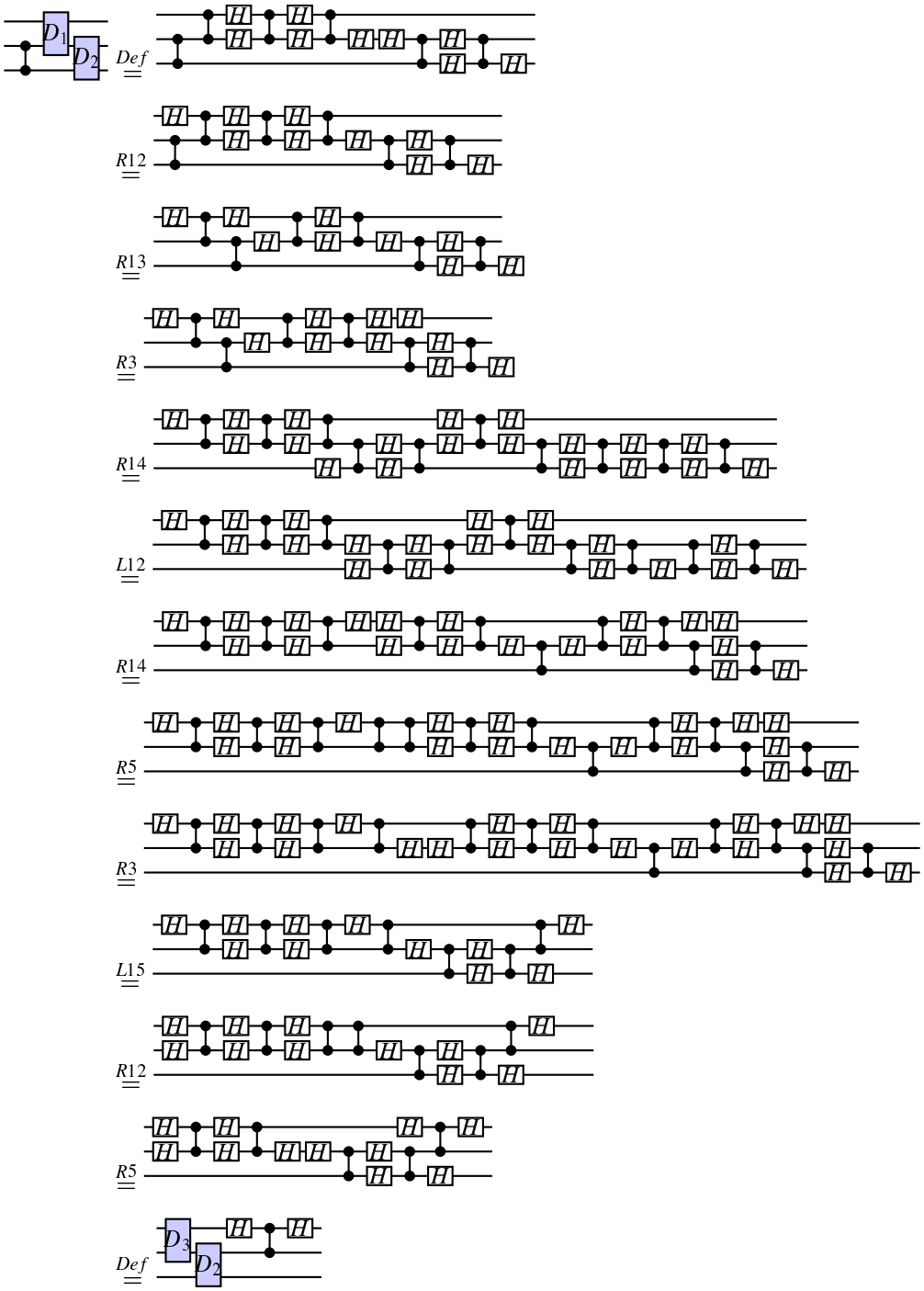

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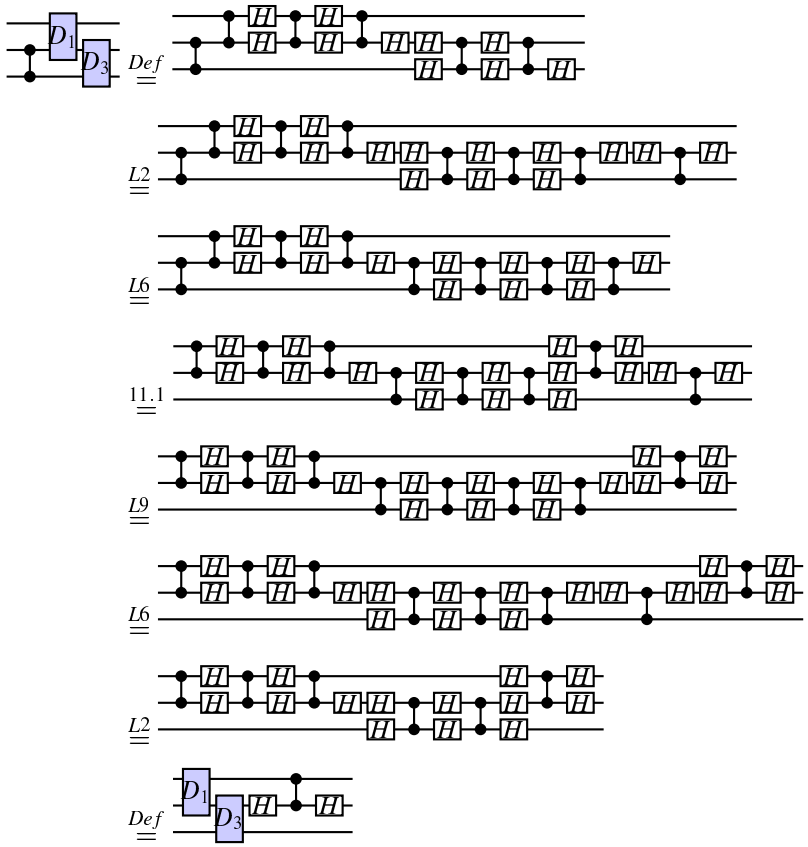

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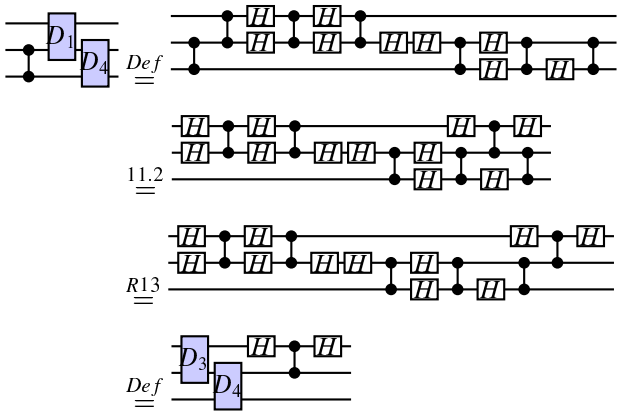

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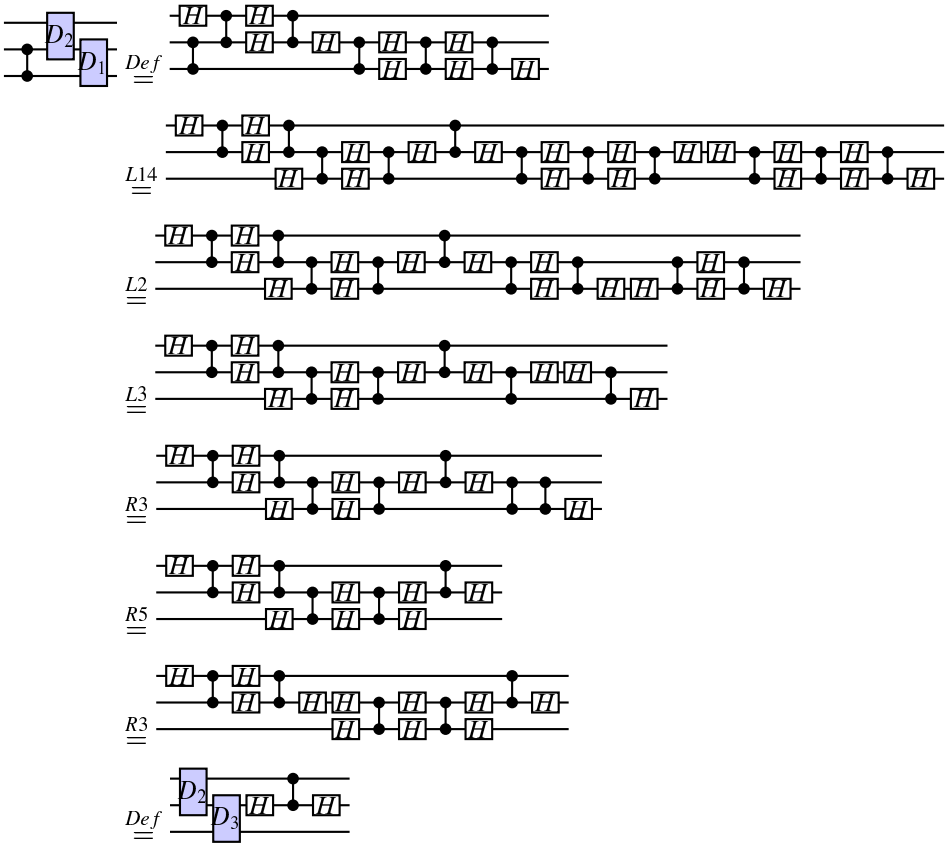

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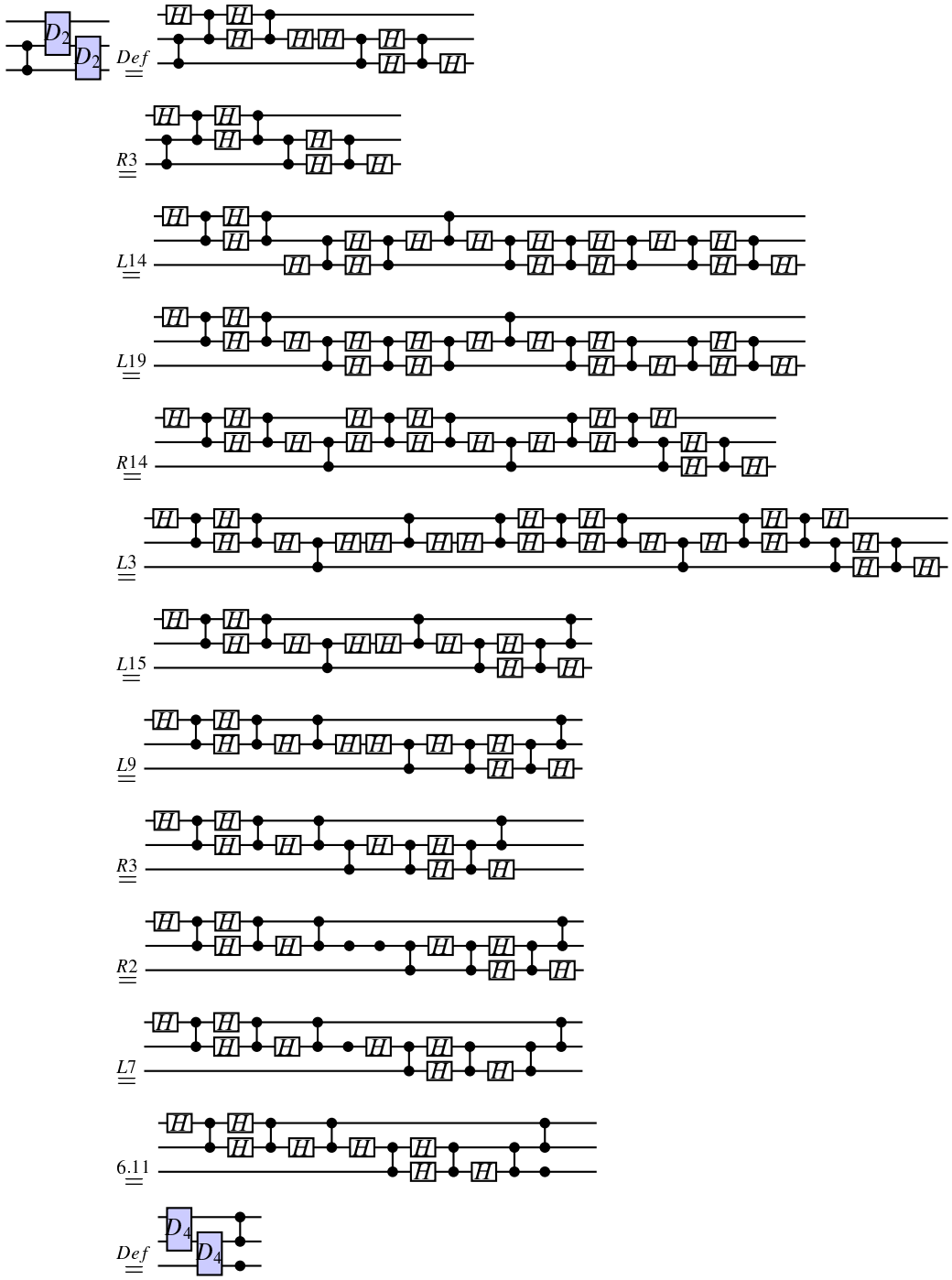

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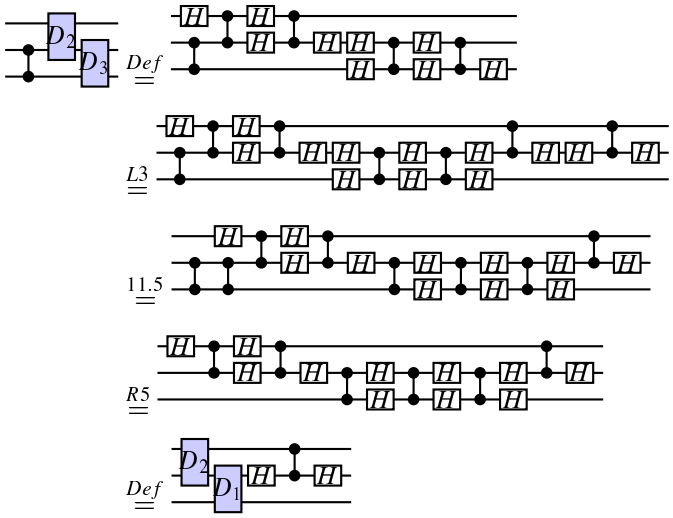

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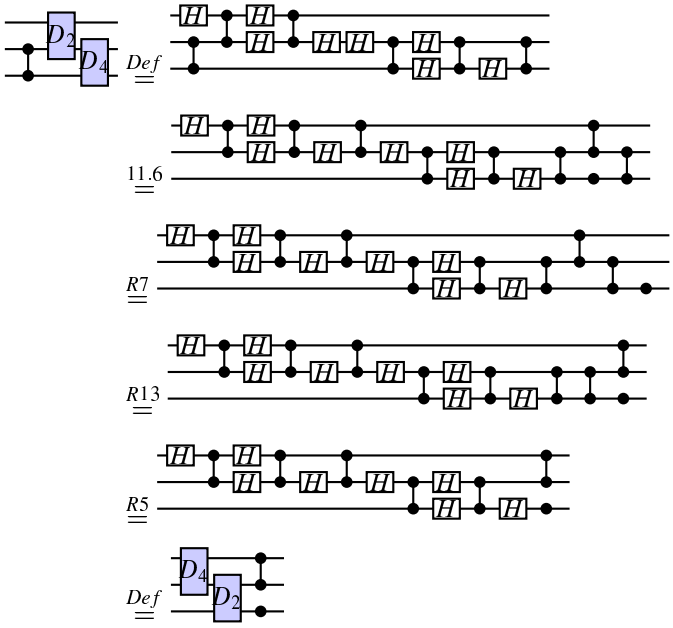

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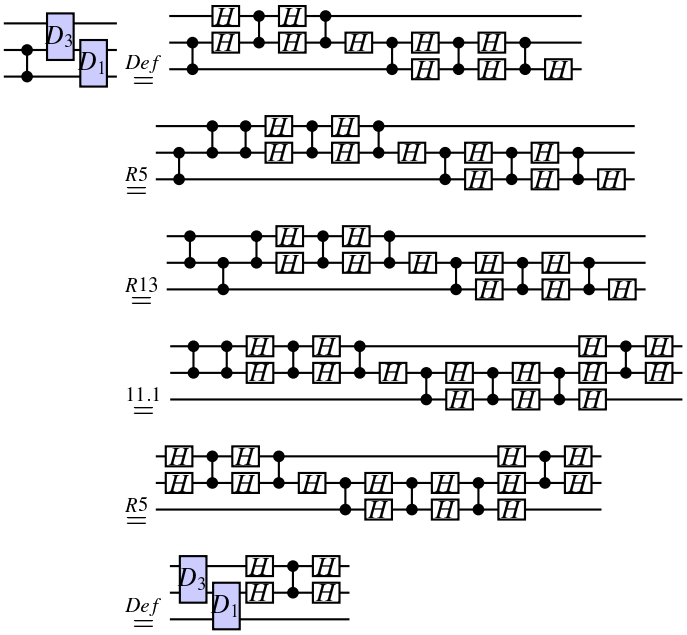

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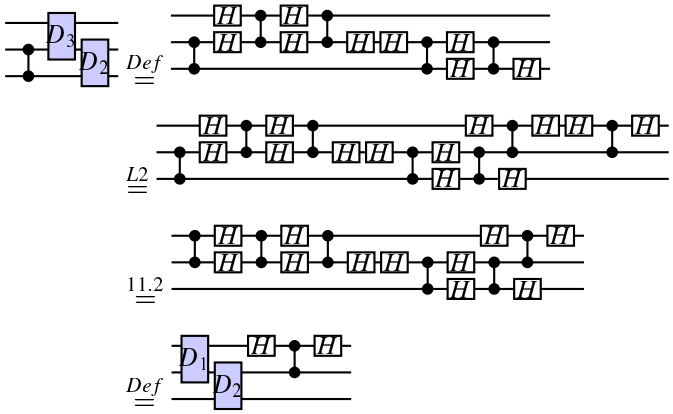

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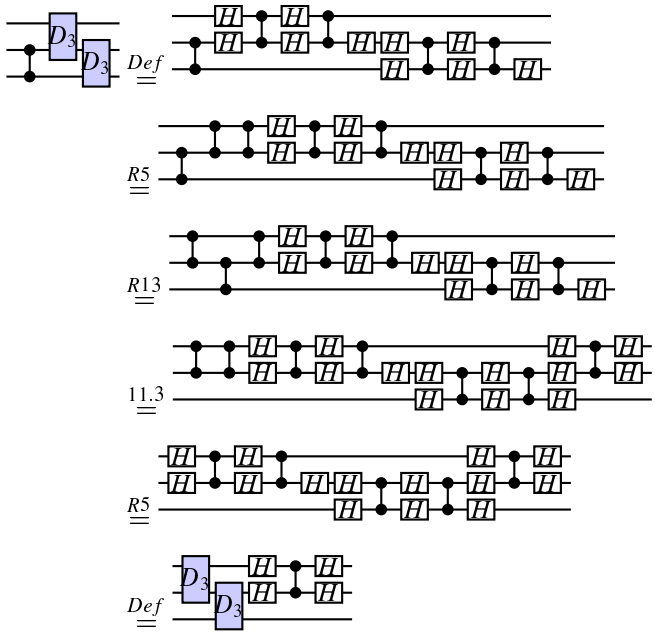

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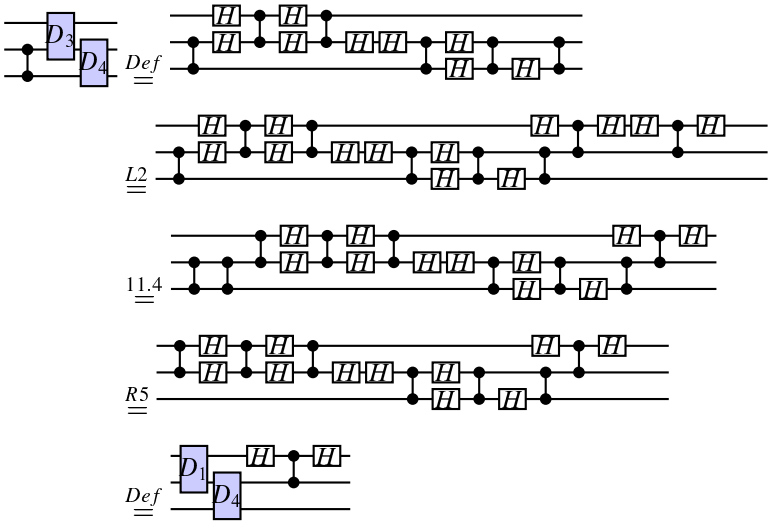

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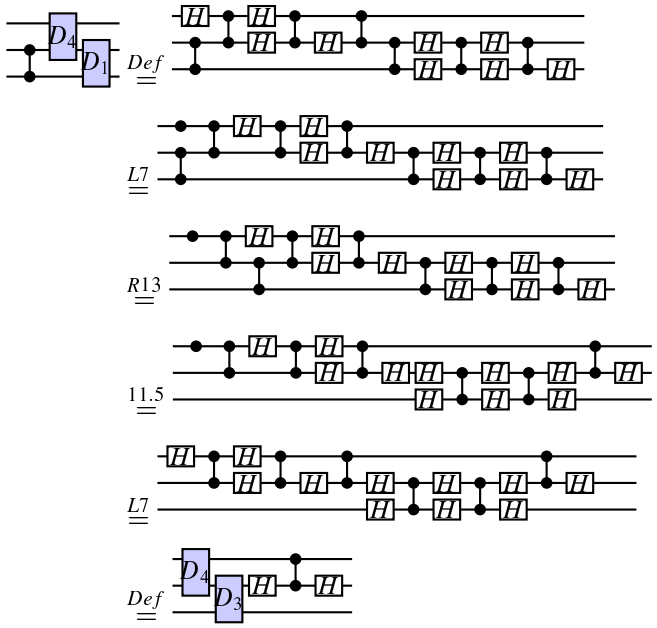
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R13
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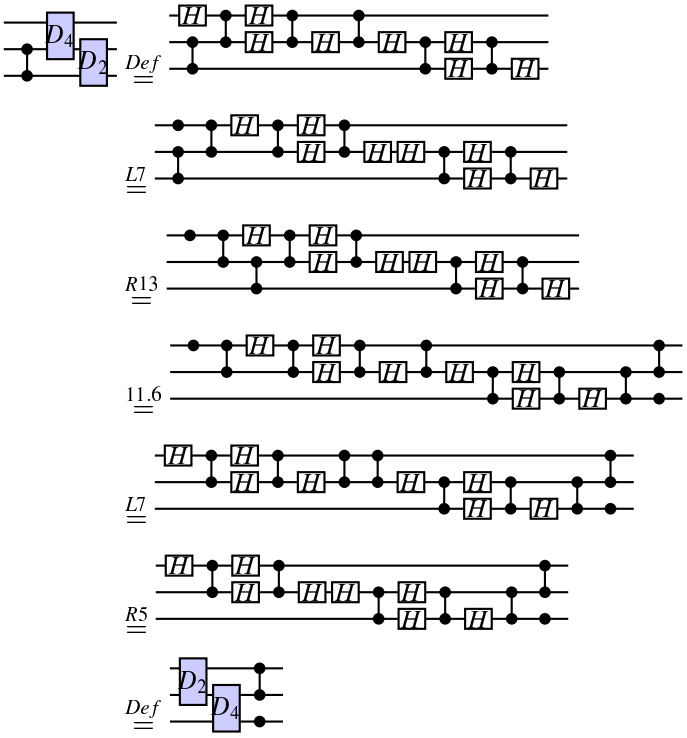
L7
D4
D3
Def

9.14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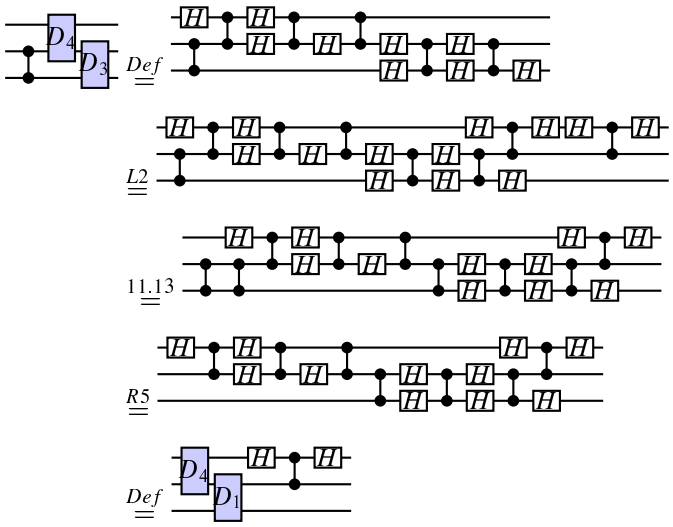
D2
Def
L7
R13
11.6
L7
R5
D2
D4
Def

9.15



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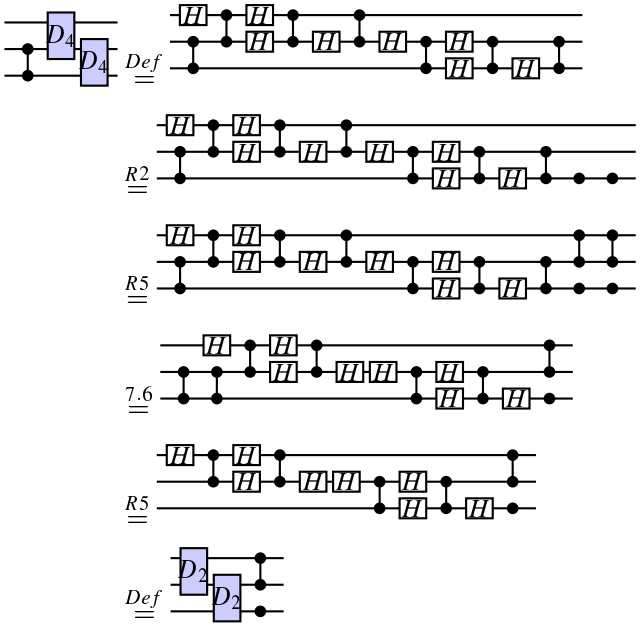
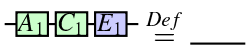
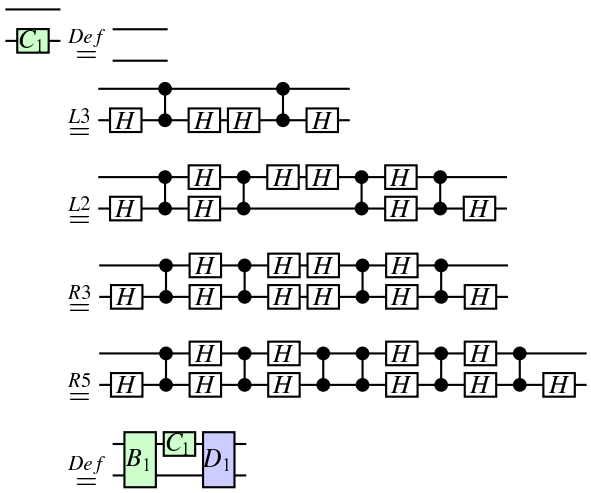


Figure 10:

10.1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