Rigidity of quantum steering and 1sDI verifiable quantum computation

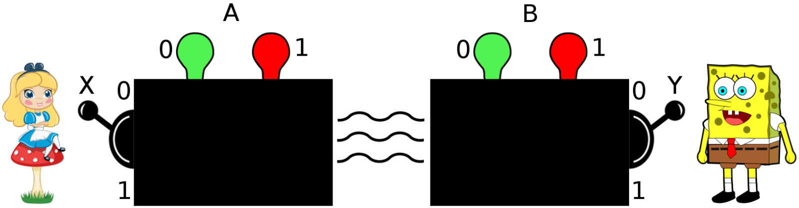
[arXiv:1512.07401]

Alexandru Gheorghiu, Petros Wallden, Elham Kashefi

8 June 2016

QPL 2016, Glasgow

Nonlocal correlations



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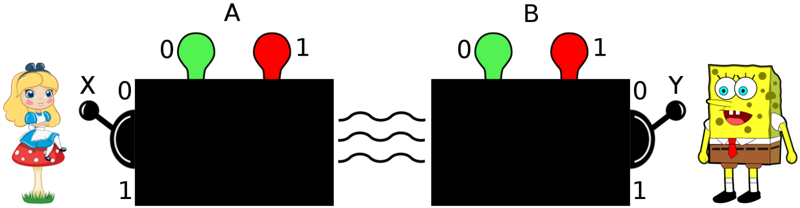

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Tsirelson's theorem (1980)

$S = 2\sqrt{2}$ is the maximum that can be achieved by QM. E.g. by having Alice and Bob share $|\phi_+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and measure:

$$A_0 = X,\ A_1 = Z,\ B_0 = (X+Z)/\sqrt{2},\ B_1 = (X-Z)/\sqrt{2}$$

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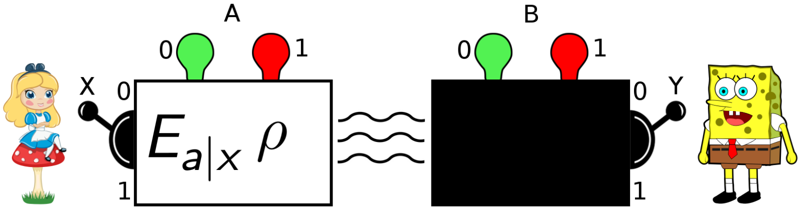
Saturating nonlocal correlations determines state and strategy!

Steering correlations



$$p(a,b|x,y) \neq \sum_{\lambda} Tr(\rho_{AB}(\lambda)(E_{a|x} \otimes I))p(b|y,\lambda)p(\lambda)$$

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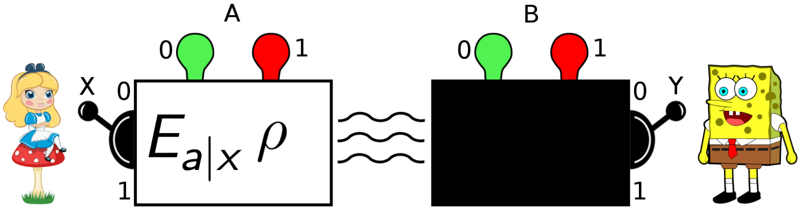

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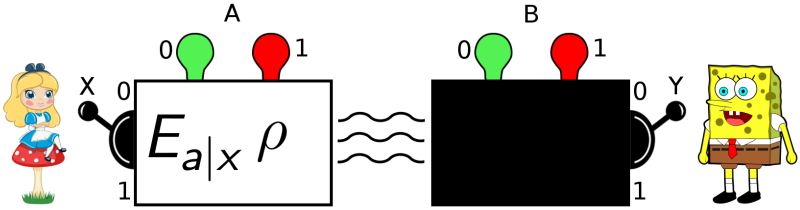

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Our main result: Converse is also true!

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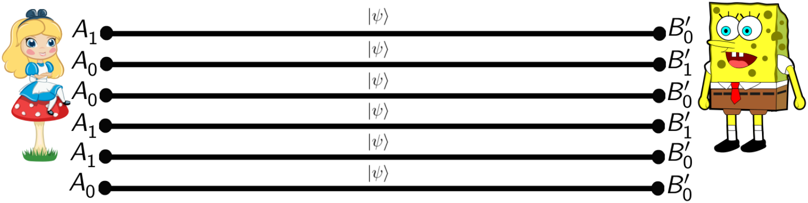
- Shared state ρ_{AB}, prepared by Bob (untrusted)
- In each round Alice and Bob measure the same state $|\psi\rangle$ (**i.i.d.**)

Self-testing i.i.d. states



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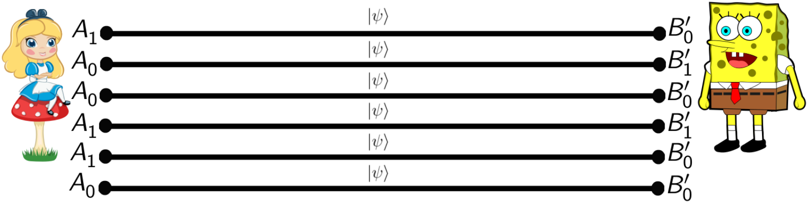

$$|\langle A_0 B'_0 \rangle + \langle A_1 B'_1 \rangle| \geq 2 - \epsilon \tag{1}$$

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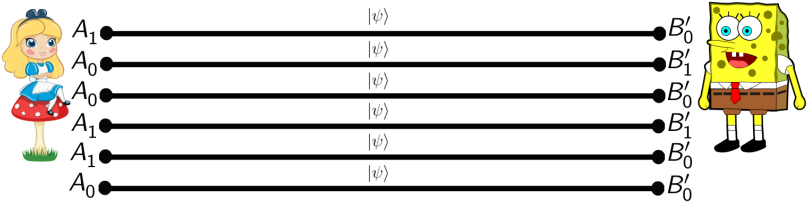

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I.i.d. self-testing theorem

If inequality 1 is satisfied, then there exists a local isometry $\Phi = I \otimes \Phi_B$ such that, for all $M_A \in \{I, A_0, A_1\}$, $N'_B \in \{I, B'_0, B'_1\}$:

$$||\Phi(M_A N'_B |\psi\rangle) - |junk\rangle M_A N_B |\phi_+\rangle || \leq O(\sqrt{\epsilon})$$

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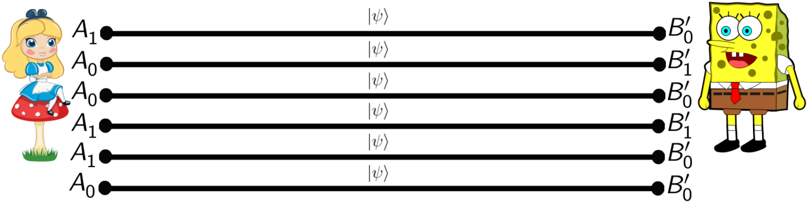

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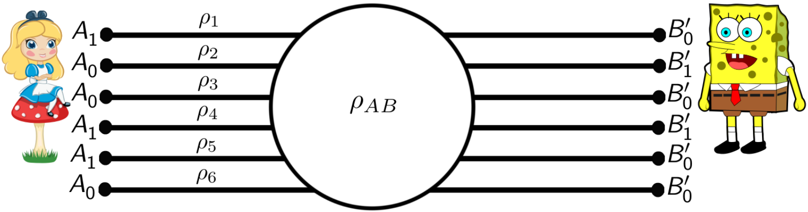
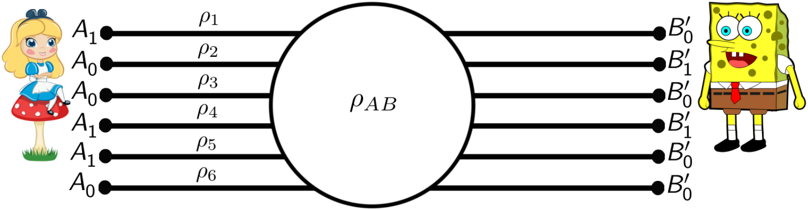
Cannot do better than $O(\sqrt{\epsilon})$!


A_1
A_0
A_0
A_1
A_1
A_0
ρ_1
ρ_2
ρ_3
ρ_4
ρ_5
ρ_6
ρ_{AB}
B'_0
B'_1
B'_0
B'_1
B'_0
B'_0

Removing i.i.d.



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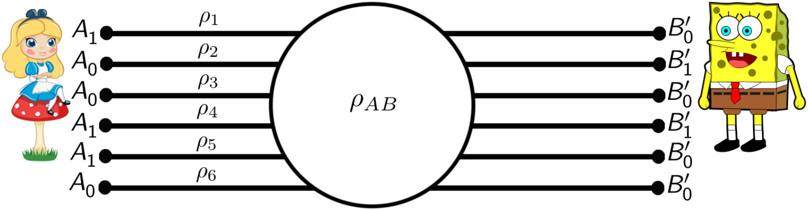

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Non-i.i.d. self-testing theorem

If inequality 1 is satisfied, then there exists a local isometry $\Phi = I \otimes \Phi_B$ such that, for $\mathcal{E}_{AB'}$ having the role of M_A, N'_B from before, we have for a randomly chosen ρ_i:

$$||\Phi(\mathcal{E}_{AB'}(\rho_i)) - \mathcal{E}_{AB}(|\phi_+\rangle\langle\phi_+|)|| \leq O(\epsilon^{1/6})$$

State and strategy determination

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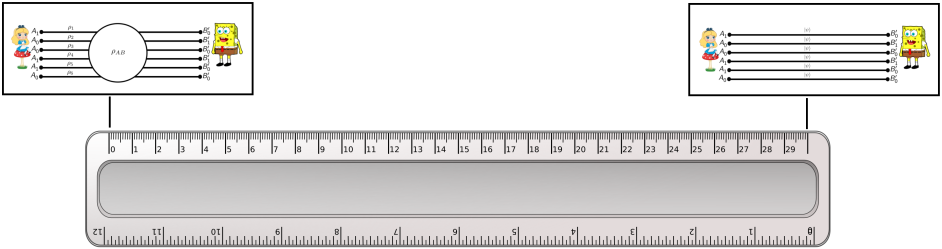
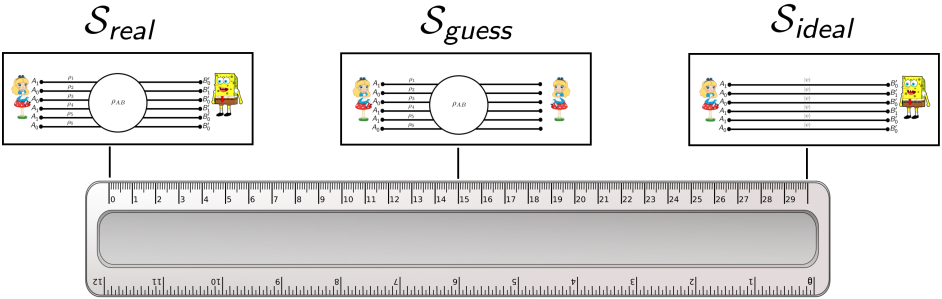
Objective: $\mathcal{S}_{real} \approx \mathcal{S}_{ideal}$

$\mathcal{S}_{real}$ $\mathcal{S}_{ideal}$



State and strategy determination - Proof sketch



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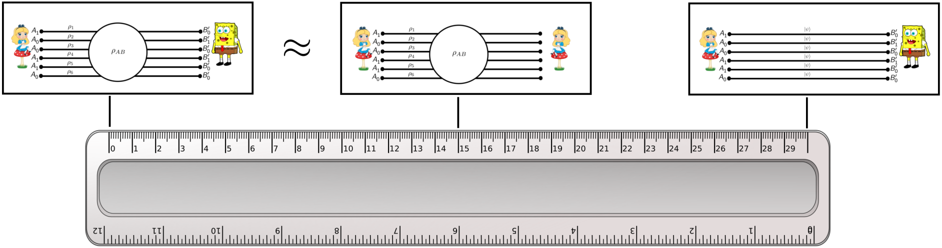
$\mathcal{S}_{real}$ $\mathcal{S}_{guess}$ $\mathcal{S}_{ideal}$



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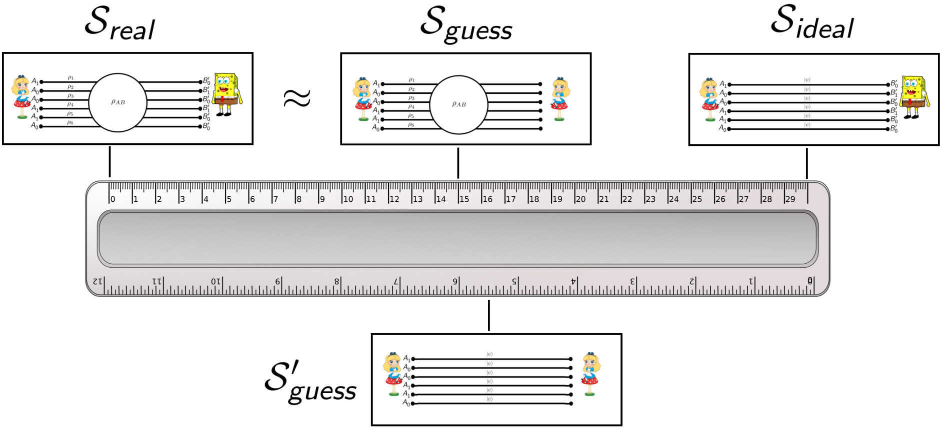

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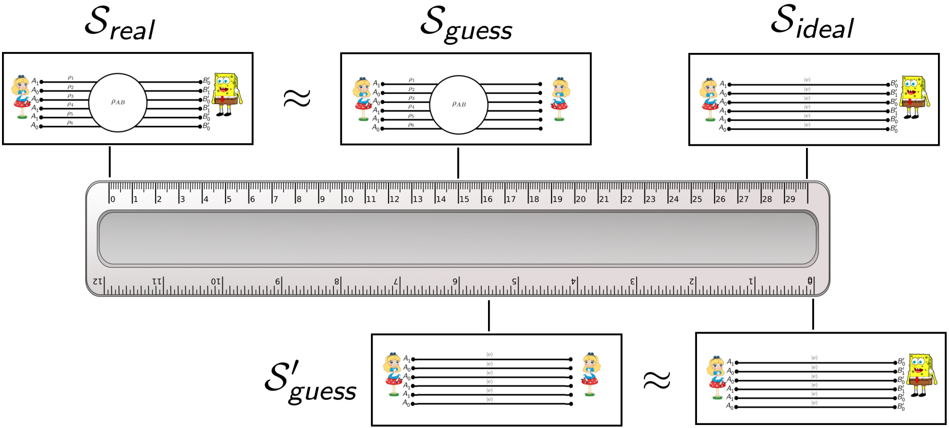
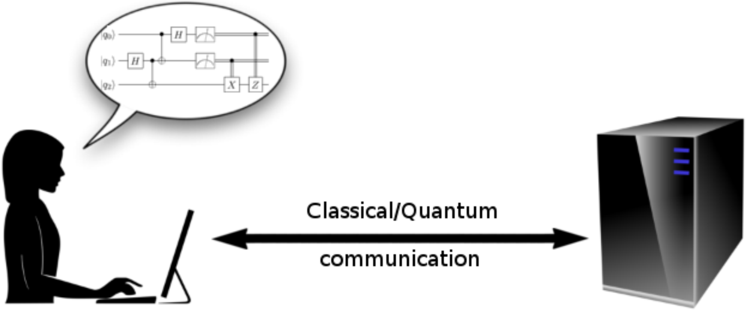


Application: verification of quantum computation



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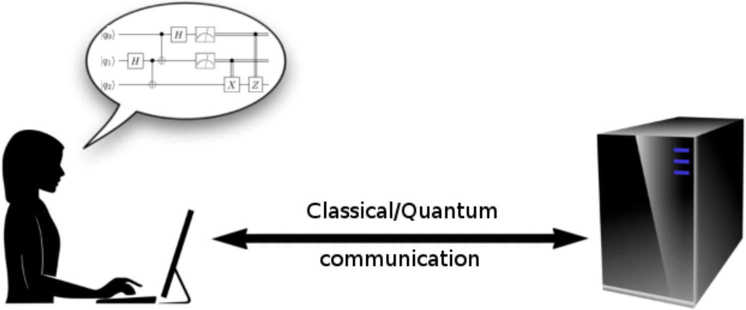

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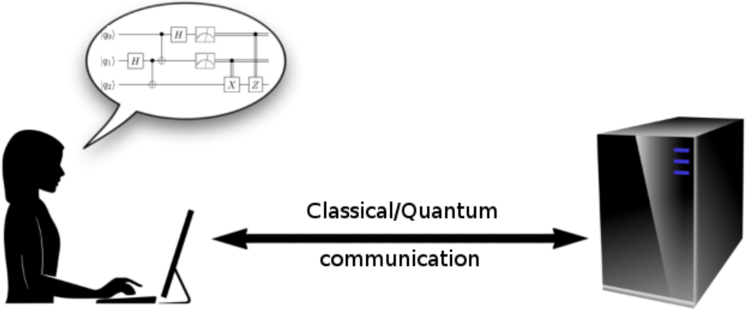

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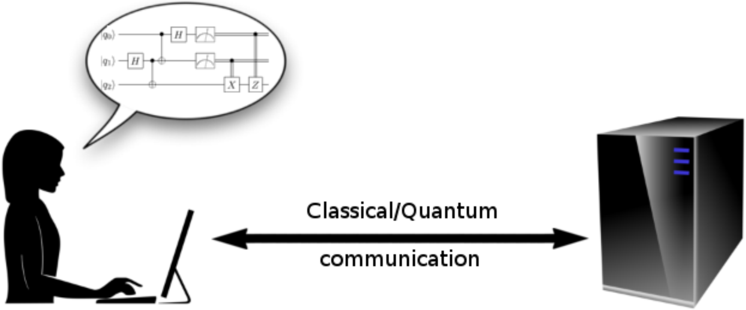

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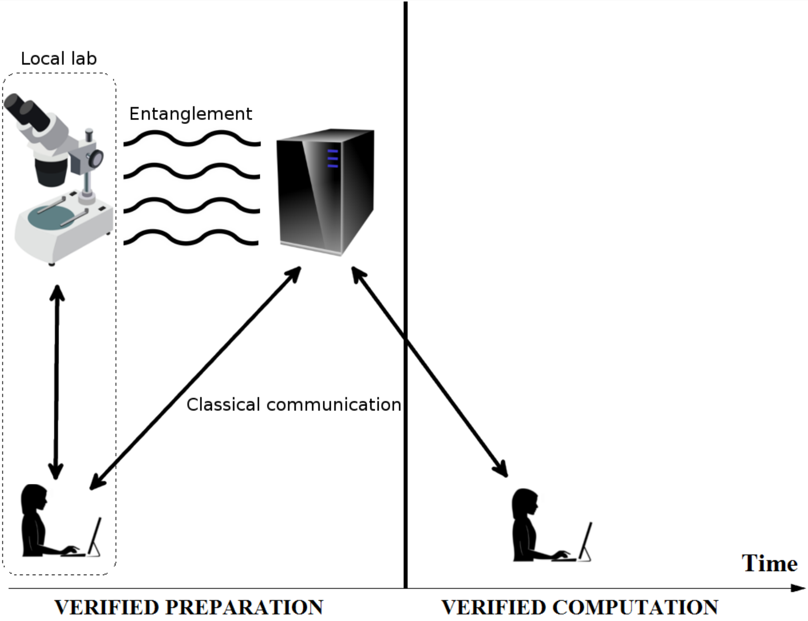

Conclusions

- Saturating correlations $\leftrightarrow$ ideal states and measurements
- I.i.d. self-testing $\rightarrow$ Non-i.i.d. self-testing $\rightarrow$ Rigidity
- Lower bounded $\Omega(\sqrt{\epsilon})$ closeness
- Tight bounds for non-i.i.d. and rigidity?
- Most natural application is quantum verification

Related work and references

Presentation based primarily on this work:
[Gheorghiu, Kashefi, Wallden, '15] - arXiv:1512.07401

Related works on **self-testing** and **rigidity**:
[Hoban, Šupić '16] - arXiv:1601.01552
[Bancal, Navascués, Scarani, Vértesi, Yang '13] - arXiv:1307.7053
[Reichardt, Unger, Vazirani '12] - arXiv:1209.0448
[McKague, Yang, Scarani '12] - arXiv:1203.2976

Related works on **verification**:
[Gheorghiu, Kashefi, Wallden '15] - arXiv:1502.02571
[Kashefi, Wallden '15] - arXiv:1510.07408
[McKague '15] - arXiv:1309.5675

Thank you!