

Bob Coecke & Aleks Kissinger

Universities of Oxford and Nijmegen

no-broadcasting in process theories

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...the full story in <15 mins

cf. H. Barnum, J. Barrett, M. Leifer and A. Wilce (2007) *Generalized no-broadcasting theorem*. PRL. arXiv:0707.0620

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no-broadcasting in process theories



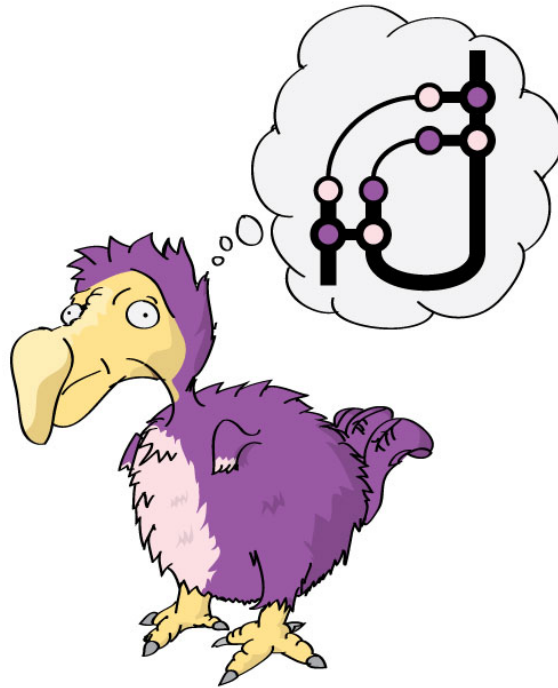
...including all math from scratch

cf. H. Barnum, J. Barrett, M. Leifer and A. Wilce (2007) *Generalized no-broadcasting theorem*. PRL. arXiv:0707.0620

Bob Coecke & Aleks Kissinger

Picturing Quantum Processes.

Cambridge University Press, 2016. (920pp)



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*Categorical quantum mechanics I:
causal quantum processes.*

arXiv:1510.05468

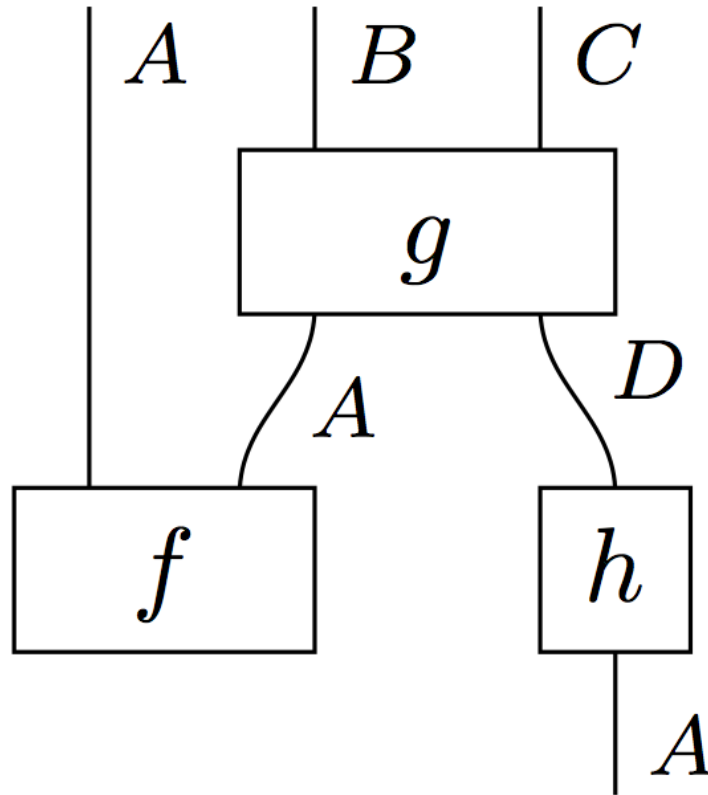
*Categorical quantum mechanics II:
classical-quantum interaction.*

arXiv:1605.08617

*Categorical quantum mechanics III:
(strong) complementarity.*

arXiv:XXX (next couple of weeks)

Diagram := boxes with ins/outs wired together



Process theory := interpretation of diagrams

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- assign to each wire a **system**
- assign to each box a **process**

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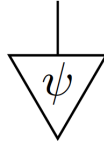
- assign to each wire a **system**
- assign to each box a **process**
- **interprets ‘wiring together’** e.g.:
 - as actual physical wires,

Process theory := interpretation of diagrams

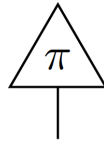
- assign to each wire a **system**
- assign to each box a **process**
- **interprets ‘wiring together’** e.g.:
 - as actual physical wires, or,
 - via decomposing in \circ and \otimes .

Special processes:

- **State** :=

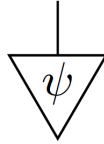


- **Effect / Test** :=

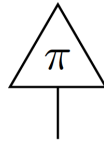


Special processes:

- **State** :=



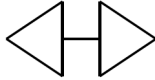
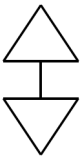
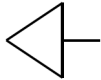
- **Effect / Test** :=



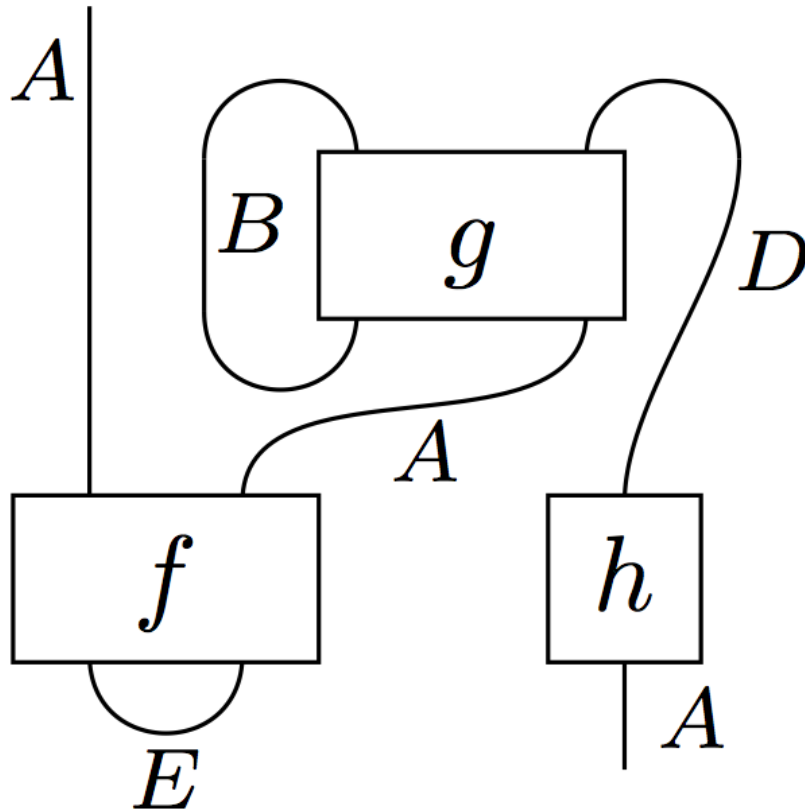
- **Number** :=



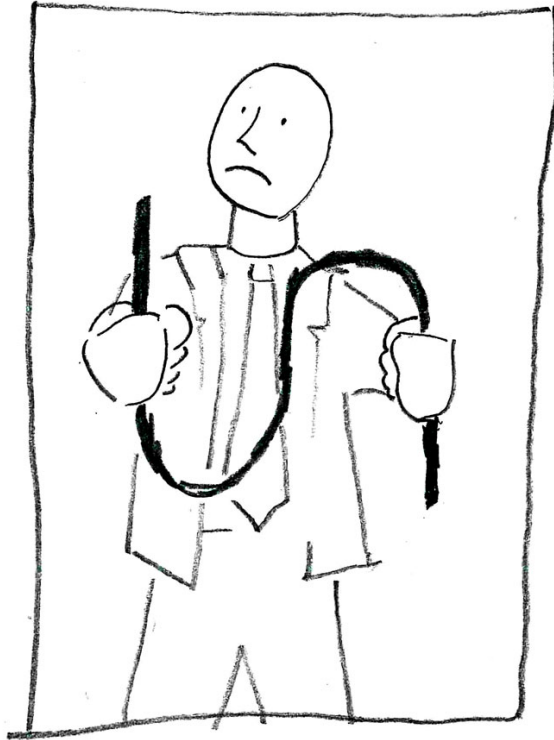
Special processes:



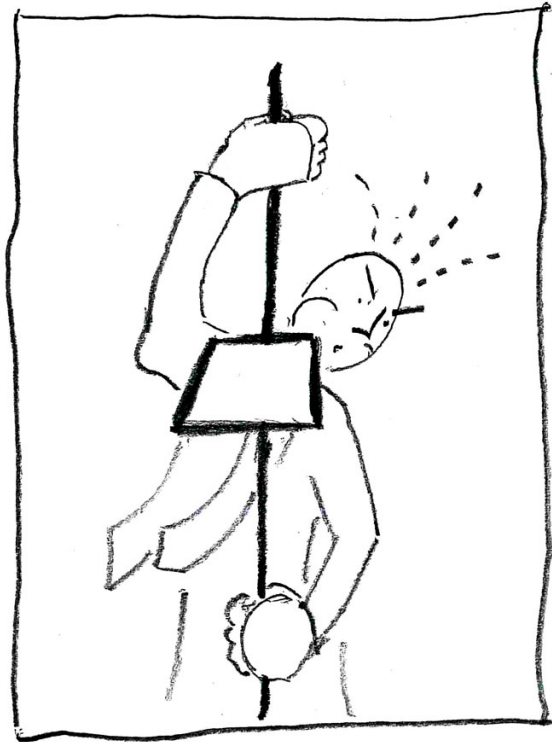
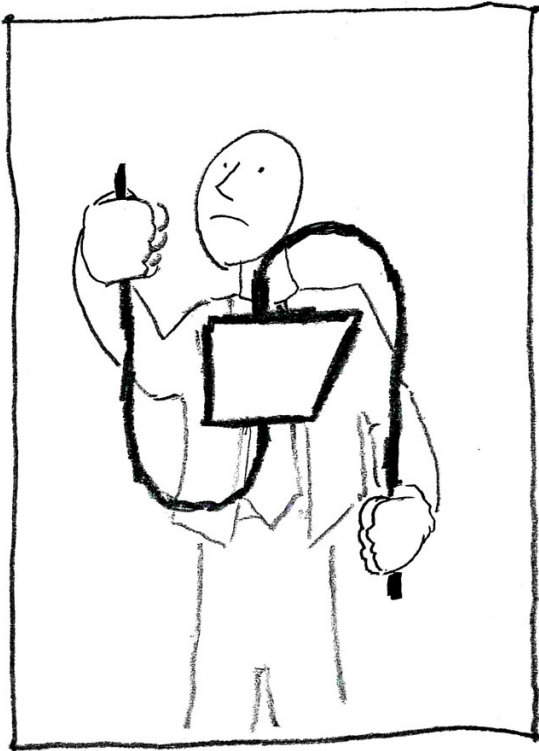
String diagram := ins and outs on equal footing



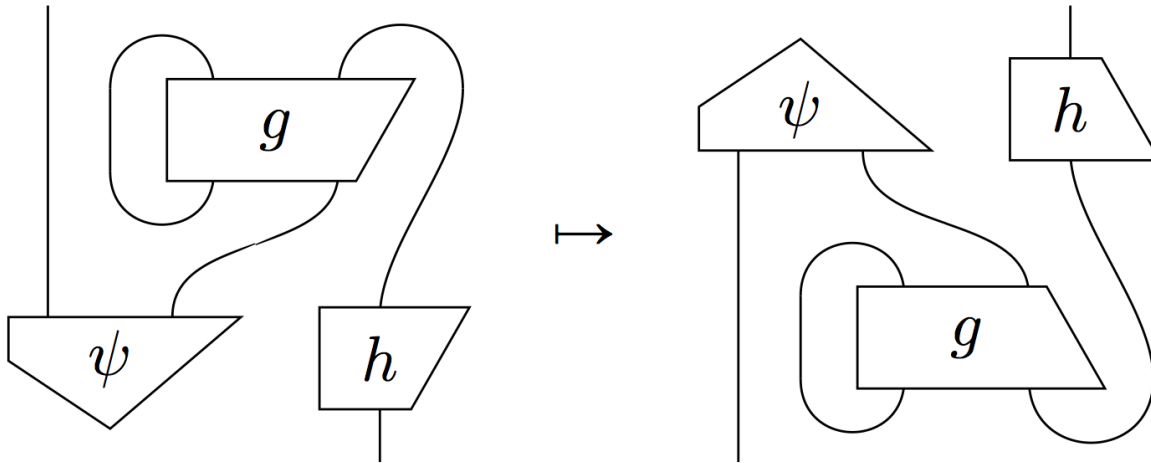
– teleportation –



– transpose –

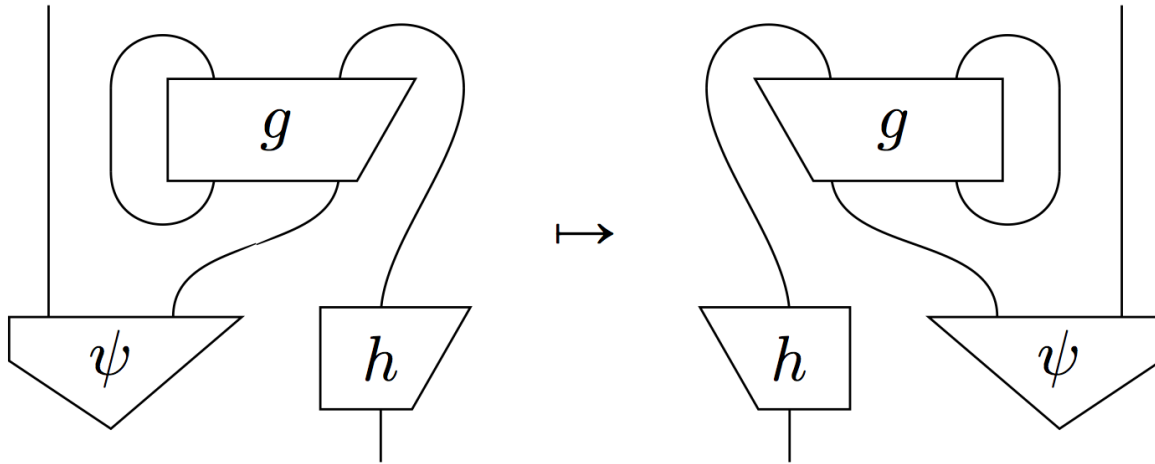


Adjoint := **H-reflection**



...conjugate

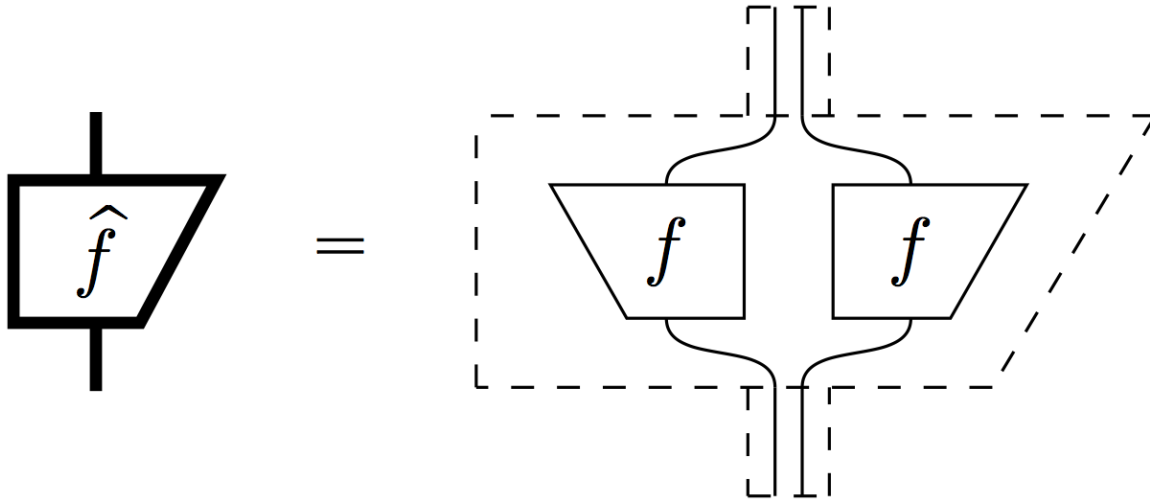
...conjugate := V-reflection



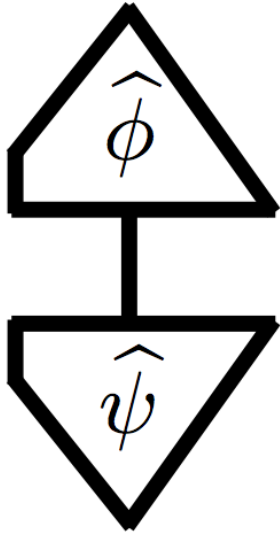
Main idea:

$$\frac{\text{classical system}}{\text{quantum system}} = \frac{\text{single wire}}{\text{double wire}}$$

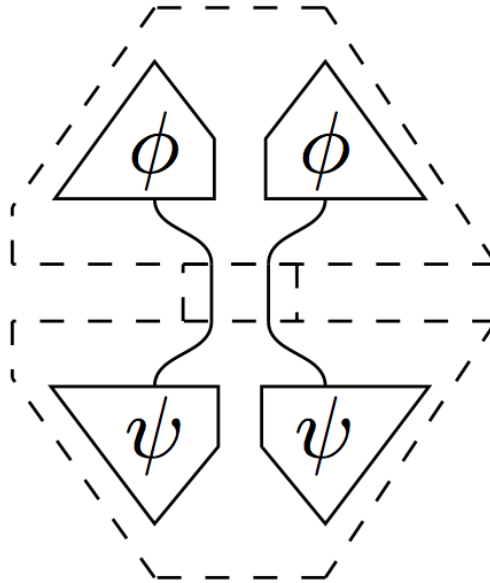
Pure quantum process :=



Born-rule ::=

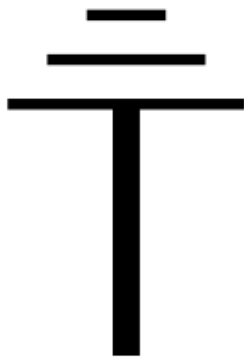


::=

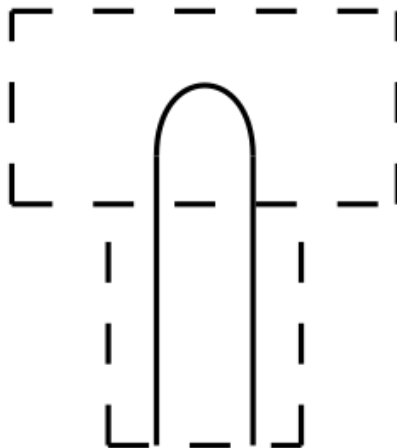


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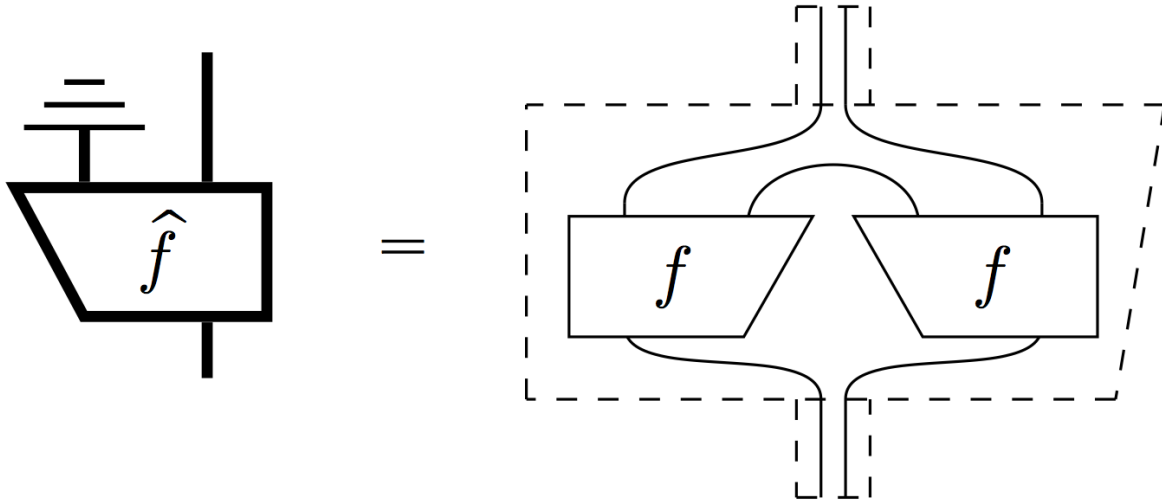
Discarding :=



:=

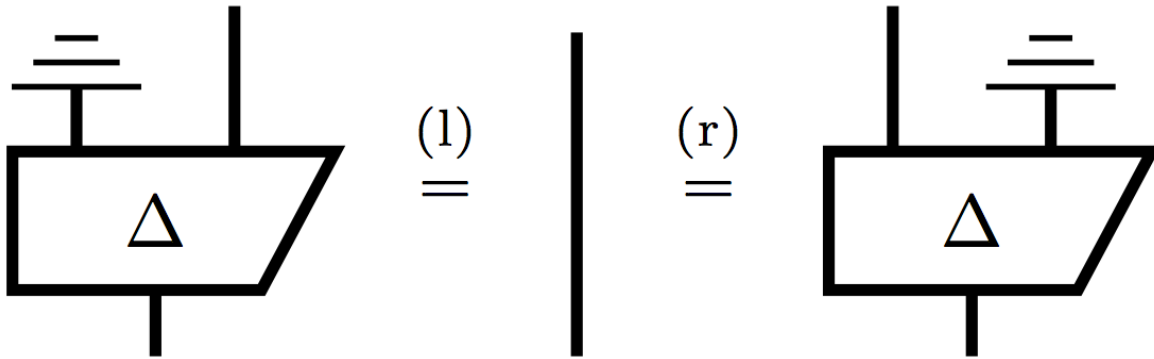


General quantum process :=

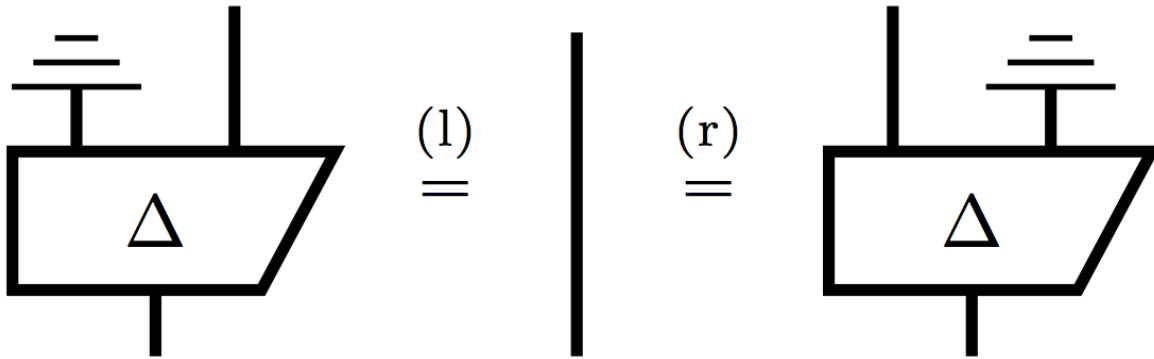


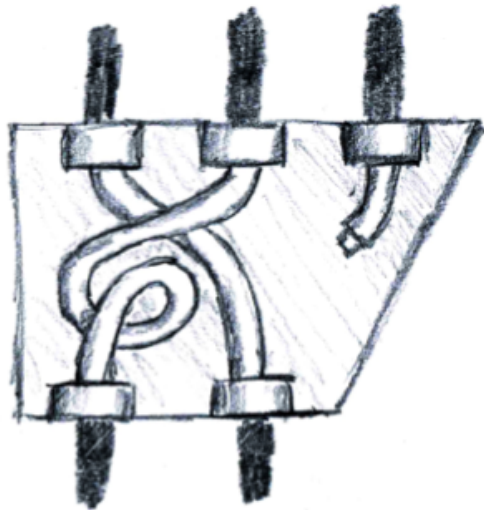
THM. No universal broadcasting

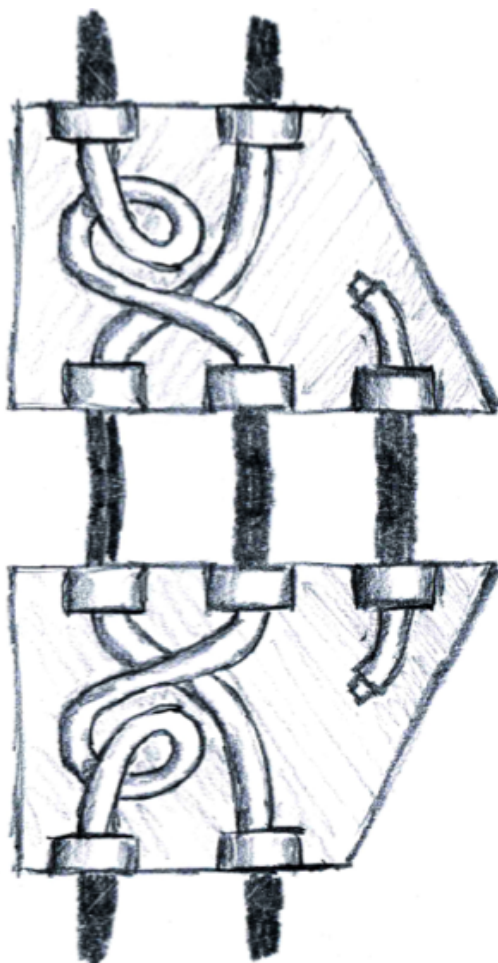
THM. No universal broadcasting i.e. no Δ s.t.:



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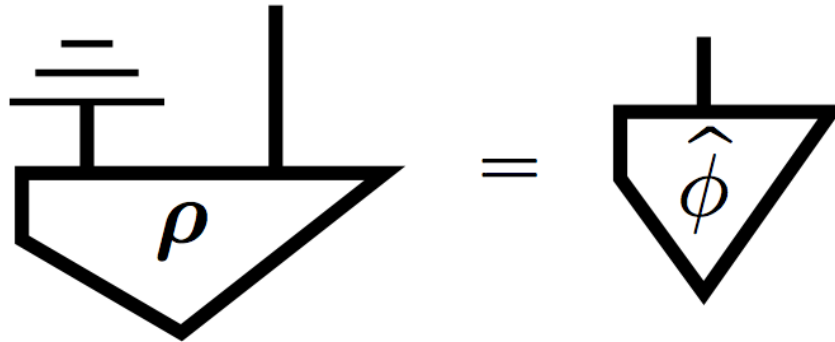




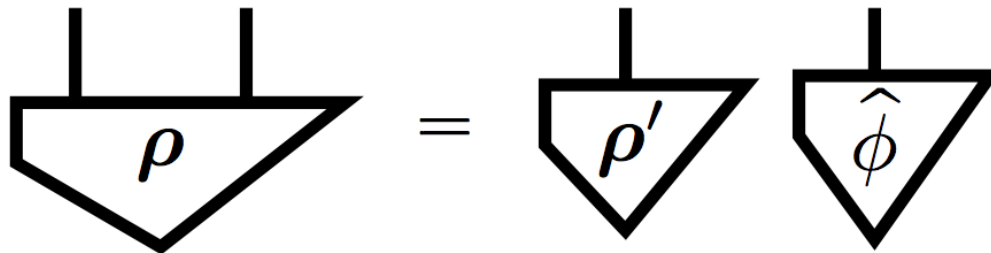


$$\left(\exists \psi, \phi : \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \diagup \\ f \\ \diagdown \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \nabla \\ \phi \\ \nabla \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \nabla \\ \psi \\ \nabla \end{array} \right) \iff \left(\exists \psi', \phi' : \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \nabla \\ f \\ \nabla \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \diagup \\ f \\ \diagdown \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \nabla \\ \phi' \\ \nabla \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \nabla \\ \psi' \\ \nabla \end{array} \right)$$

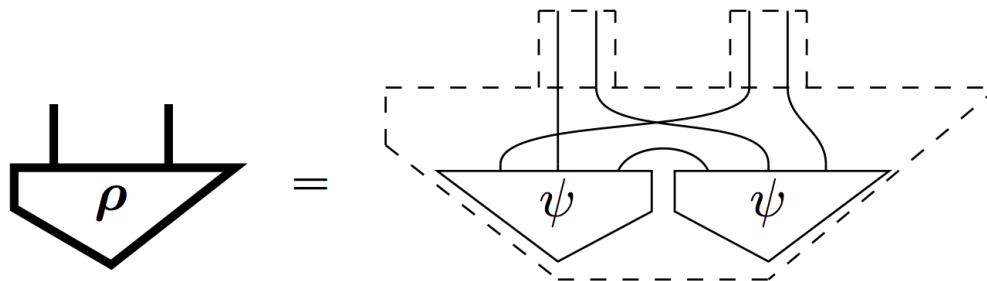
Prop. Pure reduced state:



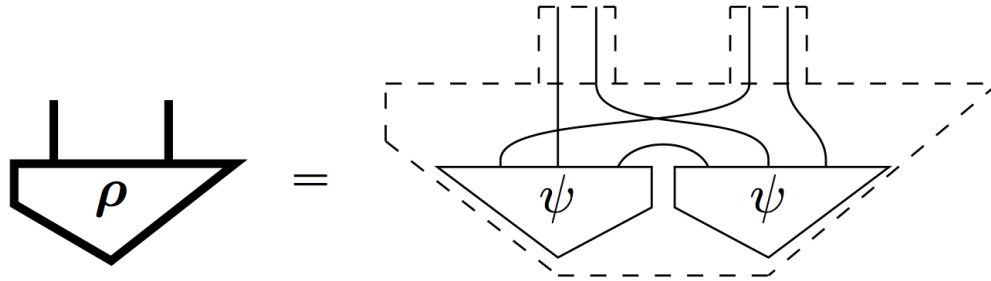
implies separation:



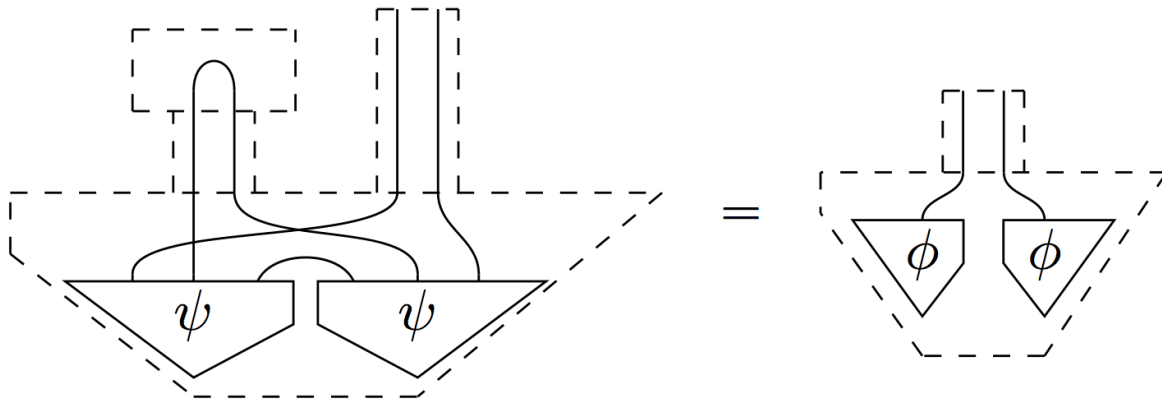
Pf. Since:



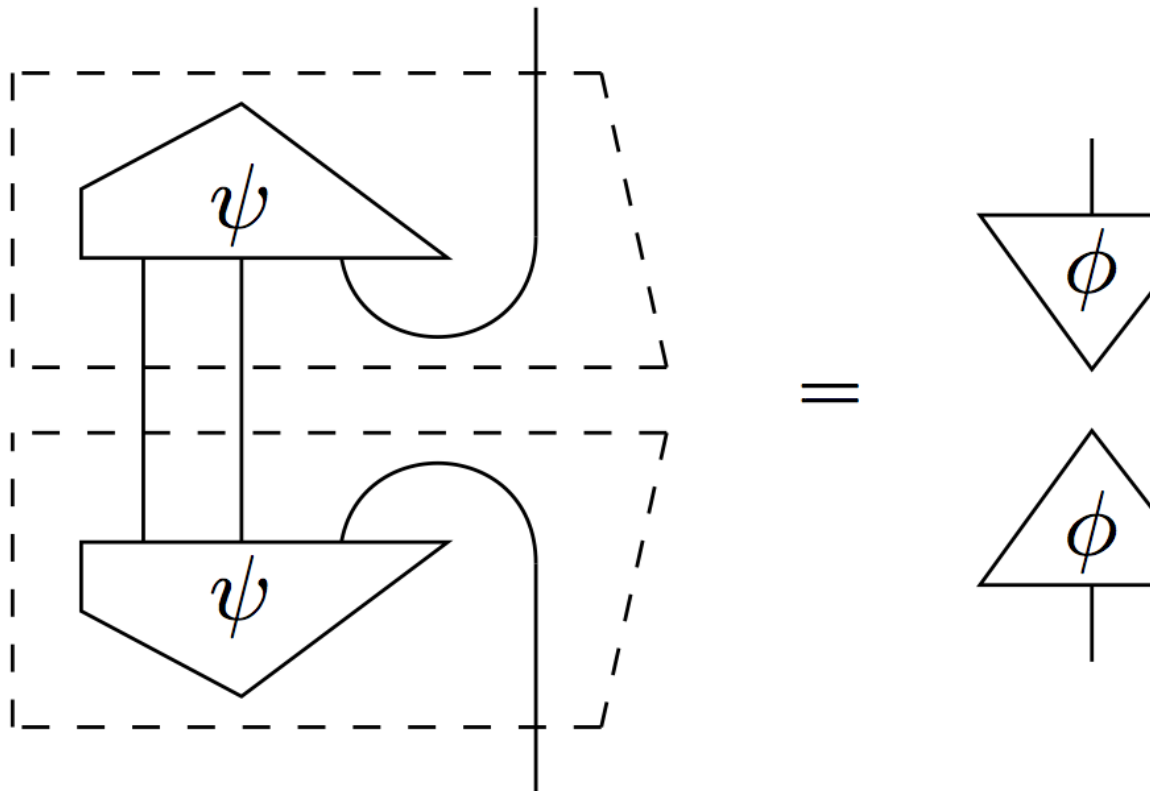
Pf. Since:



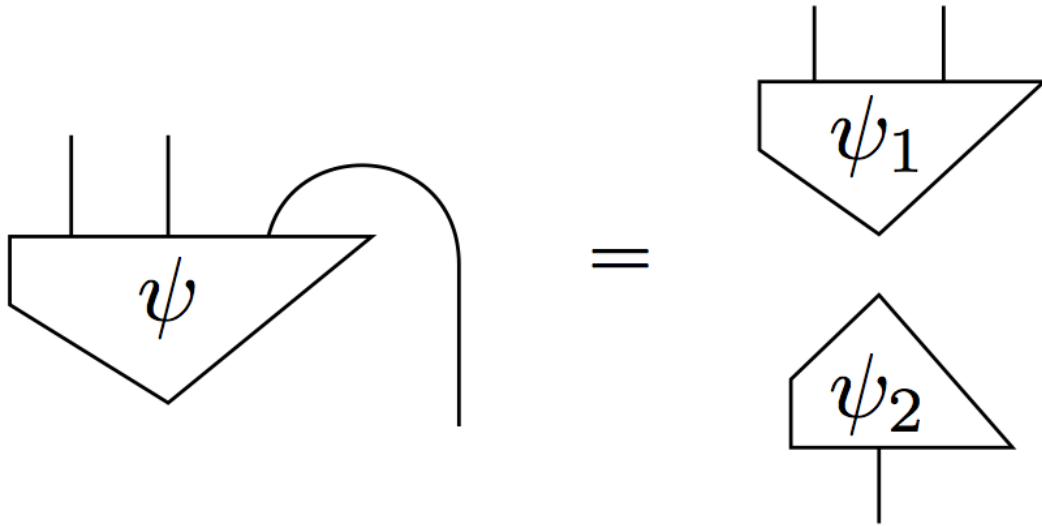
we have:



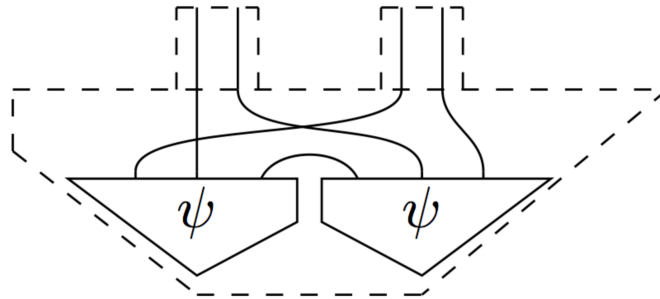
i.e.:



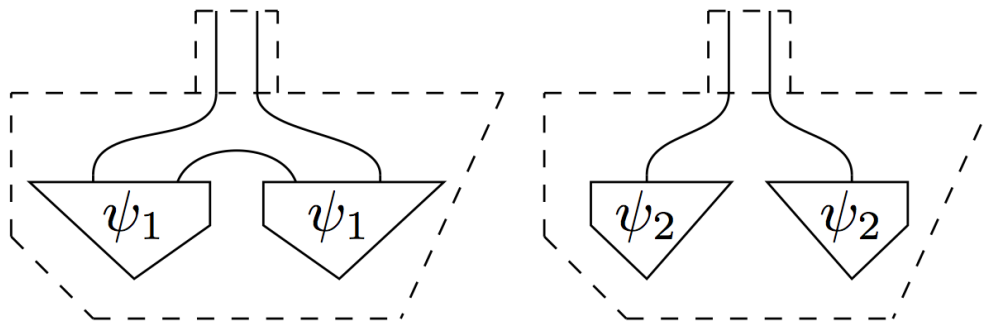
so:



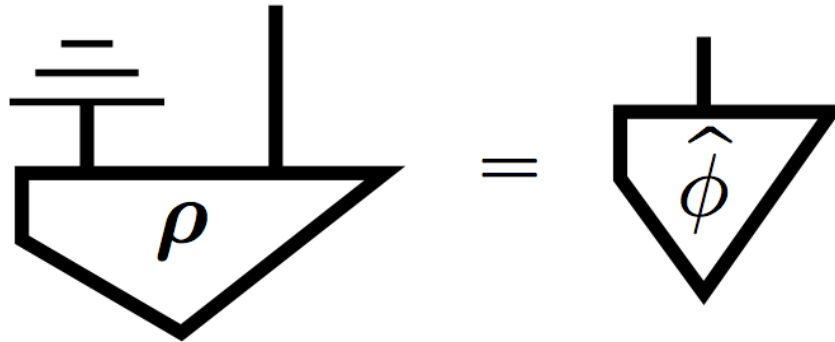
and hence:



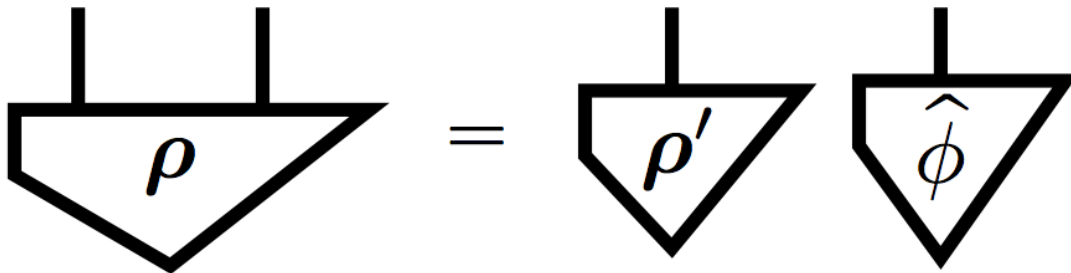
separates:



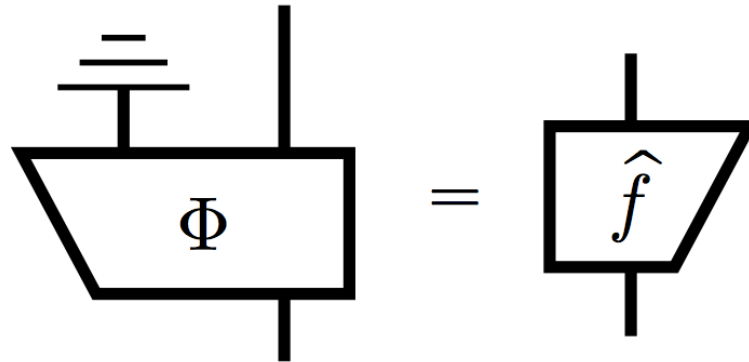
Prop. Pure reduced state:



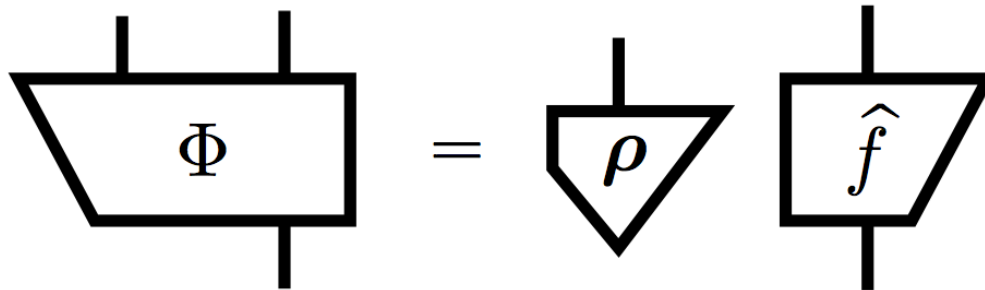
implies separation:



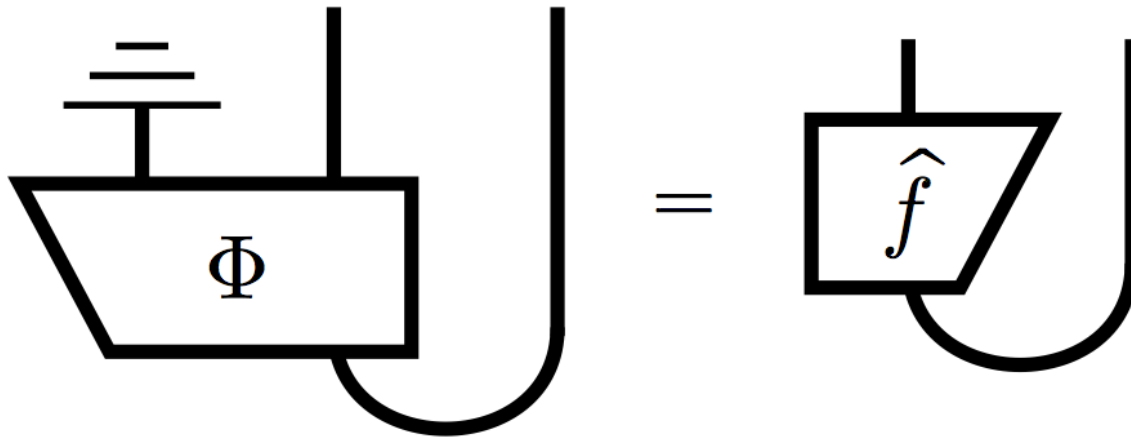
Prop. Pure reduced process:



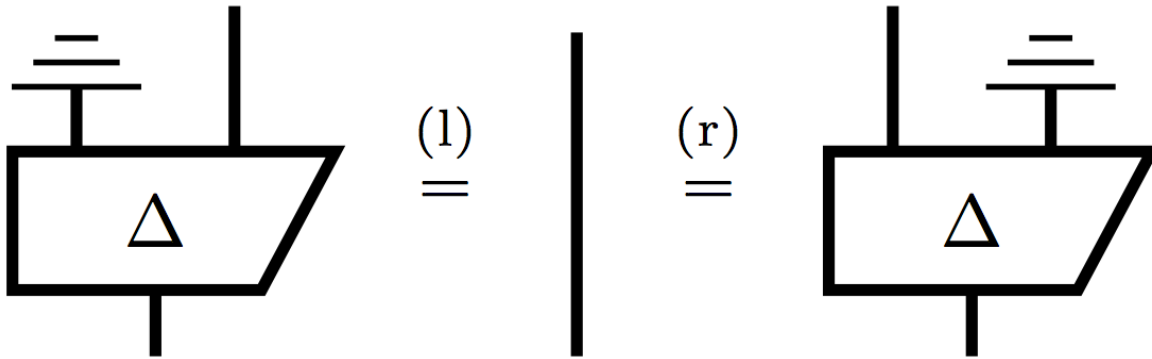
implies separation:



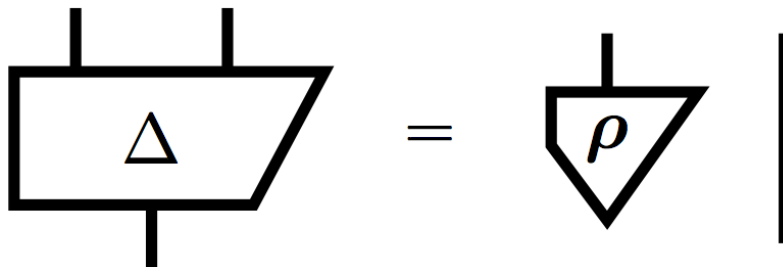
Pf. Reduces to previous:



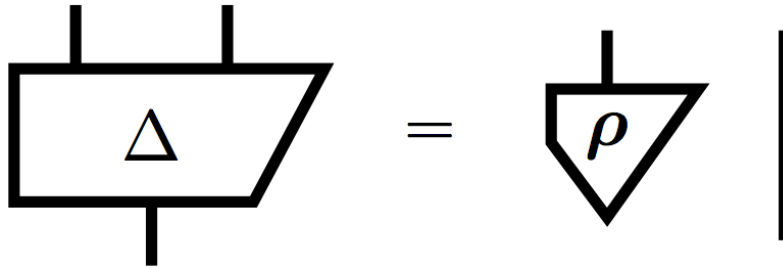
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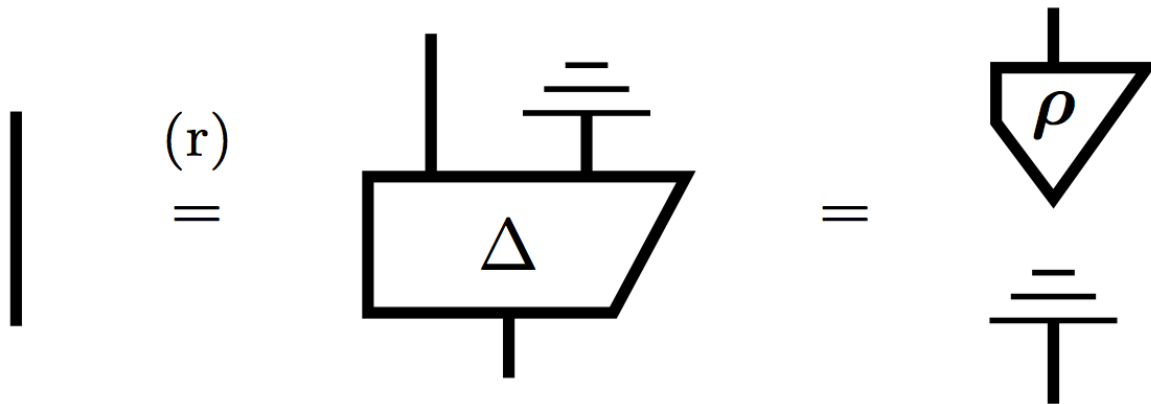
Pf. By (1) and Prop:



Pf. By (1) and Prop:



so:



RIP

(3/3/2009 – 6/6/2016)

