Causality and indefinite causal order in quantum theory

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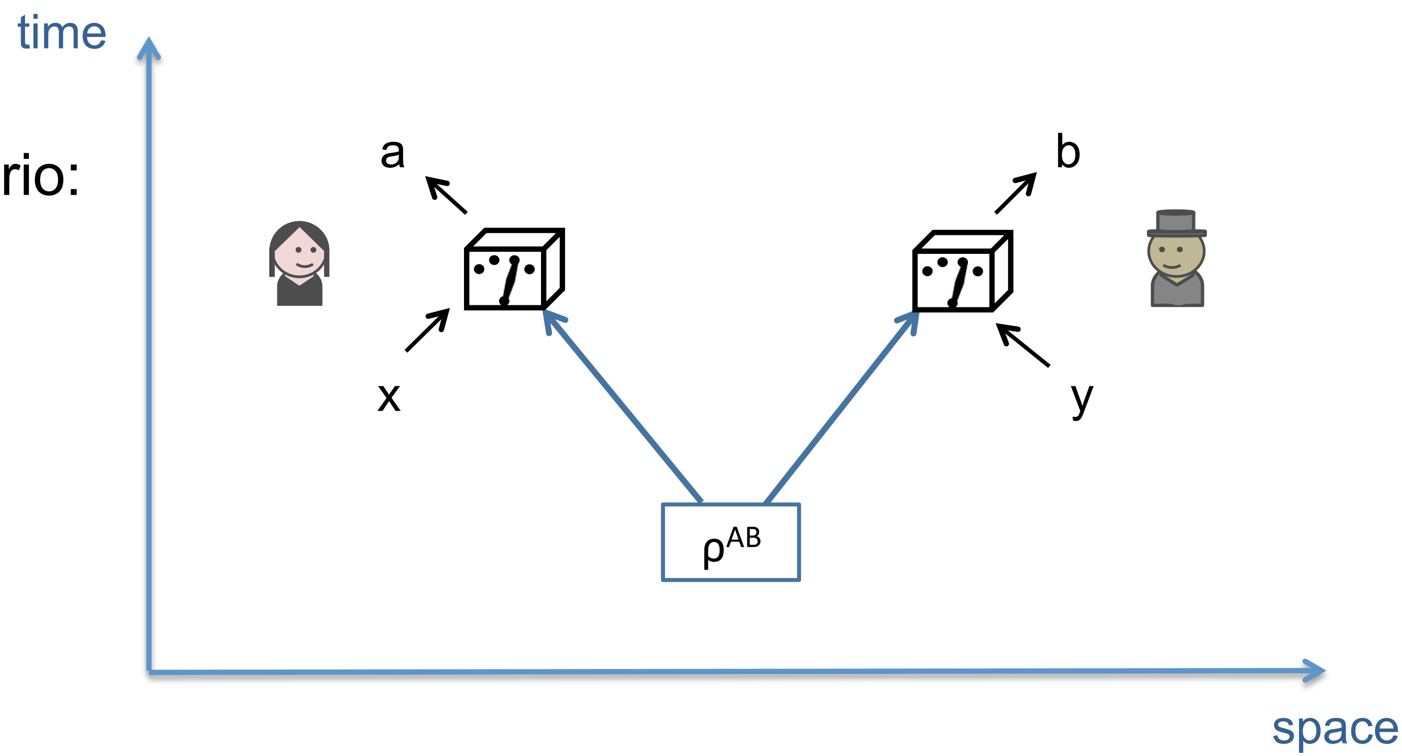
Quantum theory challenges classical notions of causality

Bell scenario:



In quantum theory: $$p(a,b|x,y) = \mathrm{Tr}[(E^A_{a|x} \otimes E^B_{b|y})\rho^{AB}]$$

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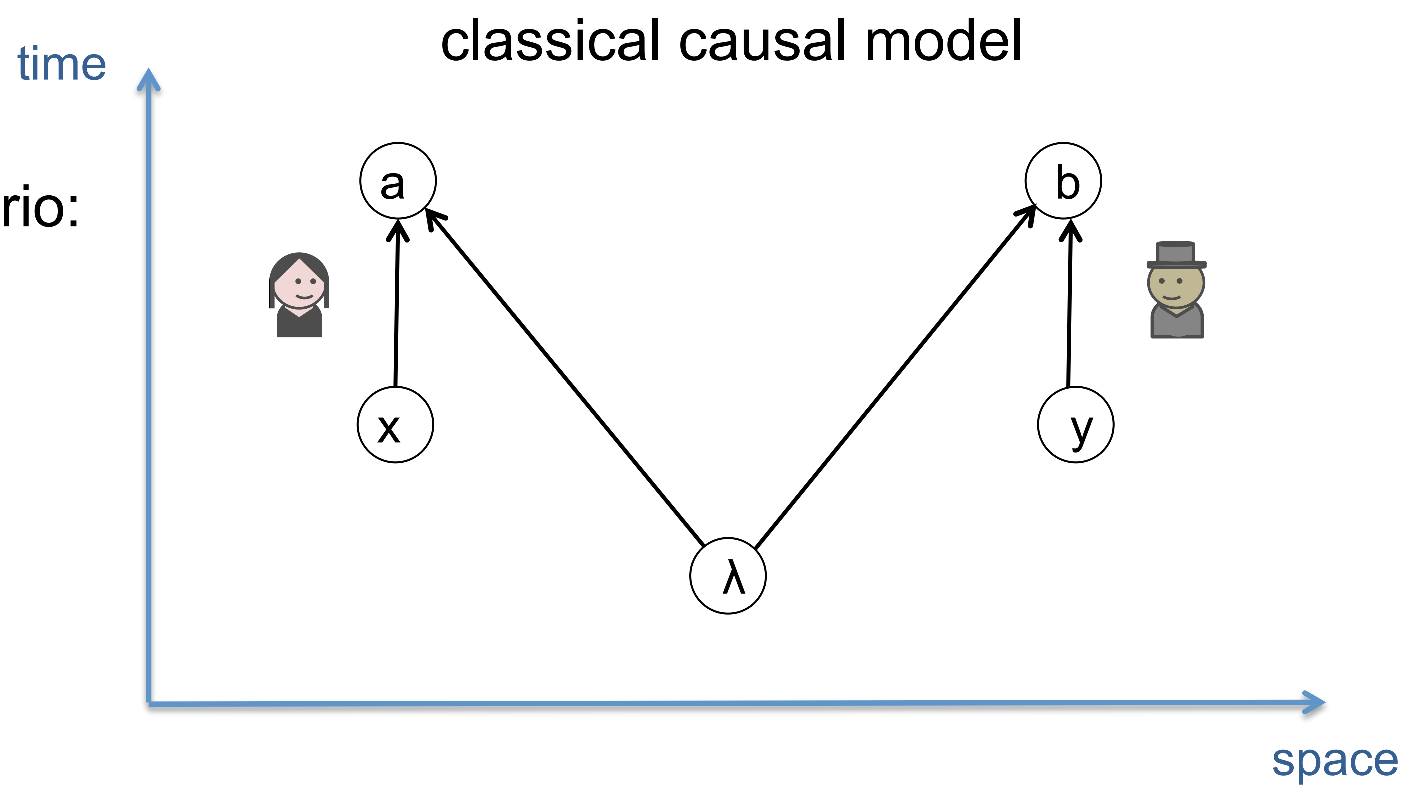

Incompatible with quantum theory!

$$p(a,b|x,y) = \sum_{\lambda} p(a|x,\lambda)p(b|y,\lambda)p(\lambda)$$

Quantum theory challenges classical notions of causality

Yet, **signaling** between space-like separated locations is **impossible**.

(QT respects the causal structure of space-time)

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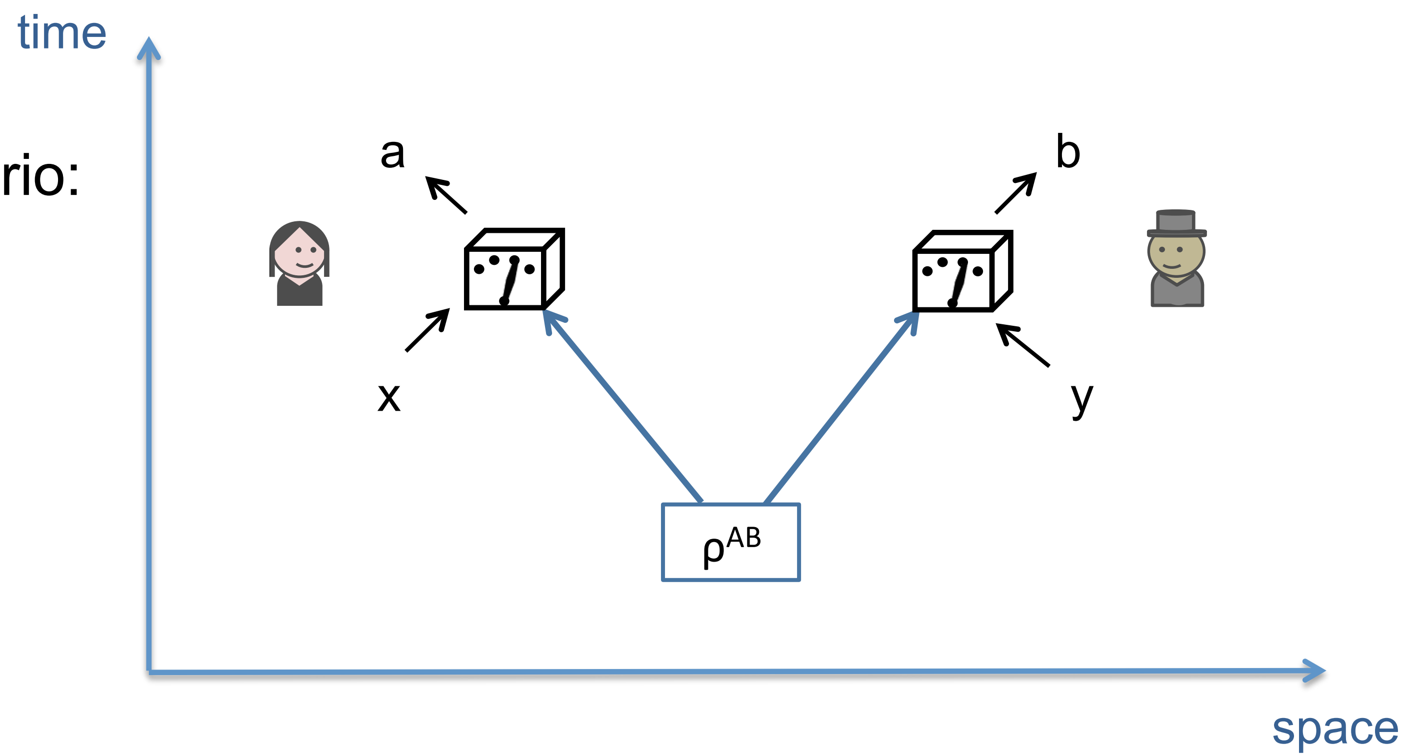

In quantum theory: $p(a|x,y) = p(a|x) \quad , \quad p(b|x,y) = p(b|y)$

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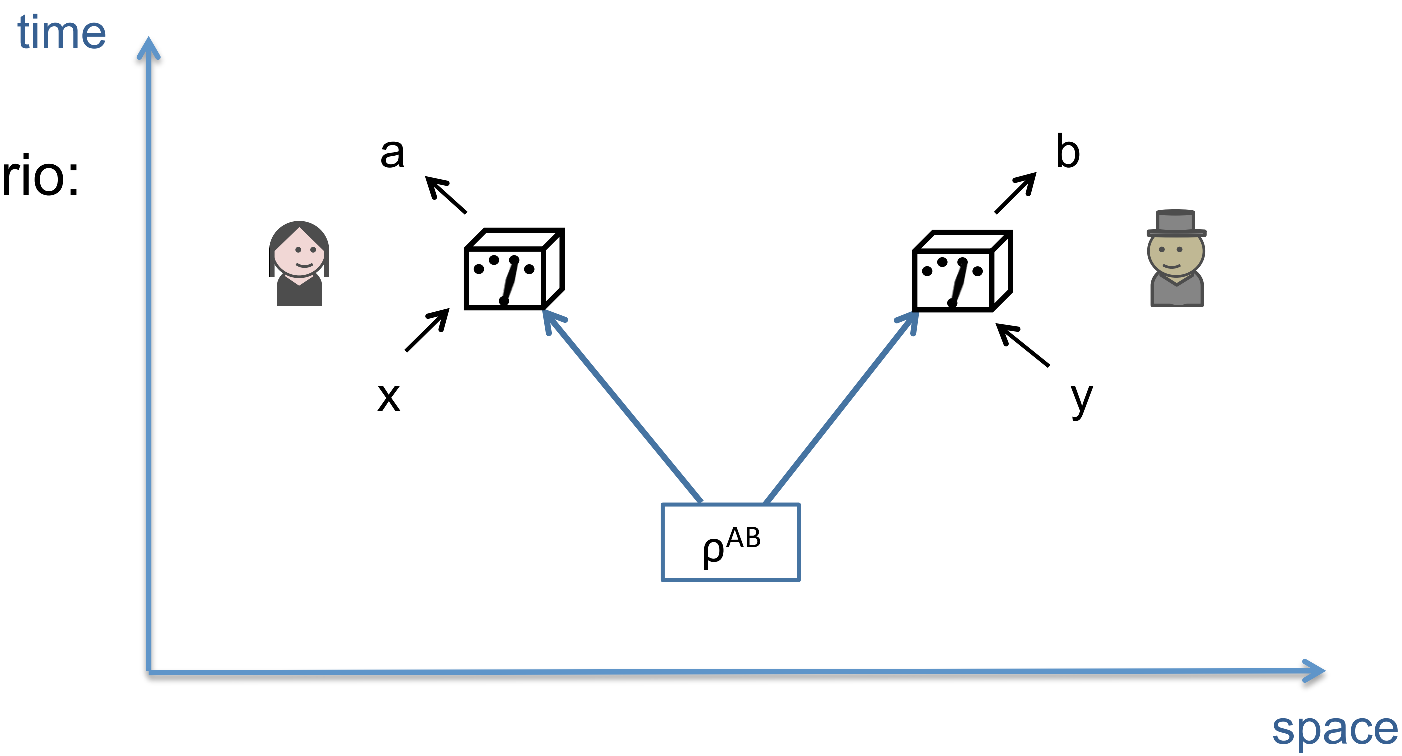
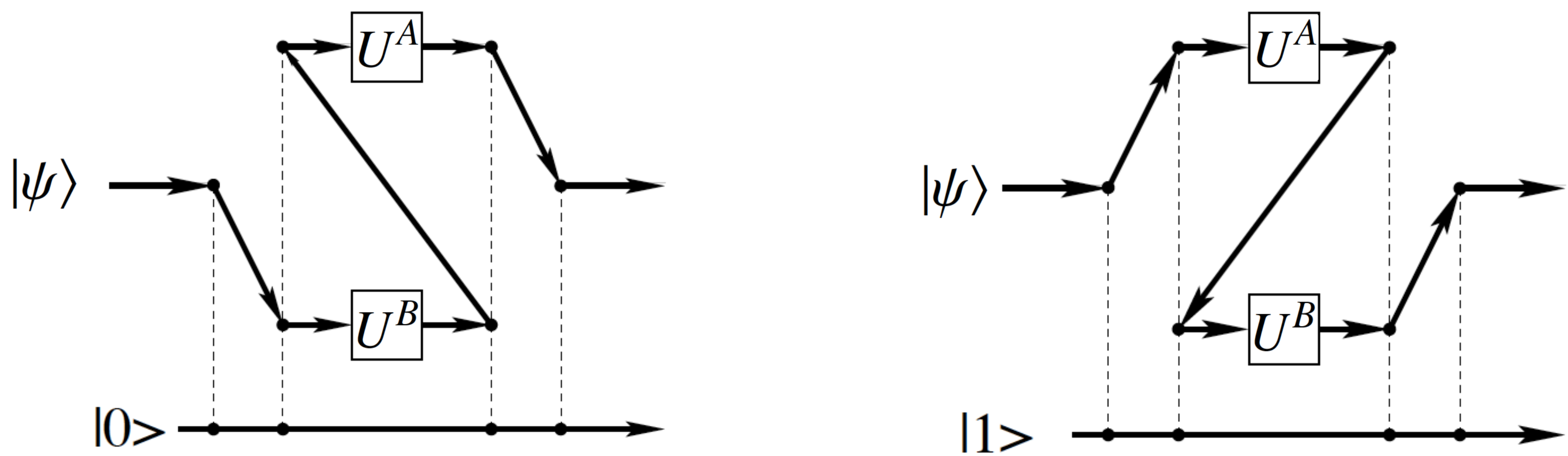


A more general, genuinely quantum, notion of causality may be needed?

Quantum theory challenges classical notions of causality

The order of operations could depend on a variable in a quantum superposition:

(indefinite causal structures?)



quantum SWITCH

$$(\alpha|0\rangle + \beta|1\rangle)|\psi\rangle \to \alpha|0\rangle U^A U^B|\psi\rangle + \beta|1\rangle U^B U^A|\psi\rangle$$

Chiribella, D'Ariano, Perinotti, and Valiron, PRA 88, 022318 (2013), arXiv:0912.0195 (2009)

Quantum theory challenges classical notions of causality

More generally, in a quantum theory of gravity, we expect scenarios with

indefinite causal structure (Hardy, http://arxiv.org/abs/gr-qc/0509120).

Questions:

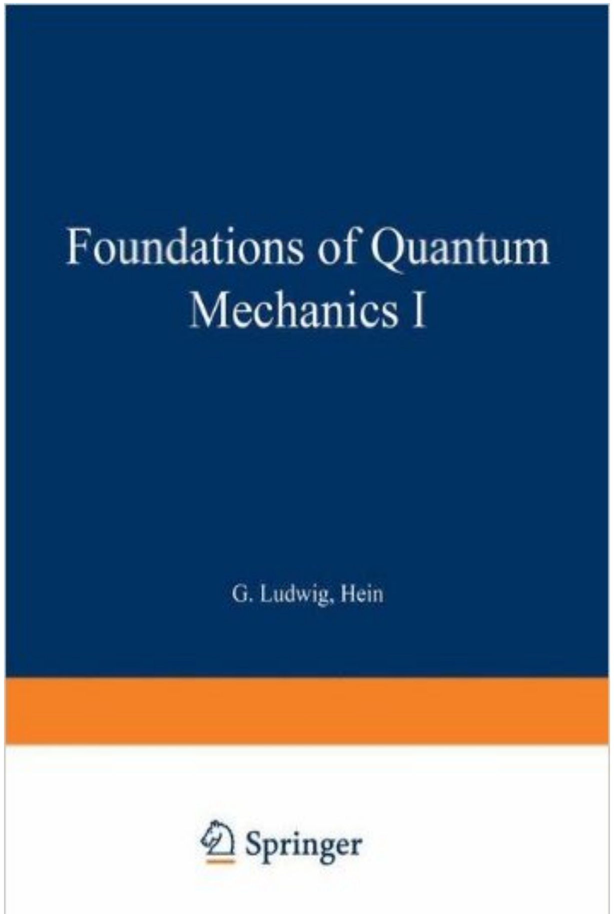
Can we generalize quantum theory such that a predefined causal structure is not assumed?

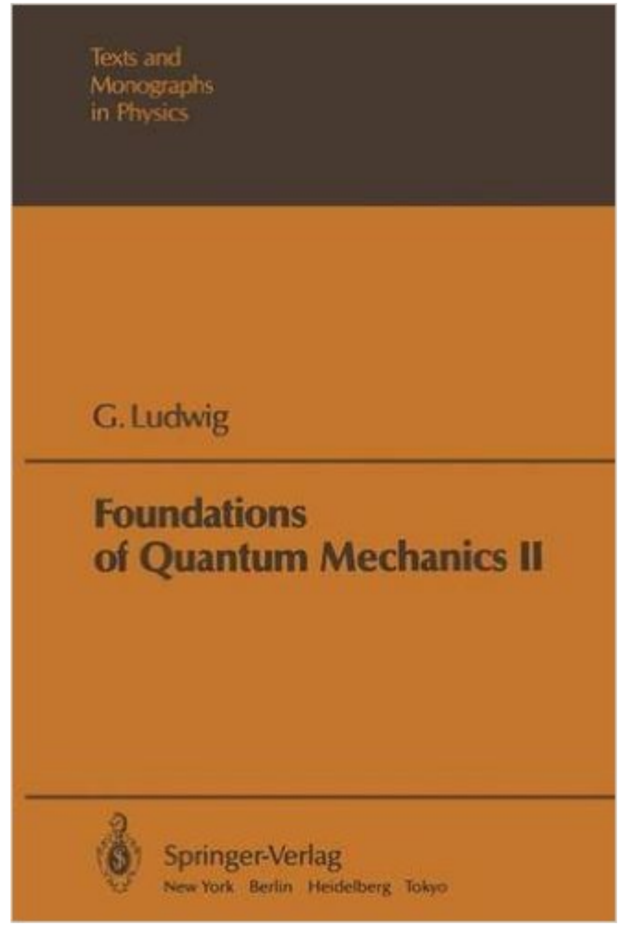
What new possibilities would follow from such a generalization?

Outline

- Quantum theory as an operational probabilistic theory in the circuit framework
- The axiom of causality and its meaning
- The process matrix framework for local operations without global causal structure
  - causal inequality violations
  - causal versus causally separable processes
  - dynamical causal relations
- A time-symmetric operational approach
- Quantum theory without any predefined causal structure

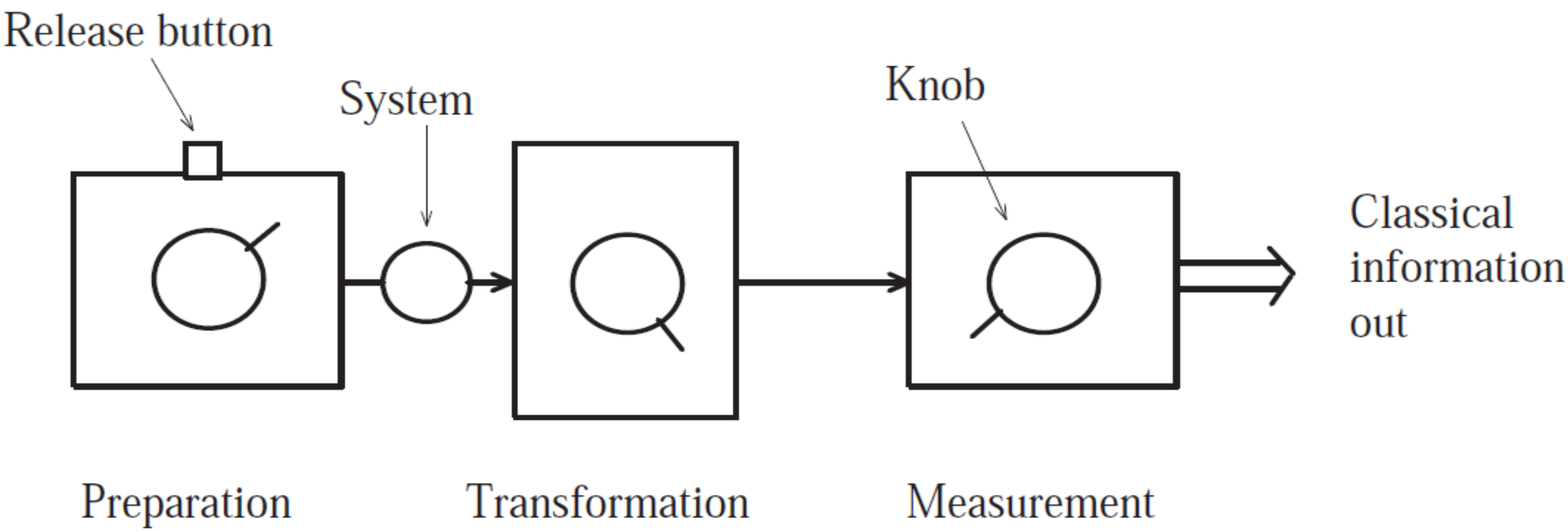
Operational Approach





Ludwig (1983, 1985)

Operational Approach



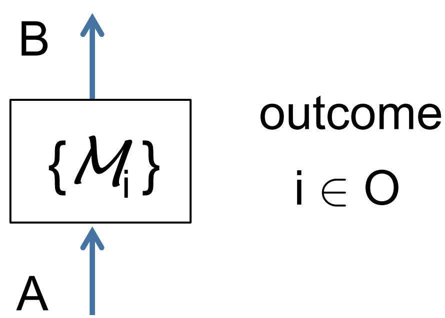
from Hardy arXiv:quant-ph/0101012 (2001)

A theory prescribes probabilities for the outcomes of operations.

Hardy (2001), Barrett (2005), Dakic and Brukner (2009), Massanes and Mülelr (2010),
Hardy (2009), Chiribella, D'Ariano, and Perinotti (2009, 2010), Hardy (2011), Barnum, Mülelr, Udedec (2014)…

The circuit framework for operational probabilistic theories

Operation (test): one use of a device with an input and an output system:



Hardy, PIRSA:09060015;
Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010) [arXiv 2009],
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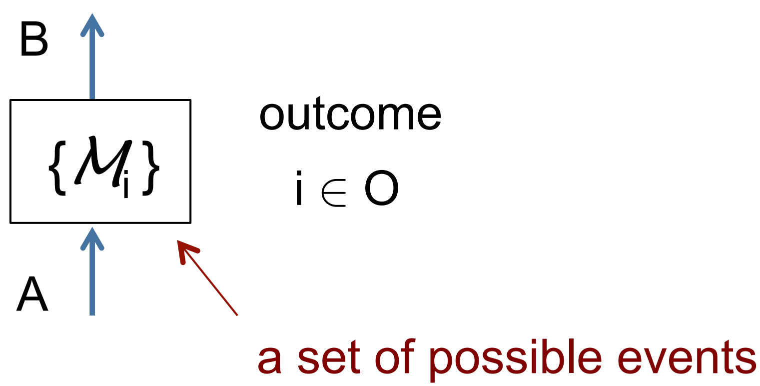

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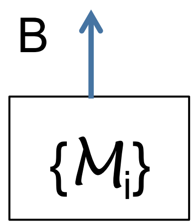
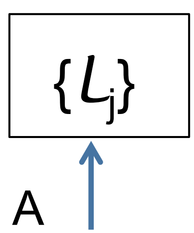
The circuit framework for operational probabilistic theories

Preparations (the input system is the trivial system I):



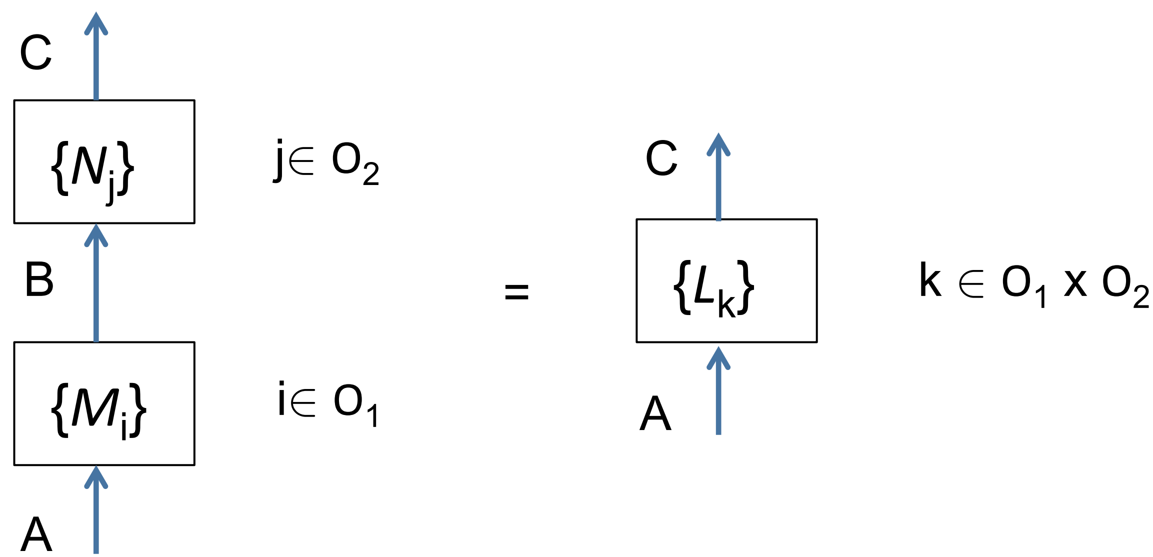
Measurements (the output system is the trivial system I):



The circuit framework for operational probabilistic theories

Operations can be *composed* in sequence and in parallel without forming loops:

Sequential composition:



For foundations of compositional theories: see, e.g., Abramsky and Coecke, Quantum Logic and Quantum Structures, vol II (2008). Coecke, Contemporary Physics 51, 59 (2010).

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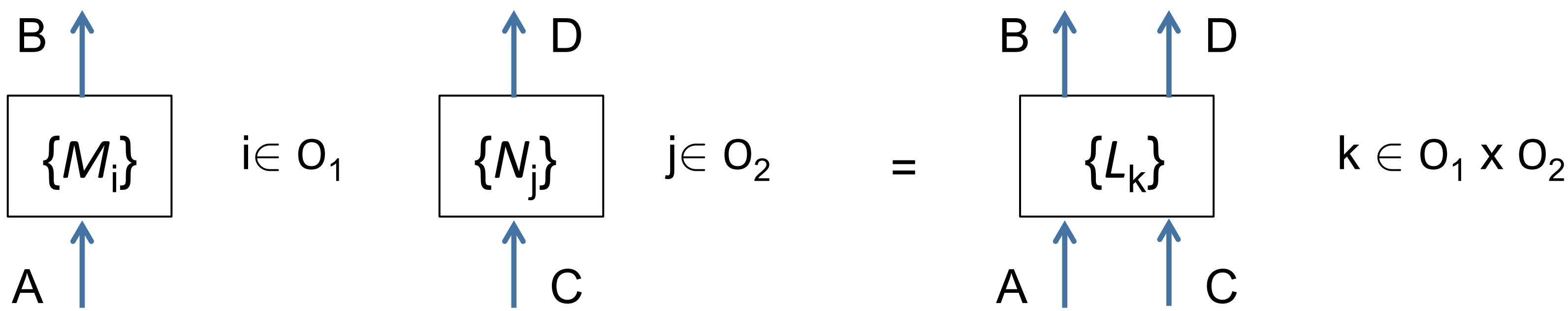
Parallel composition:



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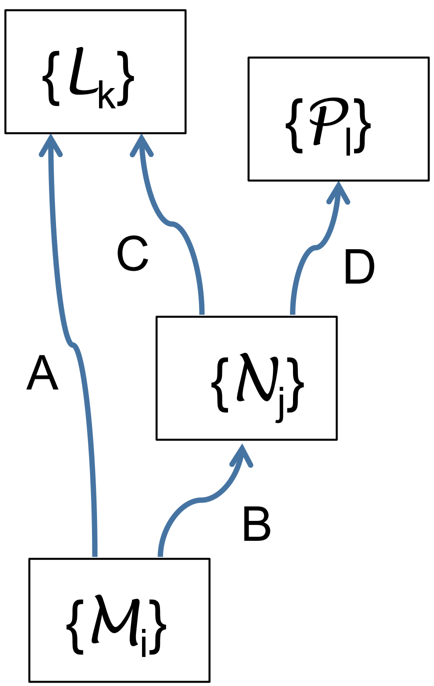
Circuit (an acyclic composition of operations with no open wires):



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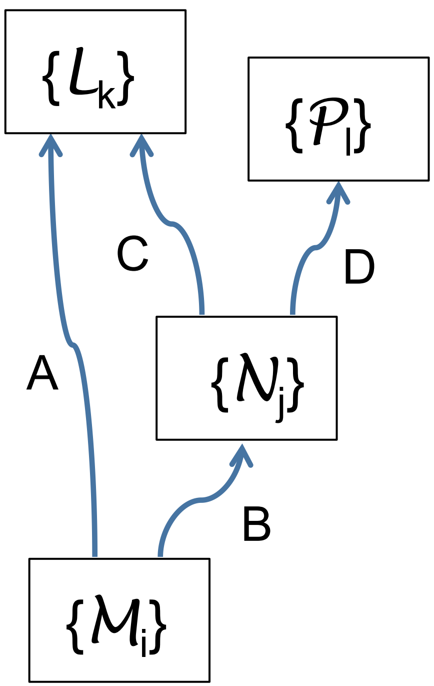


Probabilistic structure

Joint probabilities

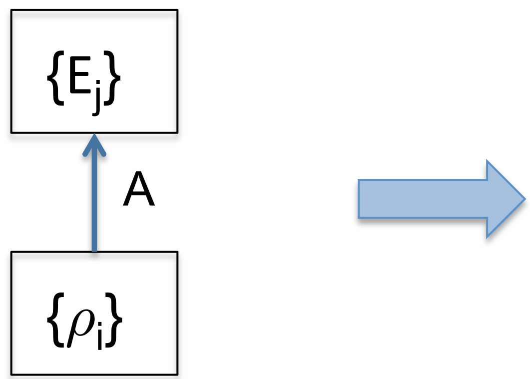
p(i, j, k, l| circuit)

p(i, j, k, l| circuit) $\geq$ 0

$\sum_{ijkl}$ p (i, j, k, l|circuit)=1

The circuit framework for operational probabilistic theories

Equivalently,



Joint probabilities

$p(i, j| \{\rho_i\},\{E_j\})$

The circuit framework for operational probabilistic theories

An OPT is completely defined by specifying all possible operations and the probabilities for all possible circuits.

If two operations yield the same probabilities for all possible circuits they may be part of, they are deemed *equivalent*.

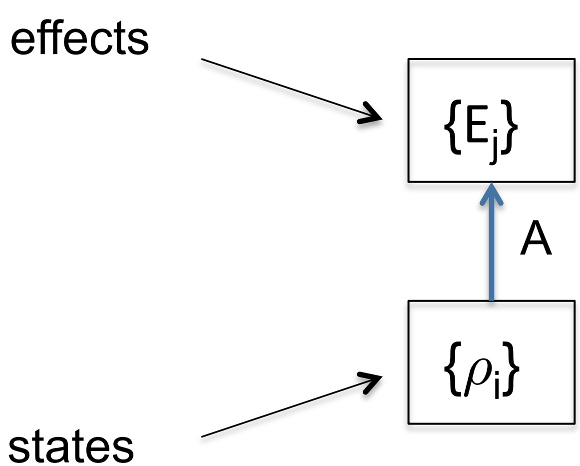
If two events (which may be part of different operations) yield the same probabilities for all possible circuits they may be part of, they are deemed *equivalent*.

States: equivalence classes of preparation events

Effects: equivalence classes of measurement events

Transformations: equivalence classes of general events from A to B

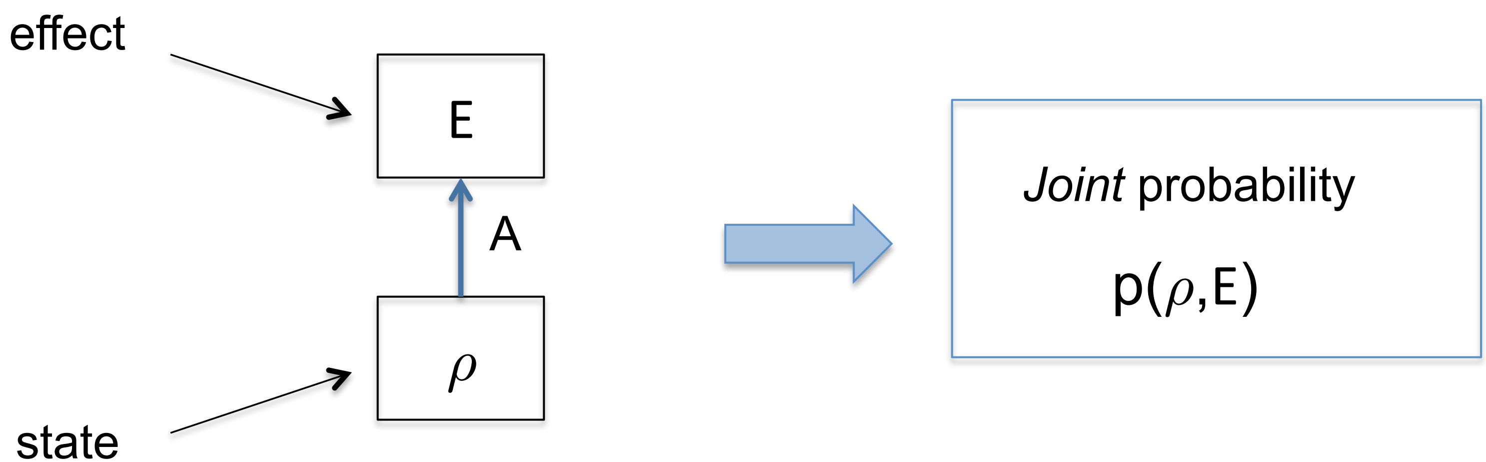
The circuit framework for operational probabilistic theories



$$p(i, j|\ \{\rho_i\},\{E_j\}) = p(\rho_i, E_j)$$

(non-contextual) function of the respective state and effect

The circuit framework for operational probabilistic theories



States are real functions on effects, and vice versa.

(elements of two dual vector spaces)

The case of standard quantum theory

System A → Hilbert space $\mathcal{H}^A$ of dimension d_A .

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Transformation from A **to** B → completely positive (CP) linear map

$$\mathcal{M}^{A\to B} \ : \ \mathcal{L}(\mathcal{H}^A) \ \to \ \mathcal{L}(\mathcal{H}^B)$$

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Operation from A **to** B → $\{\mathcal{M}_i^{A\to B}\}_{i\in O}$

where $\sum_{i\in O} \mathcal{M}_i^{A\to B} = \overline{\mathcal{M}}^{A\to B}$ is trace preserving (CPTP).

The case of standard quantum theory

State: $\rho^{I\to A}(\cdot) = \sum_{\alpha=1}^{d_A} |\psi_\alpha\rangle(\cdot)\langle\psi_\alpha|^A$, isomorphic to $\rho^A = \sum_{\alpha=1}^{d_A} |\psi_\alpha\rangle\langle\psi_\alpha|^A \geq 0$.

(non-normalized ‘density operator’)

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[Positive operator-valued measure (POVM)]

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Main probability rule: $p(\rho^{I\to A}, E^{A\to I}) = E^{A\to I} \circ \rho^{I\to A} \boxed{= \mathrm{Tr}[\rho^A E^A]}$

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(non-normalized 'density operator')

Preparation: $\{\rho_i^A\}_{i\in O}$, where $\sum_{i\in O} \mathrm{Tr}(\rho_i^A) = 1$.

(not a natural isomorphism!)

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[Positive operator-valued measure (POVM)]

choice of bilinear form!

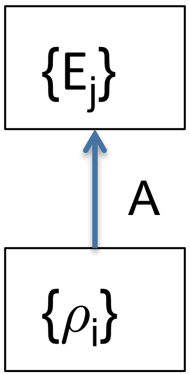
Main probability rule: $p(\rho^{I\to A}, E^{A\to I}) = E^{A\to I} \circ \rho^{I\to A}$ $= \mathrm{Tr}[\rho^A E^A]$

vector ↗ ↖ dual vector

The causality axiom

Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010), PRA 84, 012311 (2011):

Also in Ludwig (1983) (but not called causality);
Pegg, PLA 349, 411 (2006), ('**weak causality**').



$$p(i,j\,|\,\{\rho_i\},\{E_j\}) = p(\rho_i, E_j)$$

$$= Tr(\rho_i^A E_j^A)$$ in QT

The marginal probabilities of the preparation, $p(i\,|\,\{\rho_i\},\{E_j\}) = \sum_j p(\rho_i, E_j)$, are independent of the measurement:

$$p(i\,|\,\{\rho_i\},\{E_j\}) = p(i\,|\,\{\rho_i\},\{F_k\}) \qquad \forall \{\rho_i\},\{E_j\},\{F_k\}$$

'No signalling from the future'

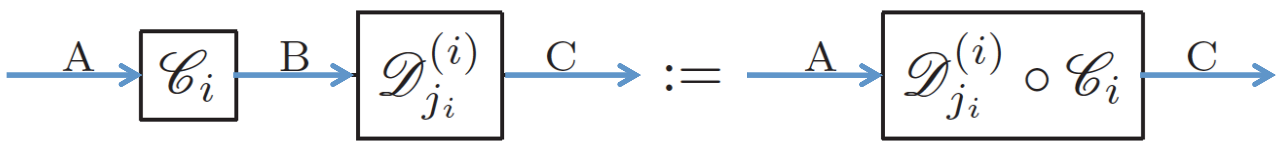
The causality axiom

Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010), PRA 84, 012311 (2011):

Some properties of causal theories:

- There is a unique deterministic effect (in quantum theory, $\mathbb{1}^A$).

- Conditioned operations are possible



- If a causal theory is not deterministic and the set of states is closed, the set of states is **convex**.

What is the axiom of causality about?

What is an operation?

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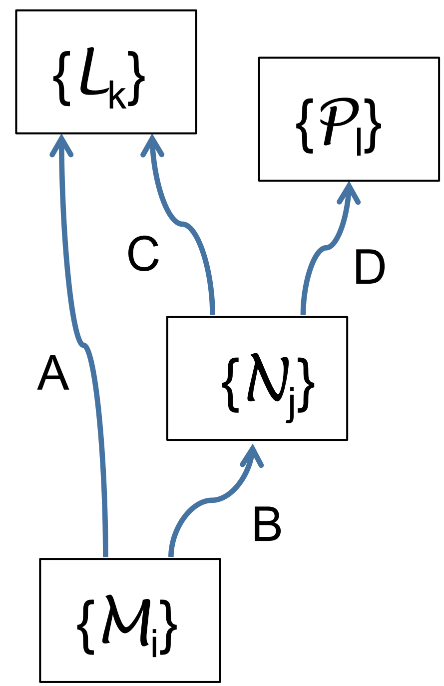
O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Two ideas:

What is an operation?

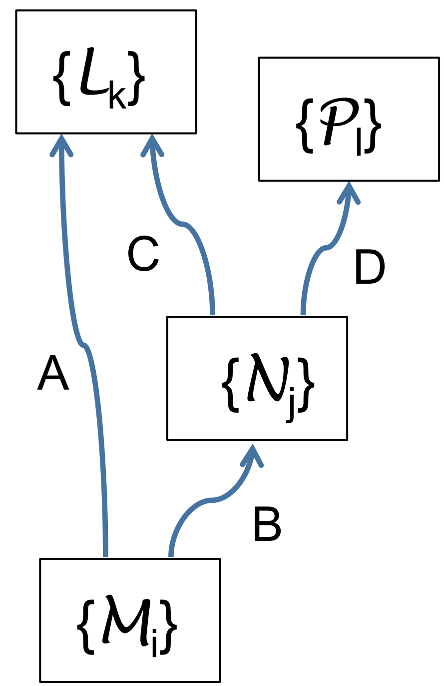


What is an operation?

Idea 1. The 'closed-box' assumption

All correlations between the events in the boxes are due to information exchange through the wires.

(The concept of circuit formalizes the idea of information exchanged via systems)



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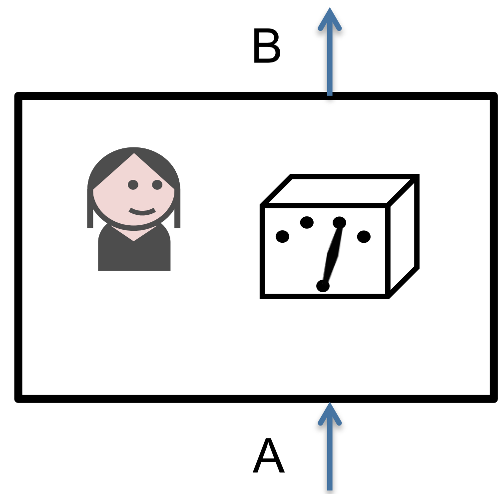
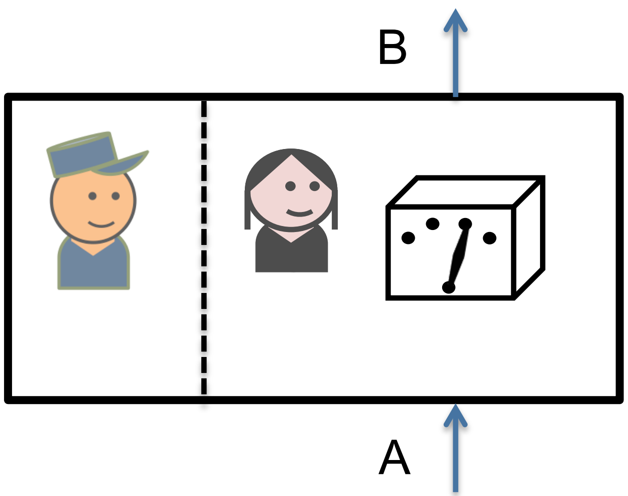


An operation could be realized inside an isolated box.

What is an operation?

The description of an operation is conditional on information.



Imagine Alice who chooses to perform one out of many possible operations $\{M^{\alpha}_{j_\alpha}\}$ with probability $p(\alpha)$ inside a closed box.

- If Charlie doesn't known the choice of Alice, he can say that the operation is $\{\{p(\alpha_1)M^{\alpha_1}_{j_{\alpha_1}}\},\{p(\alpha_2)M^{\alpha_2}_{j_{\alpha_2}}\},\cdots\}$.

- If he learns that Alice has chosen α, he can say that the operation is $\{M^{\alpha}_{j_\alpha}\}$.

(This is consistent with the Bayesian update of the probabilities of a circuit.)

→ A subset of the possible events in an operation defines another operation.

What is an operation?

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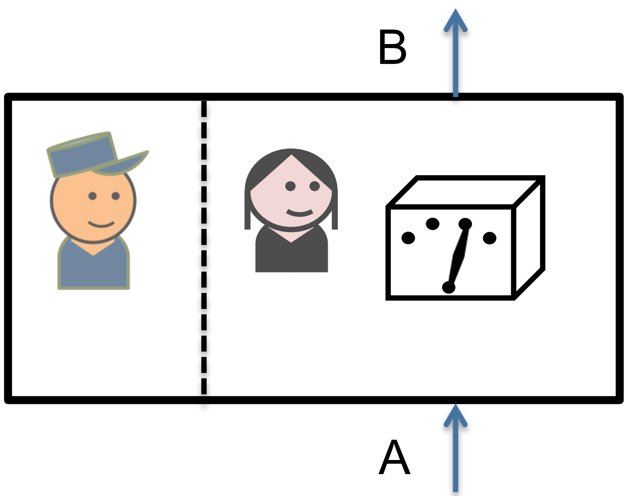

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But not all subsets of events are considered valid operations!

Why?

What is an operation?

Intuition: a valid operation can be ***chosen*** by the experimenter, but an arbitrary subset of its outcomes cannot.

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Problem: this would mean that the causality axiom is a *definition* and not an axiom. However, the axiom seems to express a non-trivial physical constraint.

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How do we formalize this?

Idea 2: The ‘no post-selection’ criterion:

The ‘choice’ of operation can be known *before* the time of the input system, irrespectively of future events.

Under this criterion, the causality axiom expresses a *nontrivial constraint*.

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The very concept of operation is *time-asymmetric* !

Does the property of causality imply an actual physical asymmetry?

(*Note:* The formal asymmetry does not automatically imply a physical asymmetry because the very concept of operation is asymmetric.)

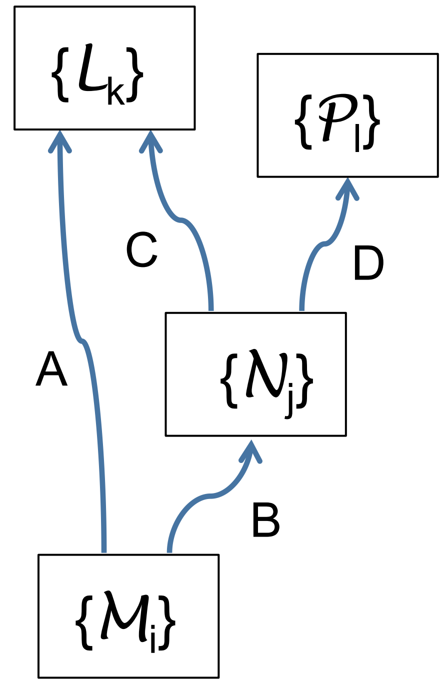
It actually does - The ‘pre-selected’ operations in the reverse time direction are all post-selected operations in the forward direction.

These time-reversed operations do not obey the causality axiom.

Physics under time reversal is not described by the usual quantum theory.

To summarize, in the circuit framework, a notion of time is presumed.

Events are equipped with a partial (causal) order coming from the circuit composition – one operation precedes another if there is a directed path from the former to the latter through the circuit.



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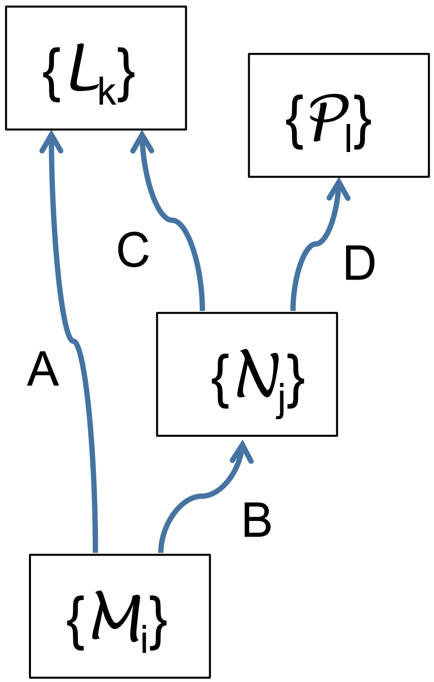
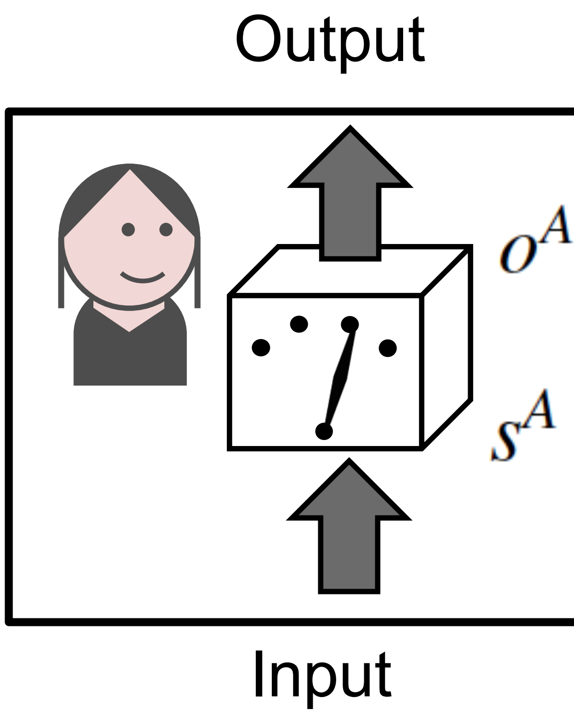


Can we understand time and causal structure from more primitive concepts?

(e.g., signaling from Alice to Bob → Alice is in the past of Bob)

The process framework



4) A system exits the lab.

3) An *outcome* o^A is obtained.

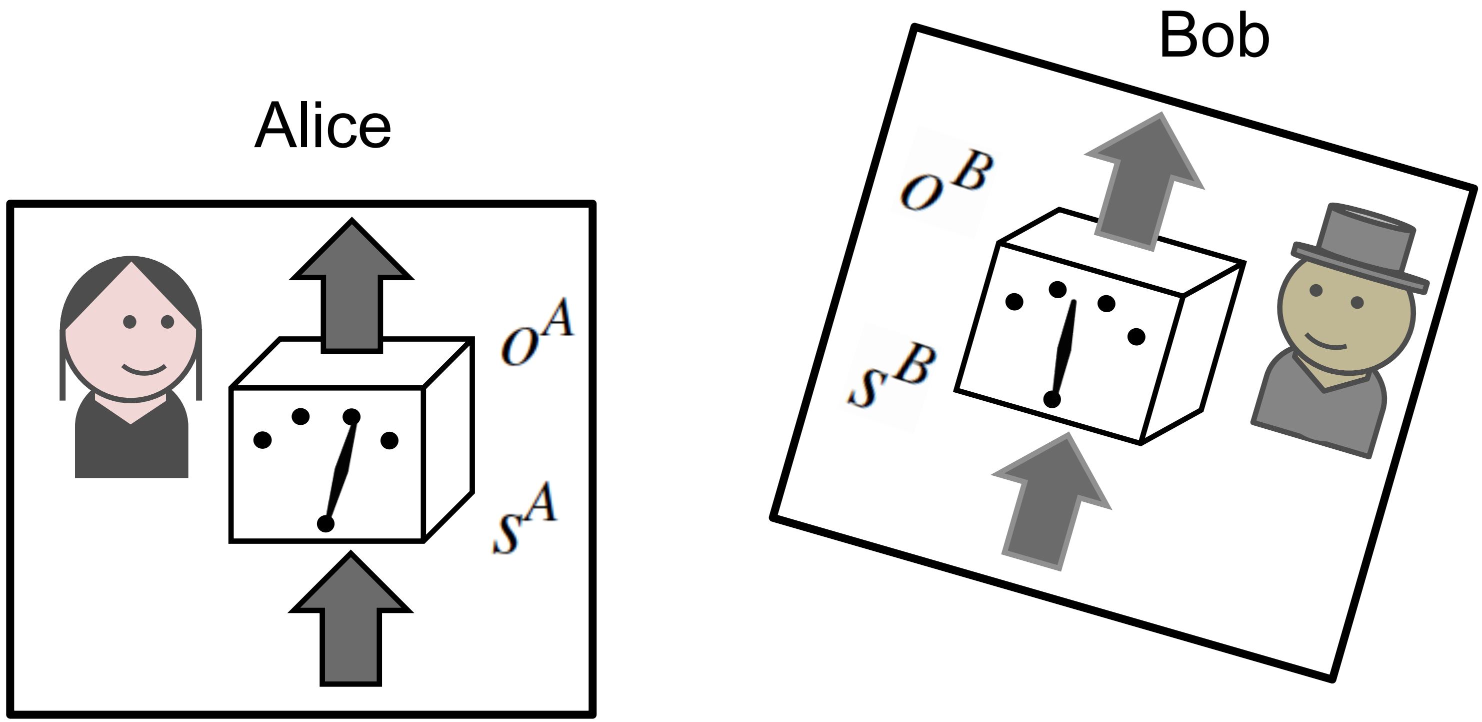
2) A *setting* s^A is chosen.

1) A system enters the lab.

A local experiment can exchange information with the outside world only via the input and output systems.

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

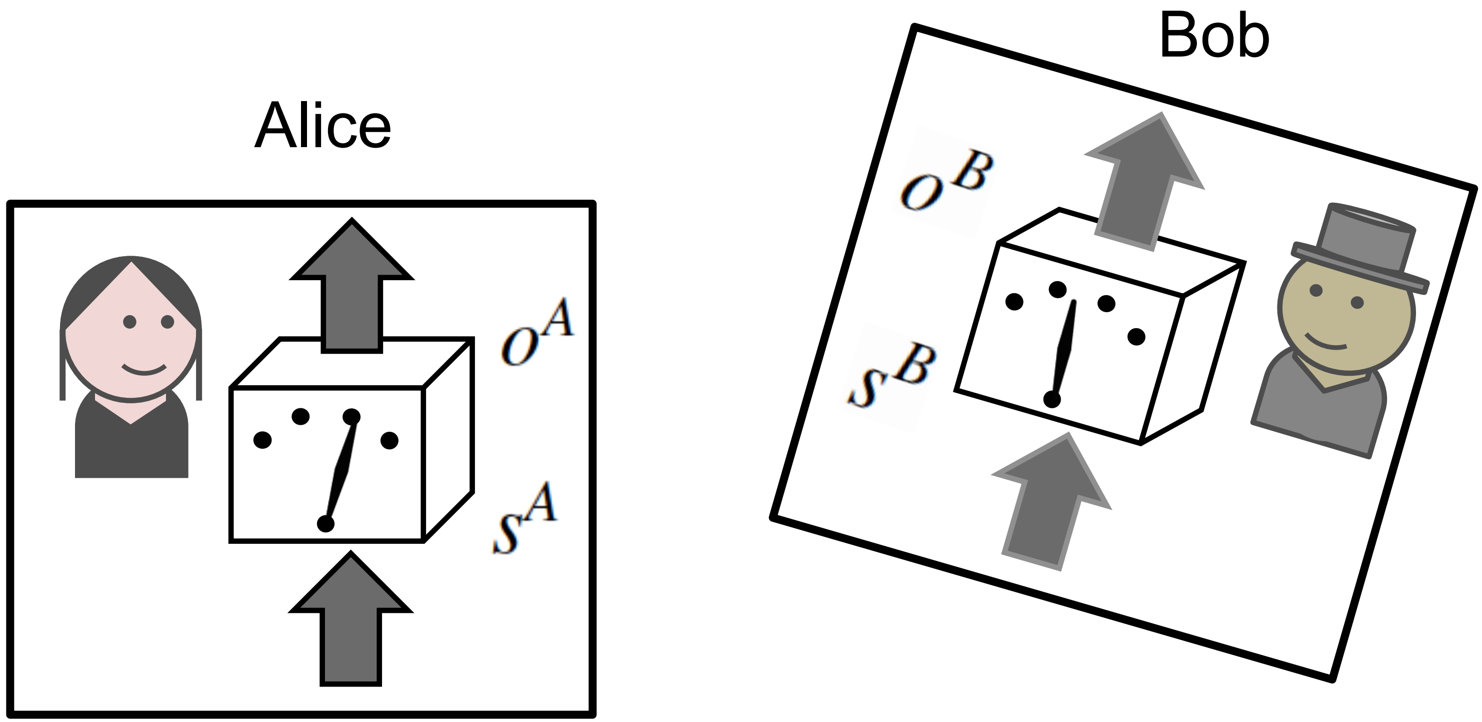
The process framework



No assumption of global causal order between the local experiments.

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

The process framework

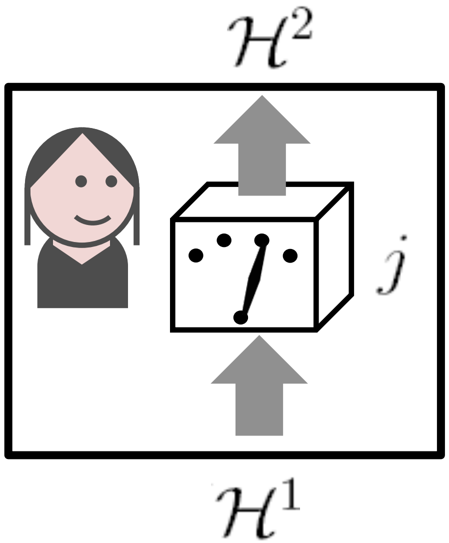


‘Process’ →
(catalogue of probabilities)

$$p(o^A, o^B, \ldots | s^A, s^B, \ldots)$$

Quantum processes

Local descriptions agree with quantum mechanics



Transformations = completely positive (CP) maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^1) \rightarrow \mathcal{L}(\mathcal{H}^2)$$

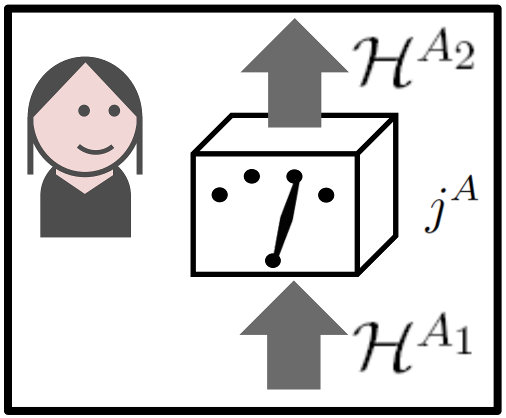
Kraus representation:

$$\mathcal{M}_j(\rho) = \sum_k E_{jk} \rho E_{jk}^\dagger$$

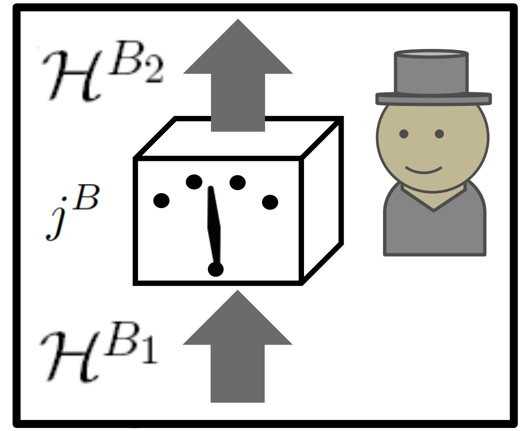
Completeness relation:

$$\sum_j \sum_k E_{jk}^\dagger E_{jk} = I$$

Quantum processes



$$\mathcal{M}^A_{j^A} : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2})$$



$$\mathcal{M}^B_{j^B} : \mathcal{L}(\mathcal{H}^{B_1}) \to \mathcal{L}(\mathcal{H}^{B_2})$$

Assumption 1: The probabilities are functions of the local CP maps,

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots)$$

Local validity of QM ➡ $P(\mathcal{M}^A, \mathcal{M}^B, \cdots)$ is **linear** in $\mathcal{M}^A$, $\mathcal{M}^B$, ...

Choi-Jamiołkowski isomorphism

CP maps		Positive semidefinite operators
$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M^{12} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

Choi-Jamiołkowski isomorphism

CP maps		Positive semidefinite operators
$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M^{12} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

$$M^{12} := [\mathcal{I} \otimes \mathcal{M}(|\Phi^+\rangle\langle\Phi^+|)]^T$$

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle \quad \in \mathcal{H}^1 \otimes \mathcal{H}^1$$

Choi-Jamiołkowski isomorphism

CP maps		Positive semidefinite operators
$\mathcal{M} : \mathcal{L}(\mathcal{H}^1) \to \mathcal{L}(\mathcal{H}^2)$	$\longleftrightarrow$	$M^{12} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

$$\mathcal{M}(\rho^1) = [\mathrm{Tr}_1(\rho^1 M^{12})]^{\mathrm{T}}$$

The process matrix

Representation

$$P(\mathcal{M}^{A}_{j^A}, \mathcal{M}^{B}_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

The process matrix

Representation

$$P(\mathcal{M}^A_{j^A}, \mathcal{M}^B_{j^B}, \cdots) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2}_{j^A} \otimes M^{B_1 B_2}_{j^B} \otimes \cdots\right)\right]$$

Process matrix

Similar to Born's rule but can describe signalling!

The process matrix

Assumption 2: The parties can share entangled input ancillas.

Conditions on W:

1. Non-negative probabilities: $W^{A_1 A_2 B_1 B_2 \cdots} \geq 0$

2. Probabilities sum up to 1:

$$\mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2 \cdots}\left(M^{A_1 A_2} \otimes M^{B_1 B_2} \otimes \cdots\right)\right] = 1$$

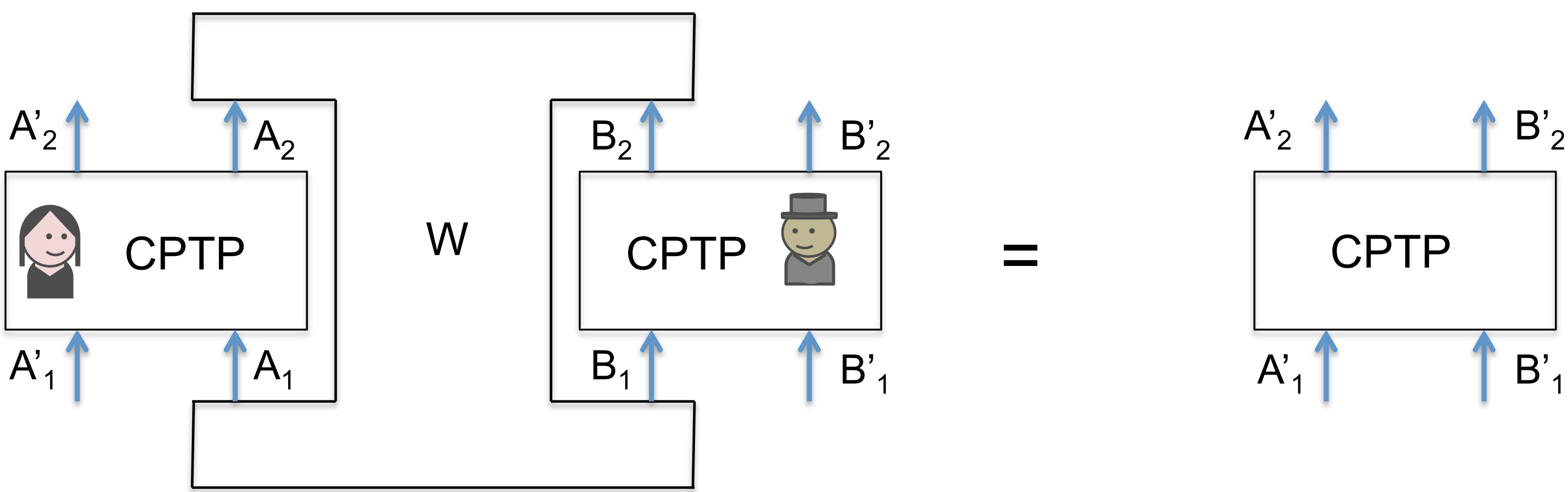
on all CPTP maps $M^{A_1 A_2}$, $M^{B_1 B_2}$, ...

Note: $M^{A_1 A_2}$ is CPTP iff $M^{A_1 A_2} \geq 0, \;\; \mathrm{Tr}_{A_2} M^{A_1 A_2} = \mathbb{1}^{A_1}$.

The process matrix

An alternative formulation as a second-order operation:

[Quantum supermaps, Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)]



Terms appearing in a process matrix

$$W^{A_1A_2B_1B_2C_1C_2\cdots} = \sum_{i,j,k,l,m,n\cdots} w_{ijklmn\cdots}\sigma_i^{A_1} \otimes \sigma_j^{A_2} \otimes \sigma_k^{B_1} \otimes \sigma_l^{B_2} \otimes \sigma_m^{C_1} \otimes \sigma_n^{C_2} \otimes \cdots$$

Hilbert-Schmidt basis: Hermitian $\{\sigma_\mu^X\}_{\mu=0}^{d_X^2-1}$, where $\sigma_0^X = \mathbb{1}^X$, $\mathrm{Tr}\sigma_\mu^X\sigma_\nu^X = d_X\delta_{\mu\nu}$

Proposition: $W^{A_1A_2B_1B_2\cdots}$ is a valid process matrix iff

1) $W^{A_1A_2B_1B_2\cdots} \geq 0$

2) In addition to the identity, it contains only terms with a non-trivial σ on X_1 and $\mathbb{1}$ on X_2 for some party $X \in \{A, B, C, \cdots\}$.

Example: bipartite case

$$W^{A_1 A_2 B_1 B_2} = \sum_{\mu_1,\ldots,\mu_4} a_{\mu_1\ldots\mu_4} \sigma_{\mu_1}^{A_1} \otimes \ldots \otimes \sigma_{\mu_4}^{B_2}$$

$\sigma_i^{A_1} \otimes \mathbb{1}^{rest}$ type A_1

$\sigma_i^{A_1} \otimes \sigma_j^{A_2} \otimes \mathbb{1}^{rest}$ type $A_1 A_2$

...

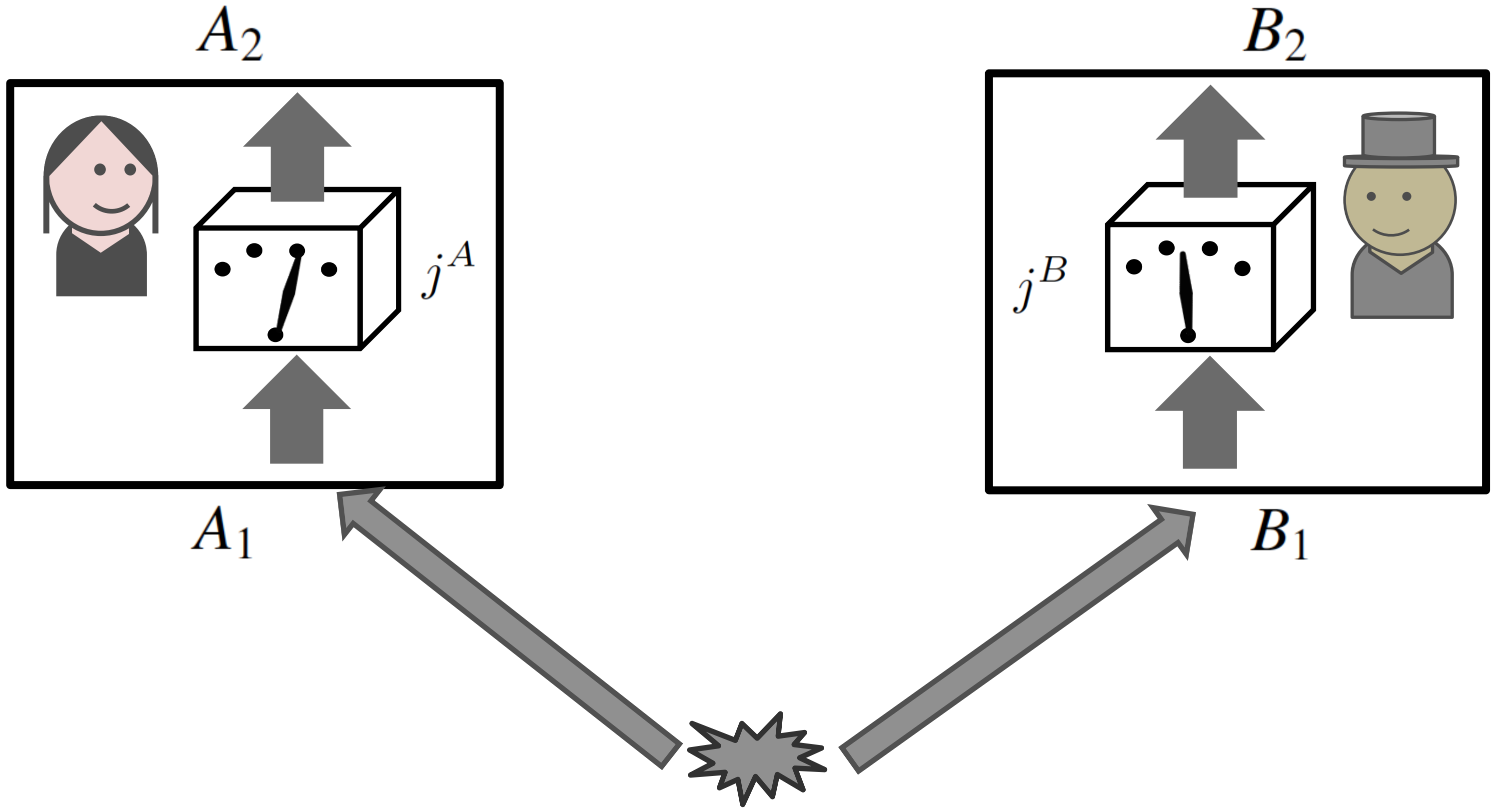
A valid process matrix:

$$W^{A_1 A_2 B_1 B_2} \geq 0$$

and contain only the identity term plus terms of type

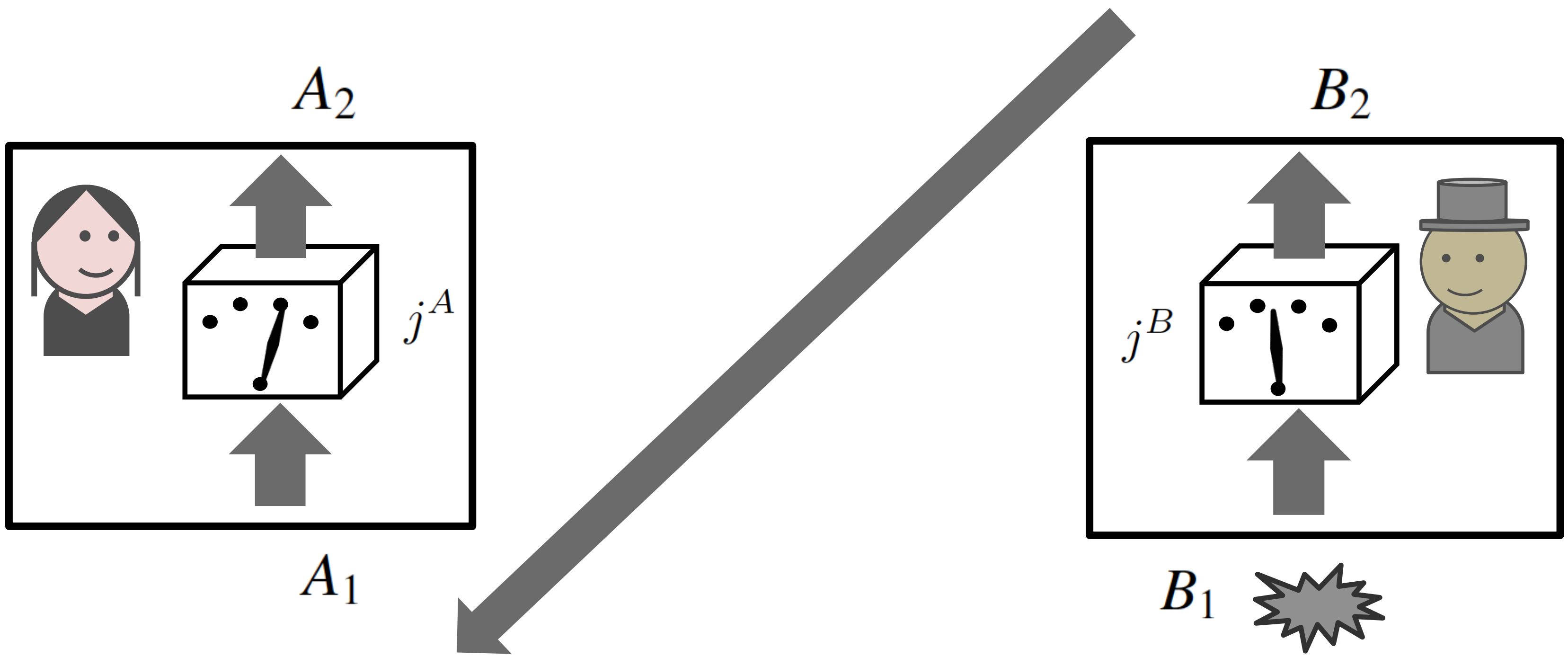
$$A_1, \quad B_1, \quad A_1 B_1, \quad A_2 B_1, \quad A_1 B_2, \quad A_1 A_2 B_1, \quad A_1 B_1 B_2$$

Example: bipartite state



$$W^{A_1 A_2 B_1 B_2} = \rho^{A_1 B_1} \otimes \mathbb{1}^{A_2 B_2}$$

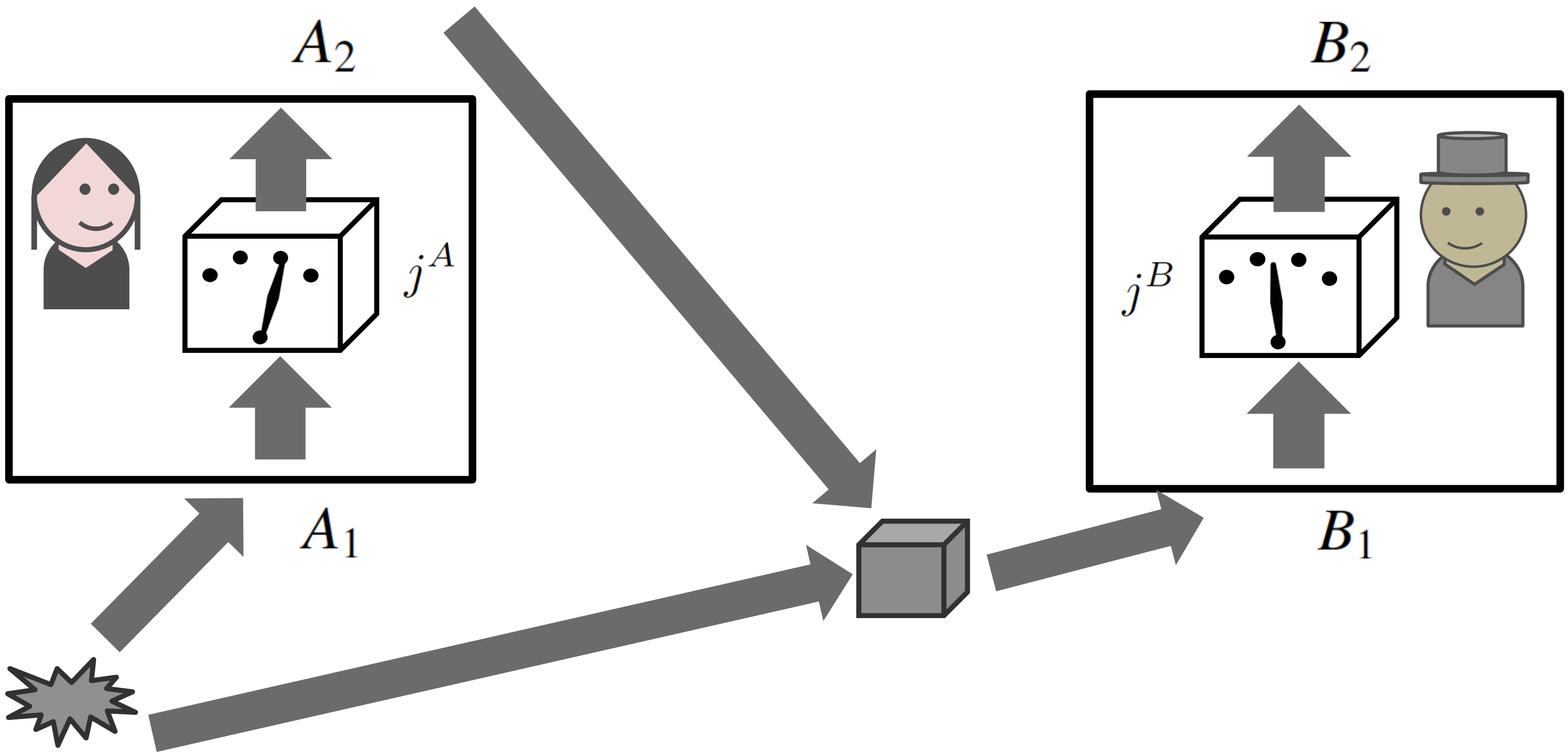
Example: channel B→A



$$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_2} \otimes (C^{A_1 B_2})^T \otimes \rho^{B_1}$$

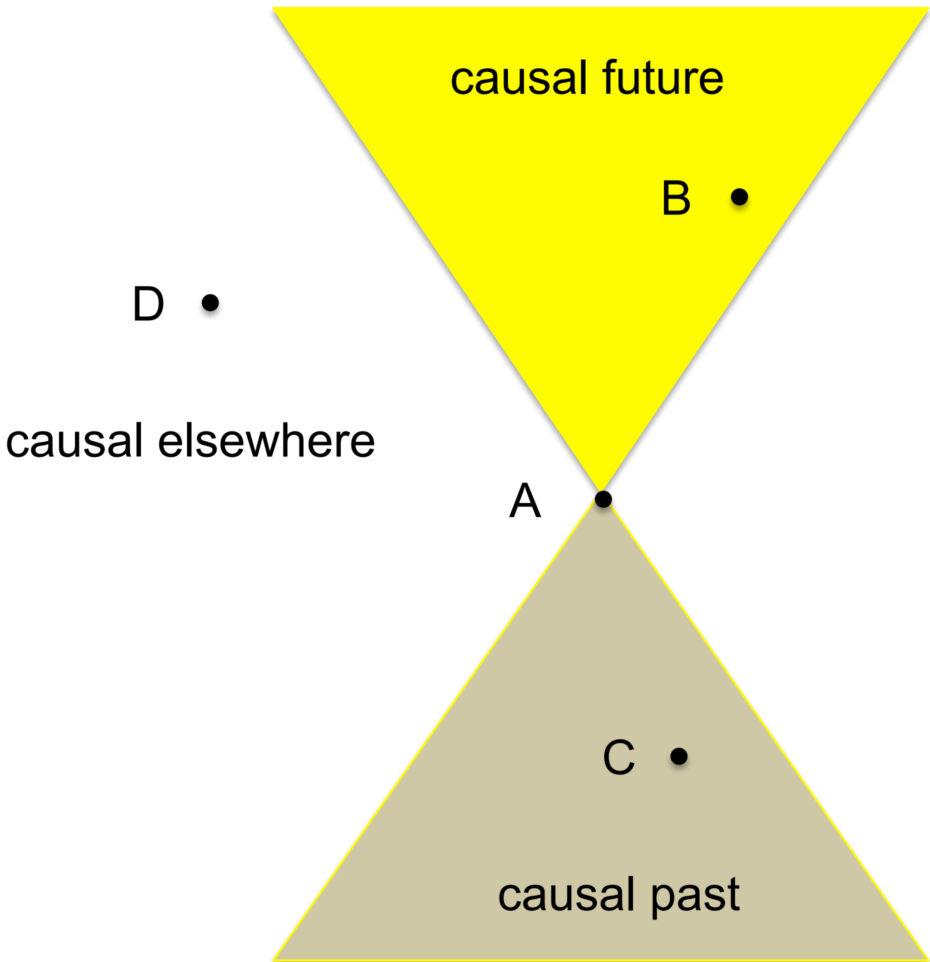
Example: channel with memory A→ B

(The most general possibility compatible with no signalling from B to A!)

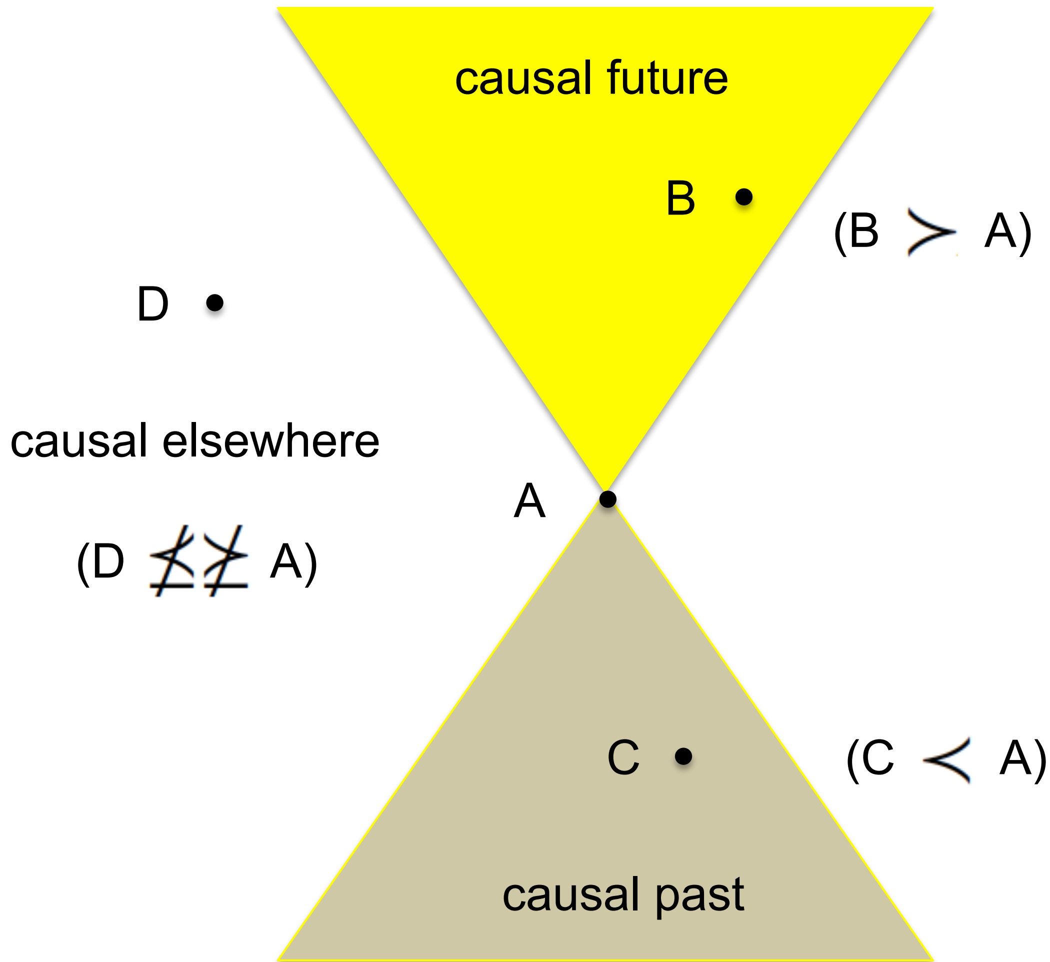


$$W^{A_1 A_2 B_1 B_2} = W^{A_1 A_2 B_1} \otimes \mathbb{1}^{B_2}$$

Causal order



Causal order



(Strict) partial order $\prec$ **:**

1) *irreflexivity*

not X $\prec$ X .

2) *transitivity*

if X $\prec$ Y and Y $\prec$ Z ,

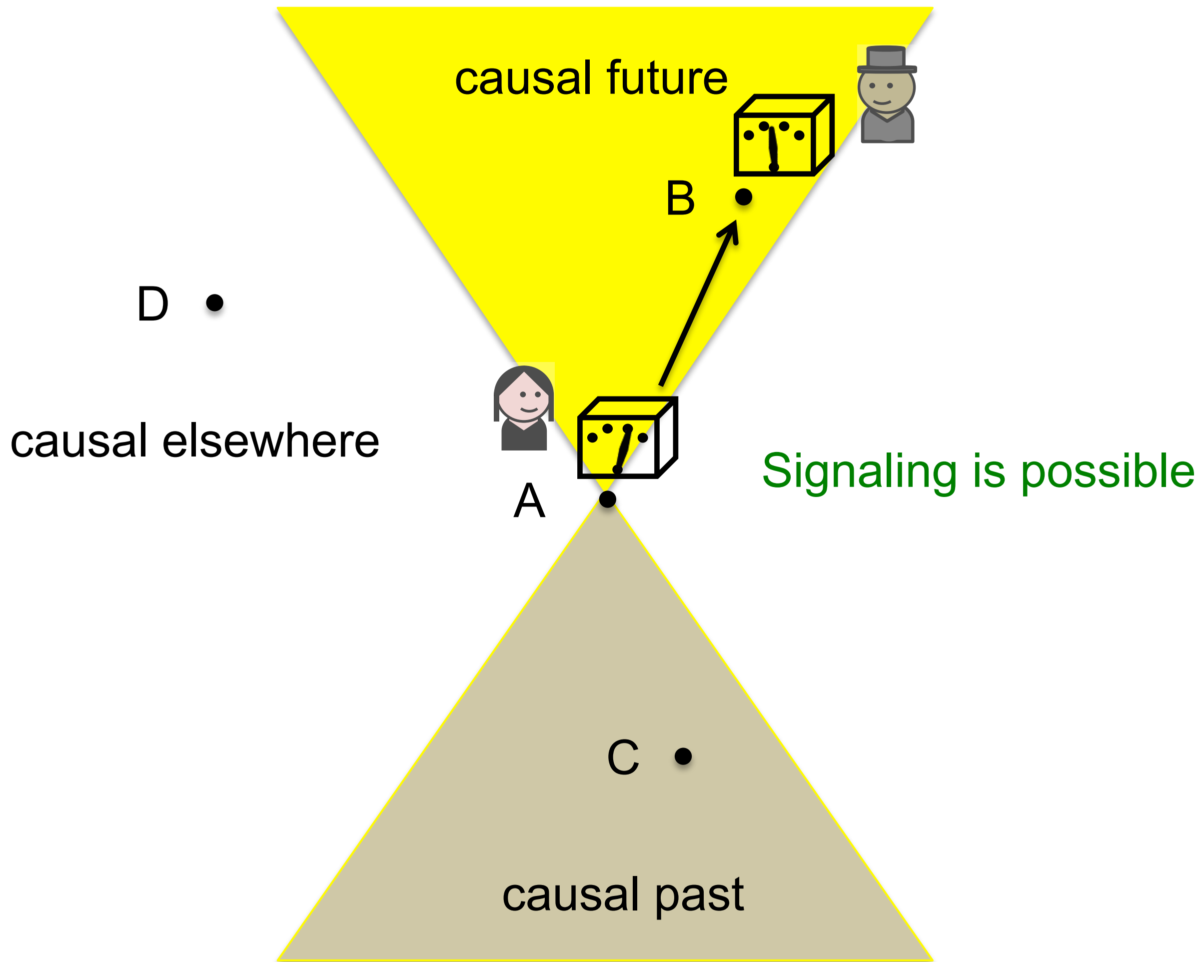
then X $\prec$ Z .

3) *antisymmetry*

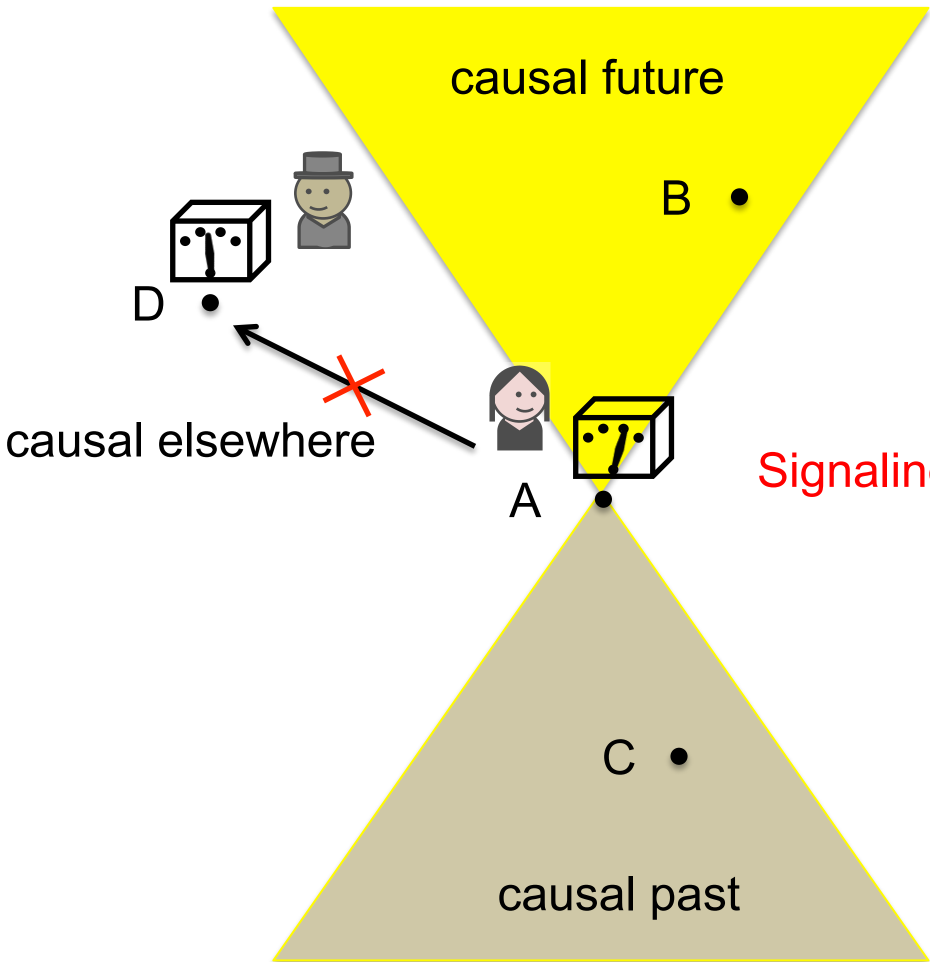
if X $\prec$ Y ,

then not Y $\prec$ X .

Causal order

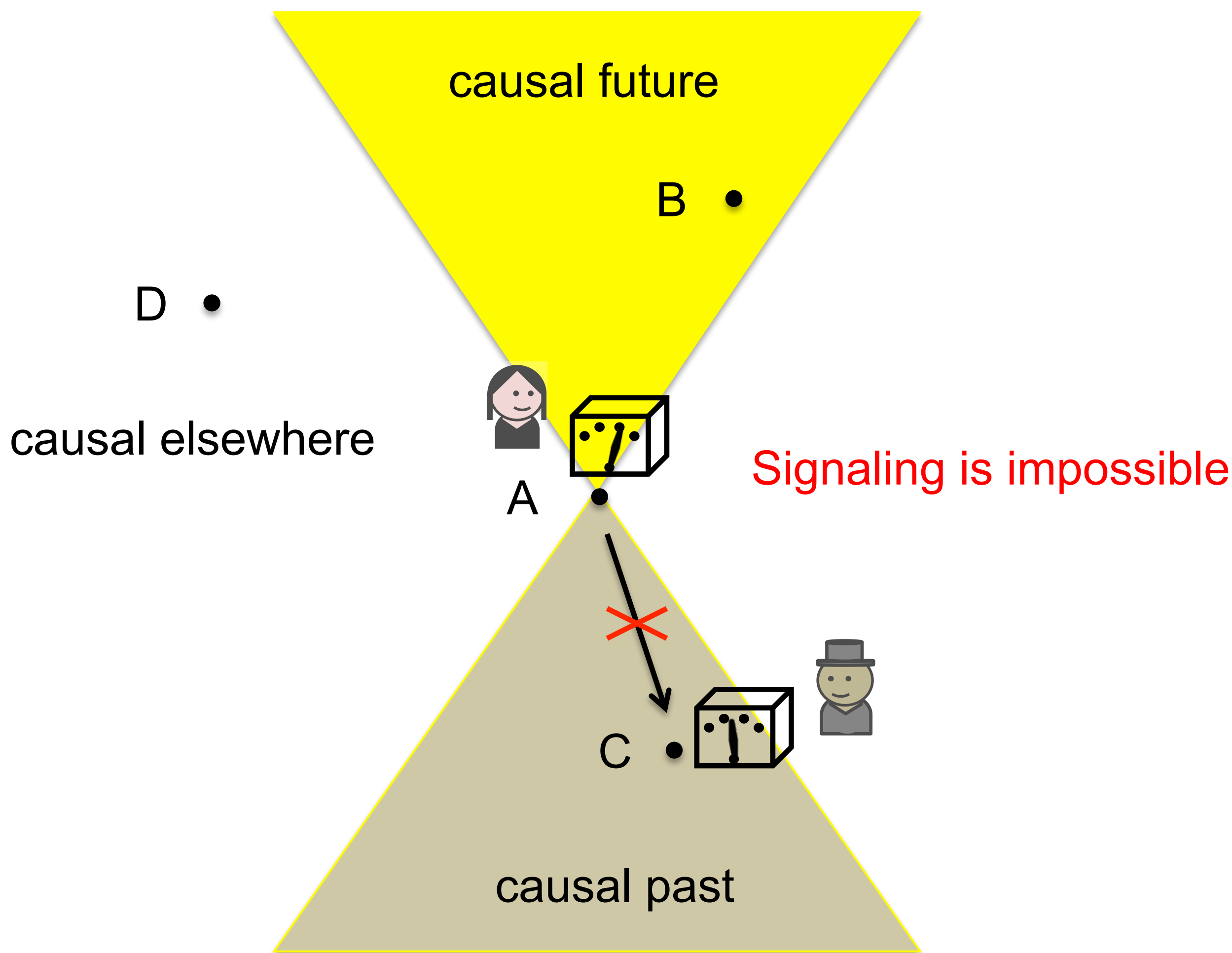


Causal order



Signaling is impossible

Causal order



Causal order

Notation: $A \npreceq B$

Alice is *not* in the causal past of Bob (hence, Alice cannot signal to Bob)

In a causal scenario, at least one of $(A \npreceq B)$ or $(B \npreceq A)$ must be true.

→ Alice cannot signal to Bob or Bob cannot signal to Alice.

Bipartite processes with causal realization

$W^{A \not\leq B}$ – no signalling from A to B (ch. with memory from B to A)

$W^{B \not\leq A}$ – no signalling from B to A (ch. with memory from A to B)

Bipartite processes with causal realization

$W^{A \not\leq B}$ – no signalling from A to B (ch. with memory from B to A)

$W^{B \not\leq A}$ – no signalling from B to A (ch. with memory from A to B)

More generally, we may conceive **causally separable** processes (probabilistic mixtures of fixed-order processes):

$$W_{cs}^{A_1 A_2 B_1 B_2} = q W^{A \not\leq B} + (1-q) W^{B \not\leq A}$$

Bipartite processes with causal realization

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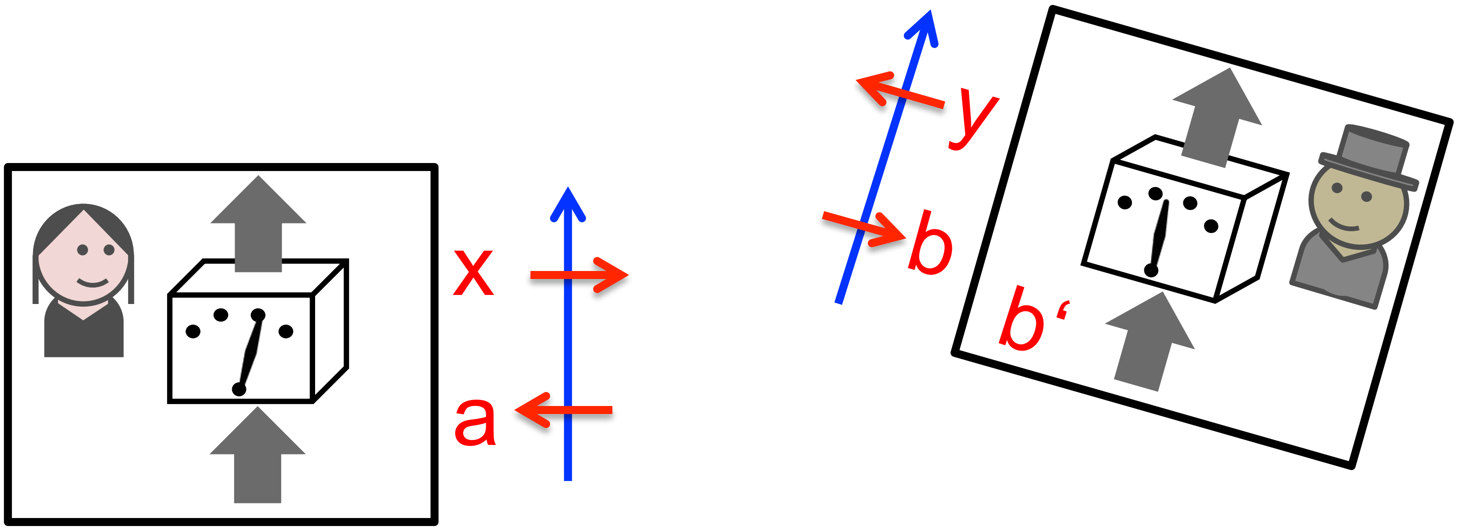
$$W_{cs}^{A_1 A_2 B_1 B_2} = q W^{A \not\preceq B} + (1-q) W^{B \not\preceq A}$$

Are all process matrices causally separable?

A causal game



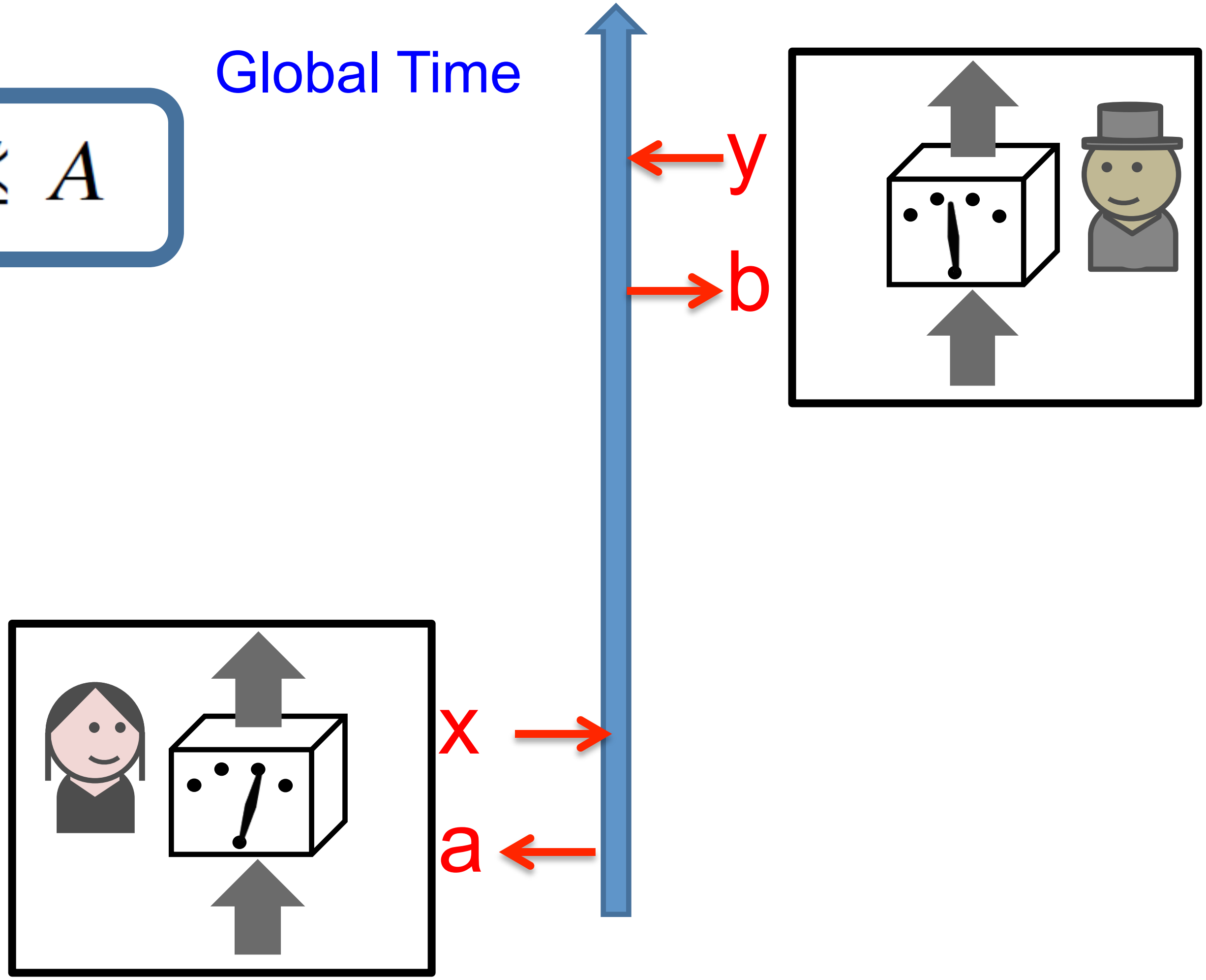
Their goal is to maximize:

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)]$$

O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

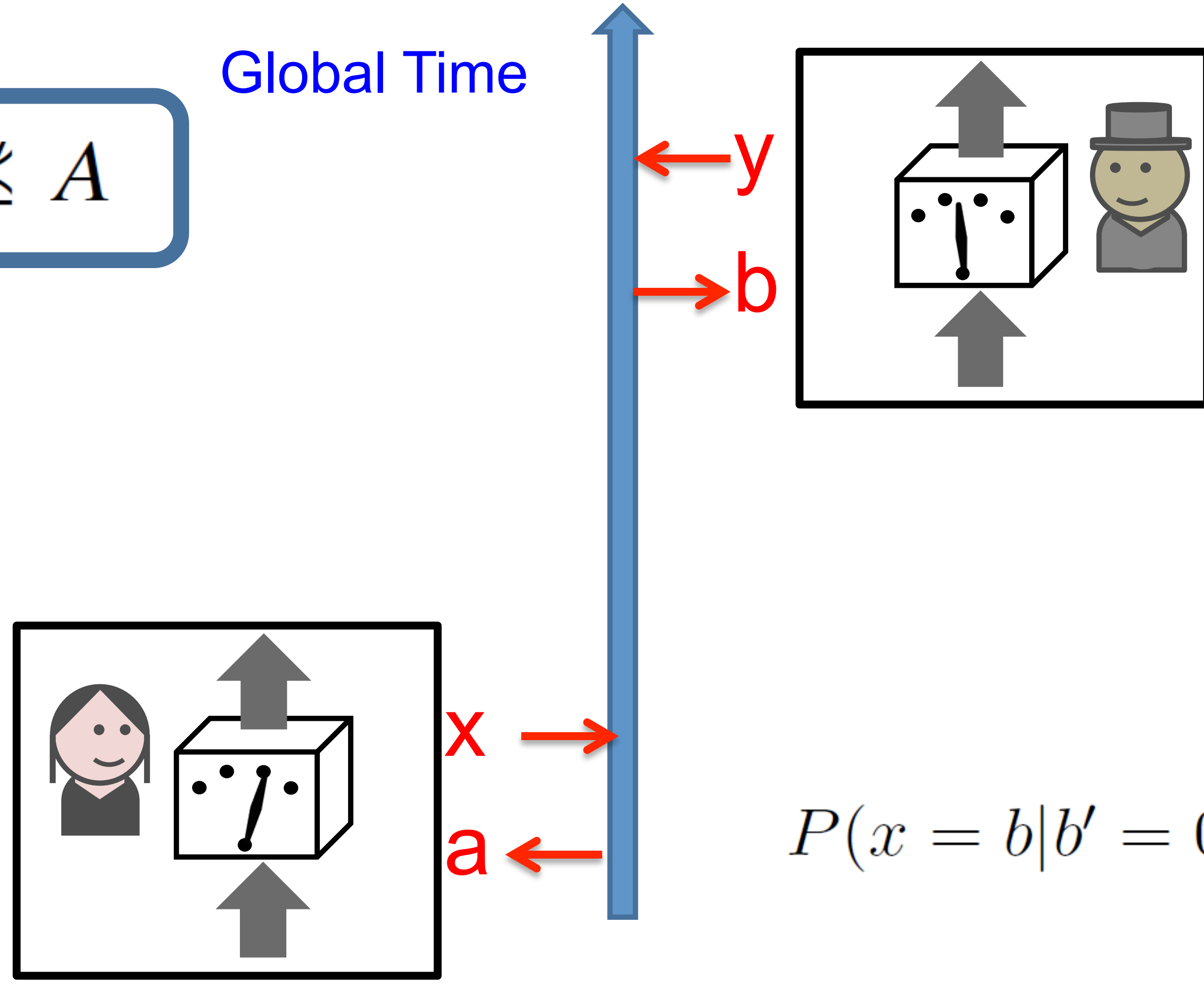
Causally ordered situation

Case $B \not\leq A$



Causally ordered situation

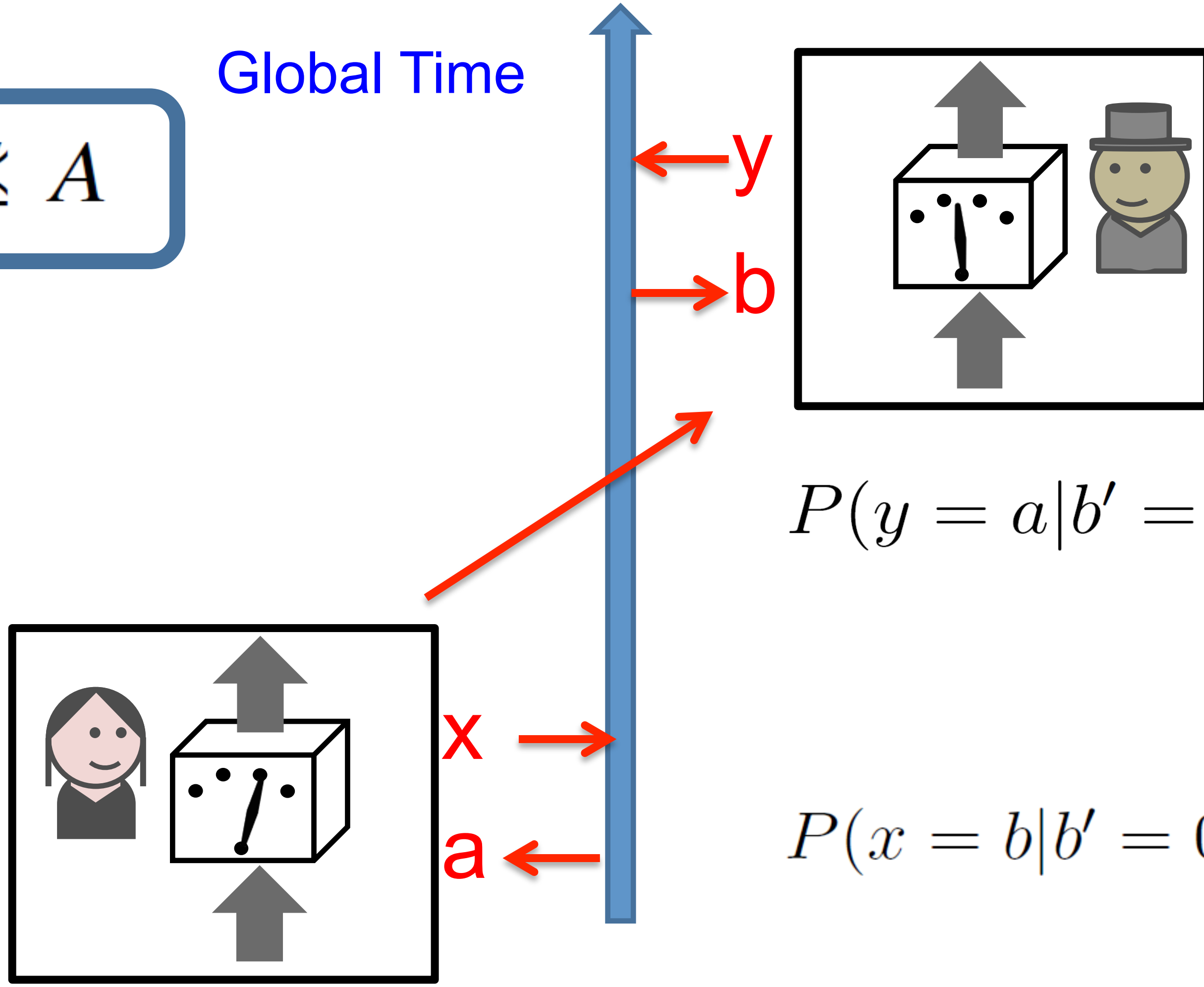
Case $B \not\leq A$



$$P(x = b | b' = 0) = 1/2$$

Causally ordered situation

Case $B \not\leq A$

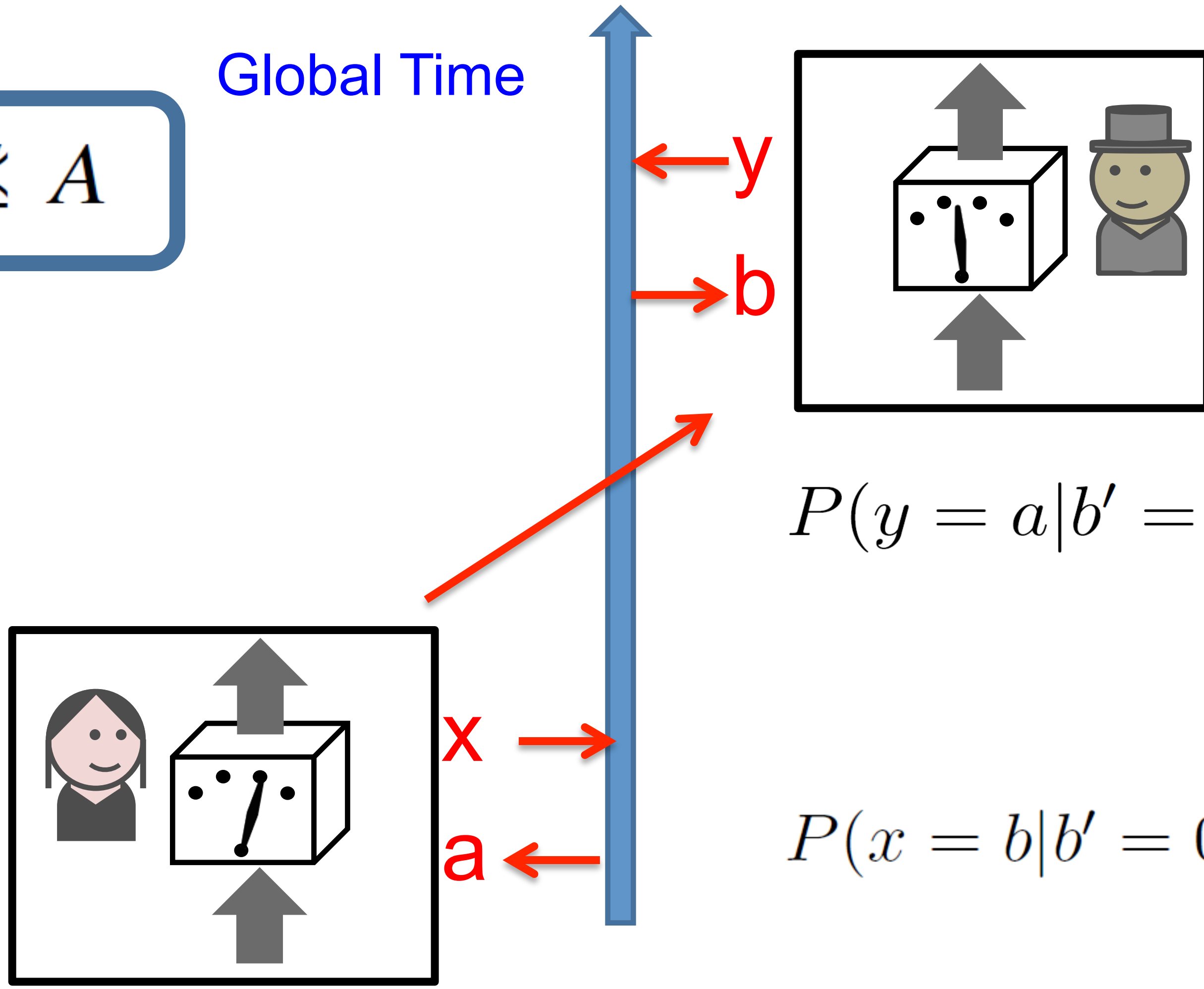


$$P(y = a | b' = 1) = 1$$

$$P(x = b | b' = 0) = 1/2$$

Causally ordered situation

Case $B \not\leq A$

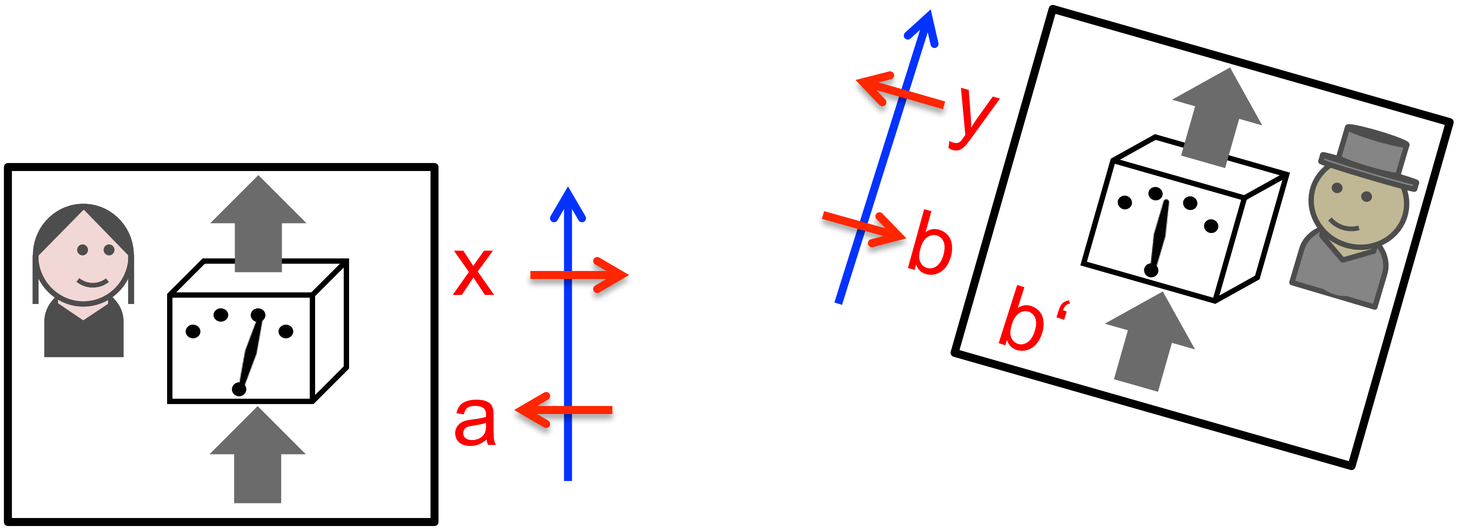


$$P(y = a|b' = 1) = 1$$

$$P(x = b|b' = 0) = 1/2$$

$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)] \leq \frac{3}{4}$$

A causal inequality

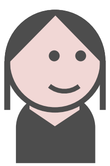


Definite causal order →

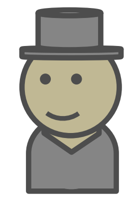
$$p_{succ} = \frac{1}{2}[P(x = b|b' = 0) + P(y = a|b' = 1)] \leq \frac{3}{4}$$

A non-causal process

Can violate the inequality with $p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$.



$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$



two-level systems

The operations of Alice and Bob do not occur in a definite order!

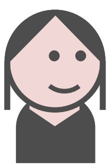
O. O., F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012).

A causally non-separable situation

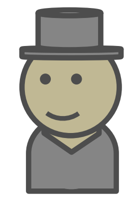
Alice always measures in the z basis and encodes the bit in the *z* basis

Alice's CP map: $|z_x\rangle\langle z_x|^{A_1} \otimes |z_a\rangle\langle z_a|^{A_2} \qquad x, a = \pm 1$

If Bob wants to receive (**b'=1**), he measures in the *z* basis



$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$



$$\langle z_\pm|\sigma_x|z_\pm\rangle = 0$$

Channel from Alice to Bob

Not seen by Bob

Bob receives the state $\widetilde{W}^{B_1 B_2} = \frac{1}{2}(\mathbb{1} + a\frac{1}{\sqrt{2}}\sigma_z^{B_1})$

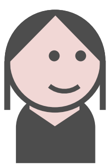
He can read Alice's bit with probability

$$P(y = a|b' = 1) = \frac{2+\sqrt{2}}{4}$$

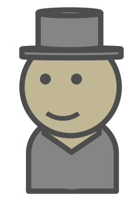
A causally non-separable situation

If Bob wants to send (***b' = 0***), he measures in the *x* basis and encodes in the *z* basis conditioned on his outcome

Bob's CP map: $|x_y\rangle\langle x_y|^{B_1} \otimes |z_{by}\rangle\langle z_{by}|^{B_2}$ $\quad y, b = \pm 1$



$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2}\right)\right]$$



$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$

Not seen by Bob

Channel from Bob to Alice, correlated with Bob's outcome

Alice receives the state $\widetilde{W}^{A_1 A_2} = \frac{1}{2}(\mathbb{1} + b\frac{1}{\sqrt{2}}\sigma_z^{A_1})$

She can read Bob's bit with probability

$$P(x = b|b' = 0) = \frac{2+\sqrt{2}}{4}$$

Other causal inequalities and violations

Simplest bipartite inequalities:

Branciard, Araujo, Feix, Costa, Brukner, NJP 18, 013008 (2016)

Multiparite inequalities:

- violation with perfect signaling

Baumeler and Wolf, Proc. ISIT 2014, 526-530 (2014)

- **violation by classical local operations:**

Baumeler, Feix, and Wolf, PRA 90, 042106 (2014)

Baumeler and Wolf, NJP 18, 013036 (2016)

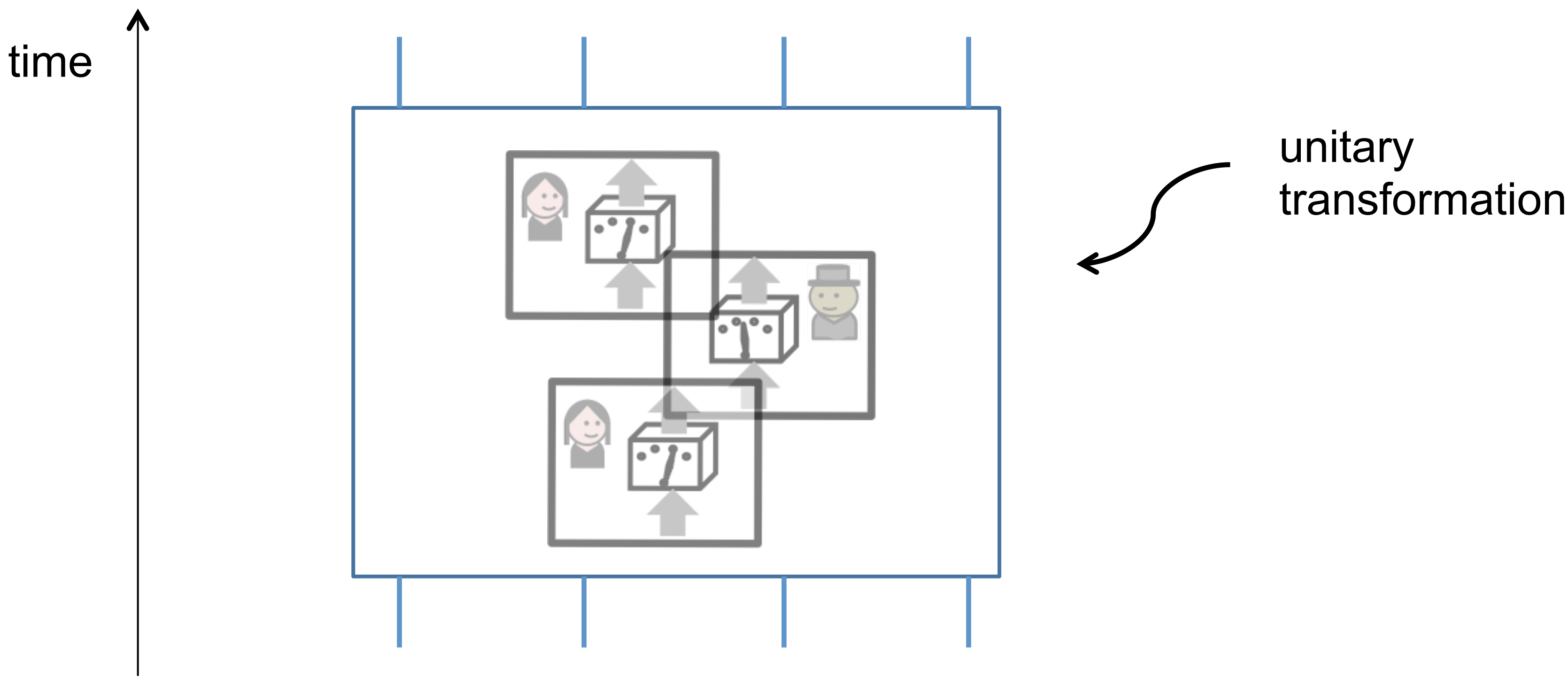
Biased version of the original inequality:

Bhattacharya and Banik, arXiv:1509.02721 (2015)

Can non-causal processes be realized physically?

Can non-causal processes be realized physically?

Not *a priori* impossible!



From the outside the experiment may still agree with standard unitary evolution in time.

The quantum switch

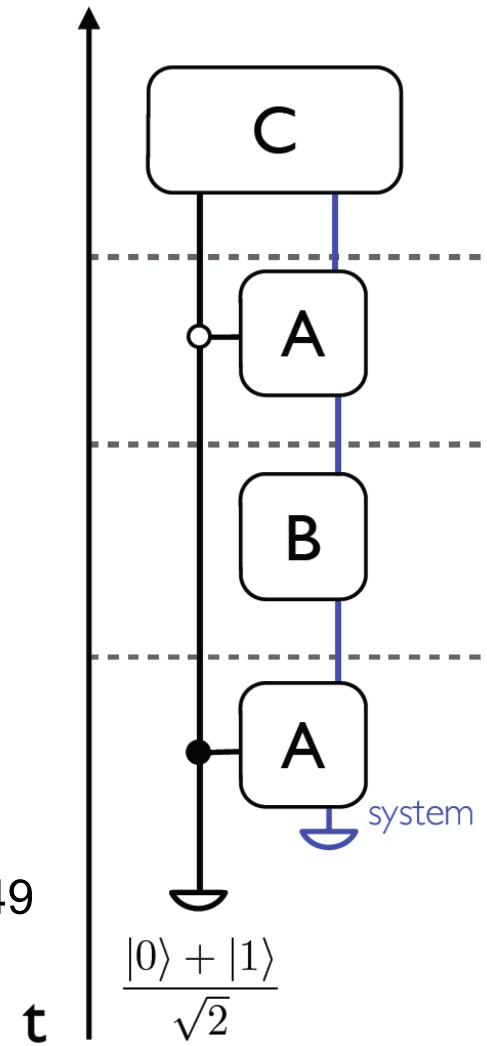
Chiribella, D'Ariano, Perinotti and Valiron, arXiv:0912.0195, PRA 2013

The *tripartite* process is not causally separable!

$$W^{A_1A_2B_1B_2C_1C_2} = |W\rangle\langle W|^{A_1A_2B_1B_2C_1C_2}$$

O. Oreshkov and C. Giarmatzi, arXiv:1506.05449

M. Araujo et al., NJP 17, 102001 (2015)



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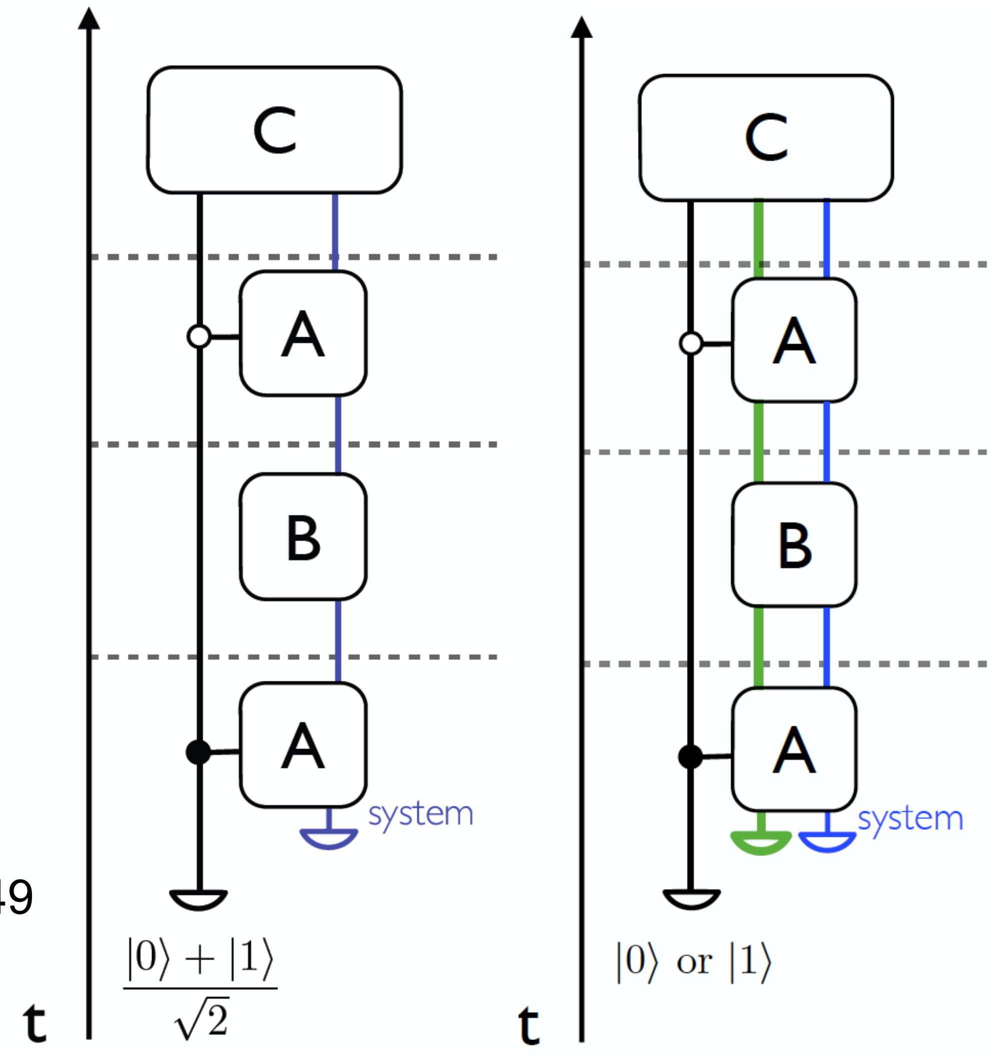
Yet, it cannot violate causal inequalities…

O. Oreshkov and C. Giarmatzi, arXiv:1506.05449

M. Araujo et al., NJP 17, 102001 (2015)

causal simulation



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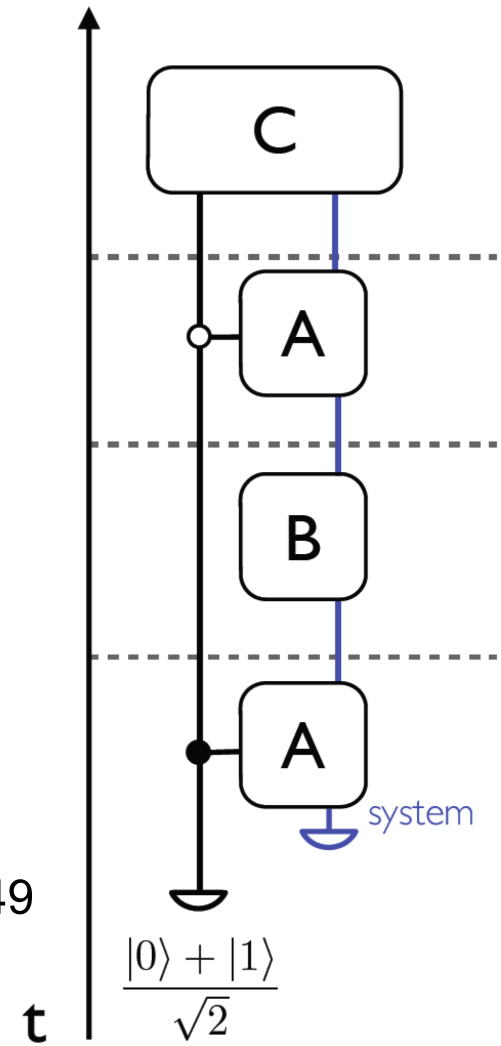
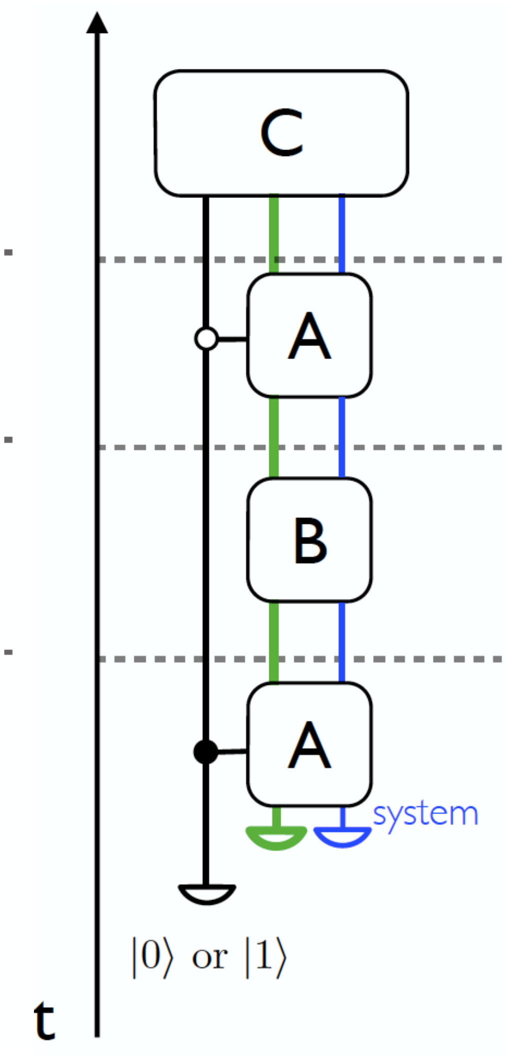
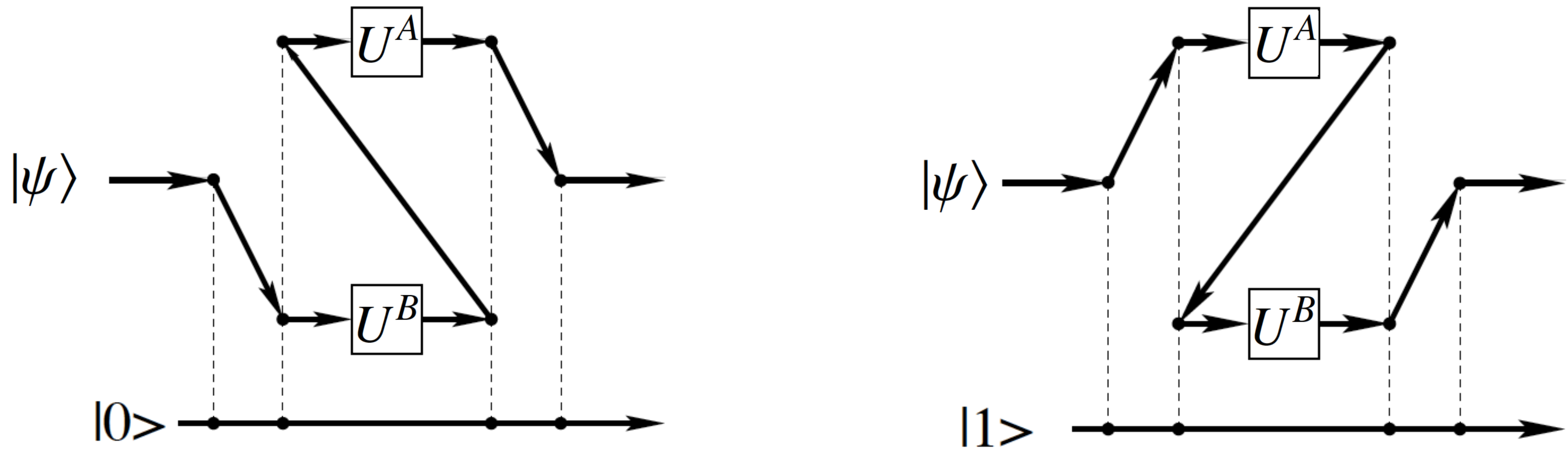


causal simulation



Advantage in black-box discrimination



quantum SWITCH

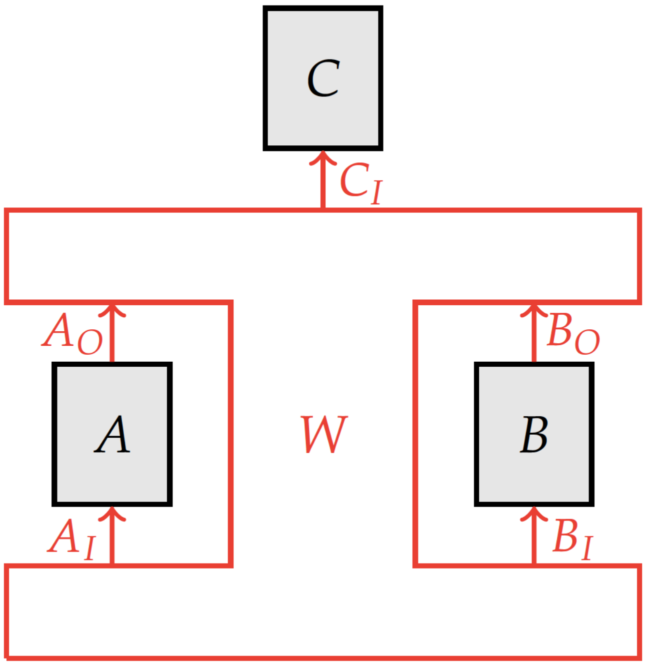
$$|+\rangle|\psi\rangle \rightarrow \frac{1}{2}|+\rangle\{U^A, U^B\}|\psi\rangle + \frac{1}{2}|+\rangle[U^B, U^A]|\psi\rangle$$

Charlie can find with certainty whether two gates commute or anti-commute, even though each gate is used only once.

Chiribella, Phys. Rev. A 86, 040301(R) (2012)

Experimental demonstration: Procopio et al., Nat. Commun. 6:7913 (2015)

Advantage in black-box discrimination



Causal witness:

$$\mathrm{tr}[S\, W^{\mathrm{sep}}] \geq 0$$

Araujo, Branciard, Costa, Feix, Giarmatzi, Brukner, New J. Phys. 17, 102001 (2015) .

Branciard, Sci. Rep. 6, 26018 (2016).

Advantage in black-box discrimination

Computations with multipartite-SWITCH

Colnaghi, D'Ariano, Perinotti, Facchini, Phys. Lett. A 376 (2012), pp. 2940--2943

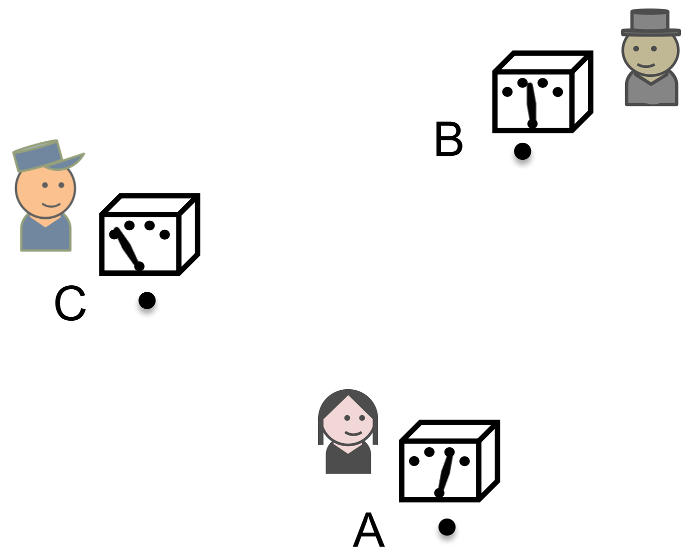
Araujo, Costa, Brukner, Phys. Rev. Lett. 113, 250402 (2014)

Communication complexity:

Guerin, Feix, Araujo, Brukner, arXiv: 1605.07372

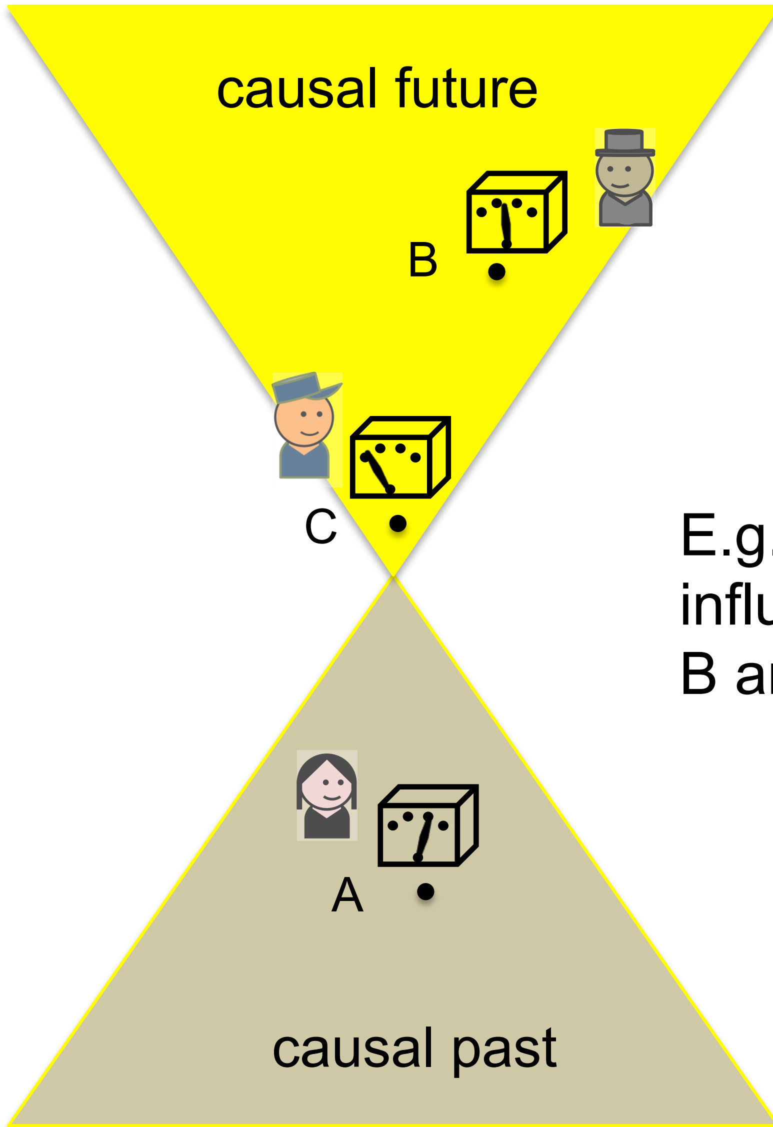
Formal theory of causality for processes

O. O. and C. Giarmatzi, arXiv:1506.05449



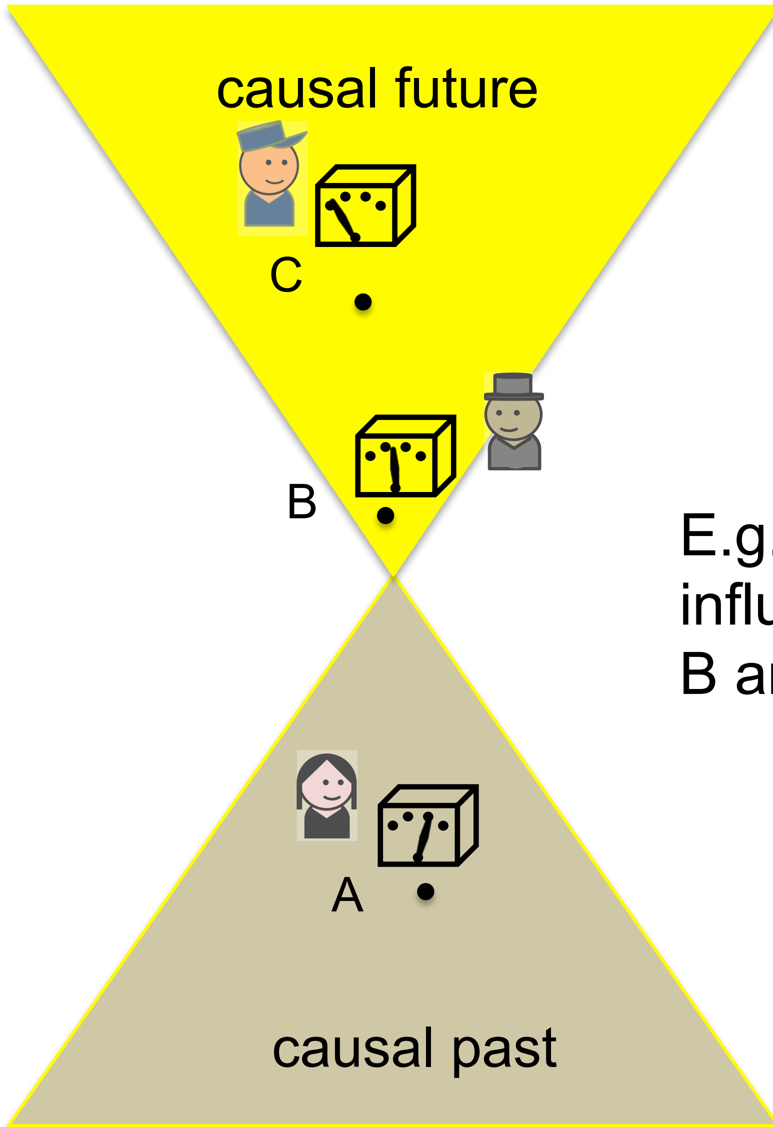
What constraints on the correlations does causality imply?

The causal order can be both *random* and ***dynamical***



E.g., the operation at A could influence the order in which B and C happen in A's future.

The causal order can be both *random* and ***dynamical***



E.g., the operation at A could influence the order in which B and C happen in A's future.

Device-independent definition of causality

O. O. and C. Giarmatzi, arXiv:1506.05449

A notion of causality should:

- **have a universal expression** (which implies the multipartite case)
- **allow of *dynamical* causal order** (a given event can influence the order of other events in its future)
- **capture our intuition of causality**

Device-independent definition of causality

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, \ldots | s^A, s^B, \ldots)\}$

Intuition: The probability for a set of events to occur outside of the causal future of Alice and for these events to have a particular causal configuration with Alice is independent of the choice of setting of Alice.

Device-independent definition of causality

O. O. and C. Giarmatzi, arXiv:1506.05449

General process: $\mathcal{W}^{A,B,\cdots} \equiv \{P(o^A, o^B, ...|s^A, s^B, ...)\}$

A process is **causal** iff there exists a probability distribution $P(\kappa(A, B, \cdots), o^A, o^B, \cdots |s^A, s^B, \cdots)$ where $\kappa(A, B, \cdots)$ is a partial order, such that for every party, e.g., *A*, and every subset *X*, *Y*, … of the other parties,

$$P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots |s^A, s^B, \cdots)$$
$$= P(\kappa(A, X, Y, \cdots), A \npreceq X, A \npreceq Y, \cdots, o^X, o^Y, \cdots |s^B, \cdots).$$

Structure of causal processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Consider $\mathcal{W}^{1,\cdots,n} \equiv \mathcal{W}^{\mathcal{A},\mathcal{B}}$

$$\mathcal{A} = \{1, \cdots, k\}$$
$$\mathcal{B} = \{k+1, \cdots, n\}$$

If no signaling from $\mathcal{B}$ to $\mathcal{A}$ → exists **reduced process** $\mathcal{W}^{\mathcal{A}}$

$$p(o^1, \cdots, o^k | s^1, \cdots, s^n) = p(o^1, \cdots, o^k | s^1, \cdots, s^k)$$

Structure of causal processes

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$$\mathcal{W}^{\mathcal{A},\mathcal{B}} \equiv \mathcal{W}^{\mathcal{B}|\mathcal{A}} \circ \mathcal{W}^{\mathcal{A}}$$

↑ **conditional process**

Structure of causal processes

O. O. and C. Giarmatzi, arXiv:1506.05449

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$$\mathcal{W}^{\mathcal{A},\mathcal{B}} \equiv \mathcal{W}^{\mathcal{B}|\mathcal{A}} \circ \mathcal{W}^{\mathcal{A}}$$

$$p(o^1, \cdots, o^n|s^1, \cdots, s^n) = p(o^{k+1}, \cdots, o^n|s^{k+1}, \cdots, s^n; s^1, o^1, \cdots, s^k, o^k)\, p(o^1, \cdots, o^k|s^1, \cdots, s^k)$$

Structure of causal processes

O. O. and C. Giarmatzi, arXiv:1506.05449

Theorem (canonical causal decomposition)**:**

$$\mathcal{W}_c^{1,\cdots,n} = \sum_{i=1}^{n} q_i \mathcal{W}^{(1,\cdots,i-1,i+1,\cdots,n)\not\preceq i}, \quad q_i \geq 0$$

where

$$\mathcal{W}^{(1,\cdots,i-1,i+1,\cdots,n)\not\preceq i} = \mathcal{W}_c^{1,\cdots,i-1,i+1,\cdots,n|i} \circ \mathcal{W}^{i}$$

(iterative formulation)

Describes causal 'unraveling' of the events in the process.

Structure of causal processes

O. O. and C. Giarmatzi, arXiv:1506.05449

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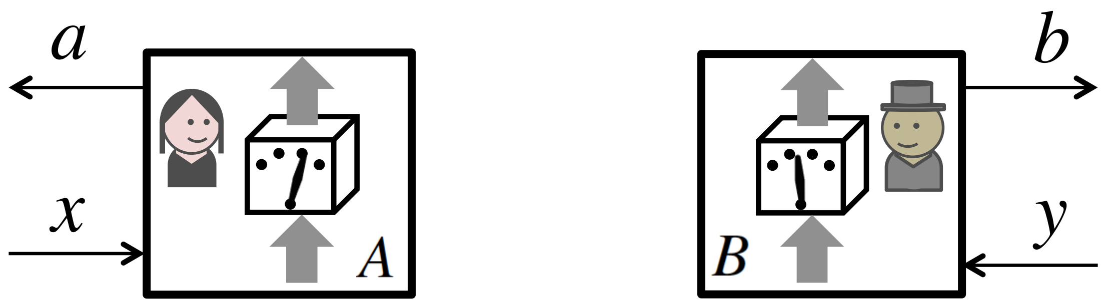
(iterative formulation)

Causal correlations form polytopes! [For the bipartite case, see Branciard *et al.*, NJP 18, 013008 (2016)]

Example of a causal inequality which is a facet:

Guess Your Neighbour's Input (GYNI) game



(bits)

$$p(x,y) = p(x)p(y), \qquad p(x) = 1/2, \quad p(y) = 1/2$$

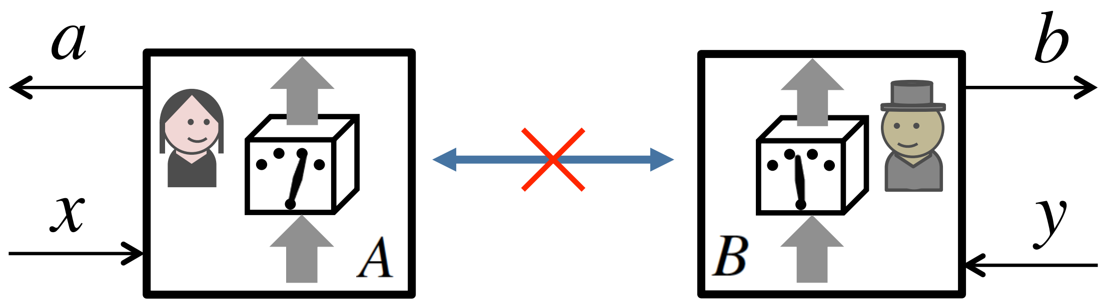
Goal: maximize $p(a = y, b = x)$

Branciard, Araujo, Feix, Costa, Brukner, New J. Phys. 18, 013008 (2016)

Example of a causal inequality which is a facet:

Guess Your Neighbour's Input (GYNI) game

Causal order $\kappa(A,B) = [A \not\preceq \not\succeq B]$



(bits)

$$p(x,y) = p(x)p(y), \qquad p(x) = 1/2, \quad p(y) = 1/2$$

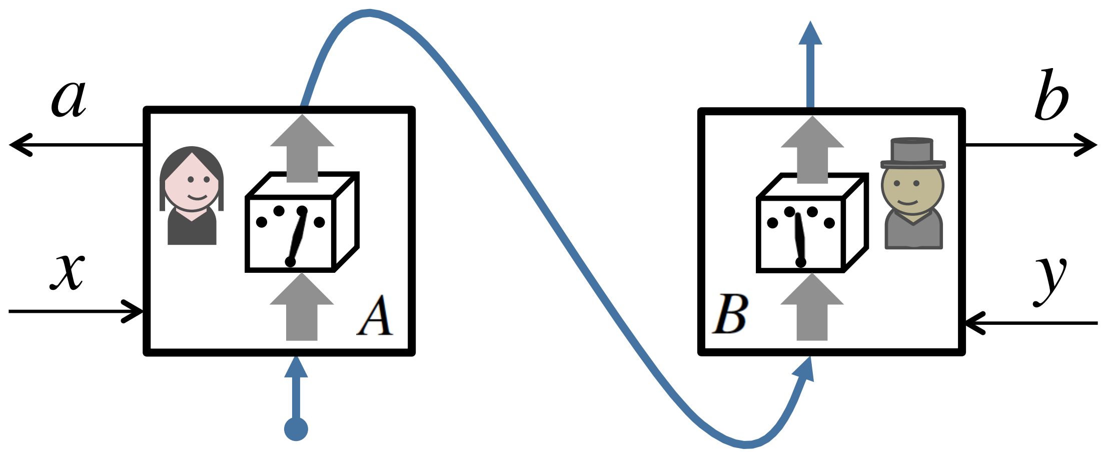
$$p(a = y, b = x) \leq 1/2$$

Branciard, Araujo, Feix, Costa, Brukner, New J. Phys. 18, 013008 (2016)

Example of a causal inequality which is a facet:

Guess Your Neighbour's Input (GYNI) game

Causal order $\kappa(A, B) = [A \prec B]$



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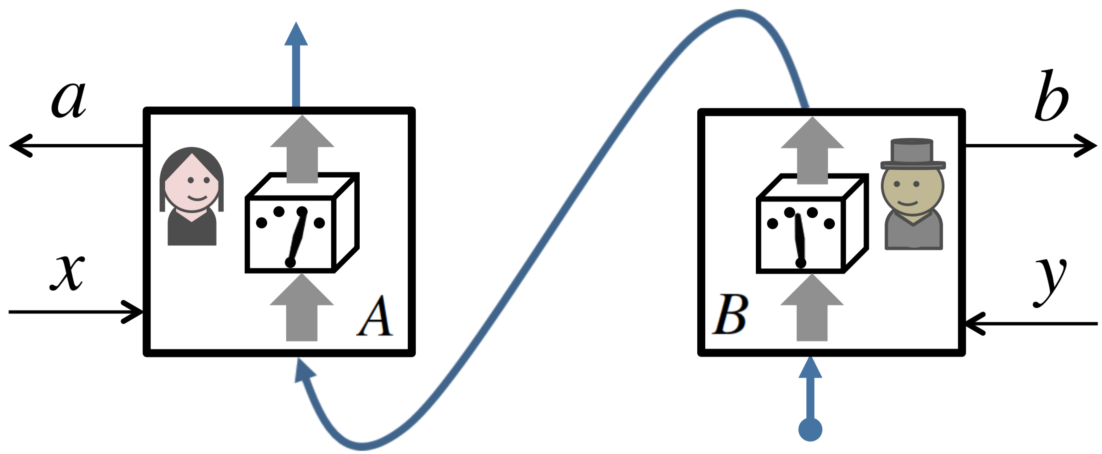
Branciard, Araujo, Feix, Costa, Brukner, New J. Phys. 18, 013008 (2016)

Example of a causal inequality which is a facet:

Guess Your Neighbour's Input (GYNI) game

Causal order $\kappa(A,B) = [B \prec A]$



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Branciard, Araujo, Feix, Costa, Brukner, New J. Phys. 18, 013008 (2016)

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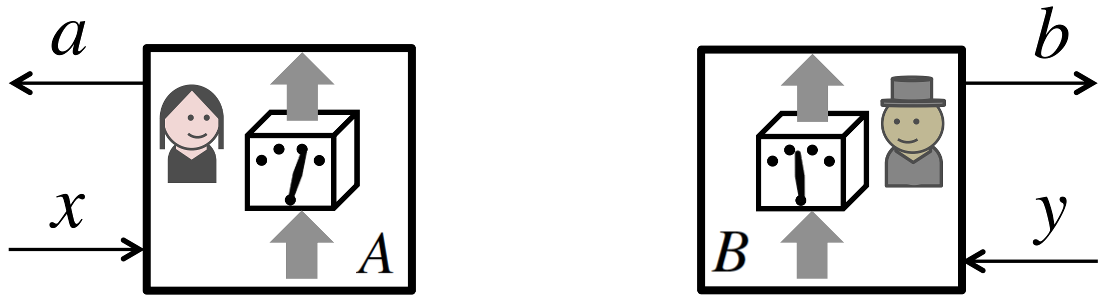


(bits)

$$p(x,y) = p(x)p(y), \qquad p(x) = 1/2, \quad p(y) = 1/2$$

There exists a process matrix which allow $p(a = y, b = x) > 1/2$.

Branciard, Araujo, Feix, Costa, Brukner, New J. Phys. 18, 013008 (2016)

Causal separability

O. O. and C. Giarmatzi, arXiv:1506.05449

A *quantum* process is called **causally separable** iff it can be written in a canonical causal form

$$\mathcal{W}_c^{1,\cdots,n} = \sum_{i=1}^{n} q_i \mathcal{W}^{(1,\cdots,i-1,i+1,\cdots,n)\nleq i}, \quad q_i \geq 0$$

where

$$\mathcal{W}^{(1,\cdots,i-1,i+1,\cdots,n)\nleq i} = \mathcal{W}_c^{1,\cdots,i-1,i+1,\cdots,n|i} \circ \mathcal{W}^i$$

with every process in this decomposition being a valid quantum process.

(analogy with Bell local and separable quantum states)

Agrees with the bipartite concept $W^{A_1A_2B_1B_2} = qW^{B\nleq A} + (1-q)W^{A\nleq B}$

Causal but causally nonseparable process

Chiribella, D'Ariano, Perinotti and Valiron, arXiv:0912.0195, PRA 2013

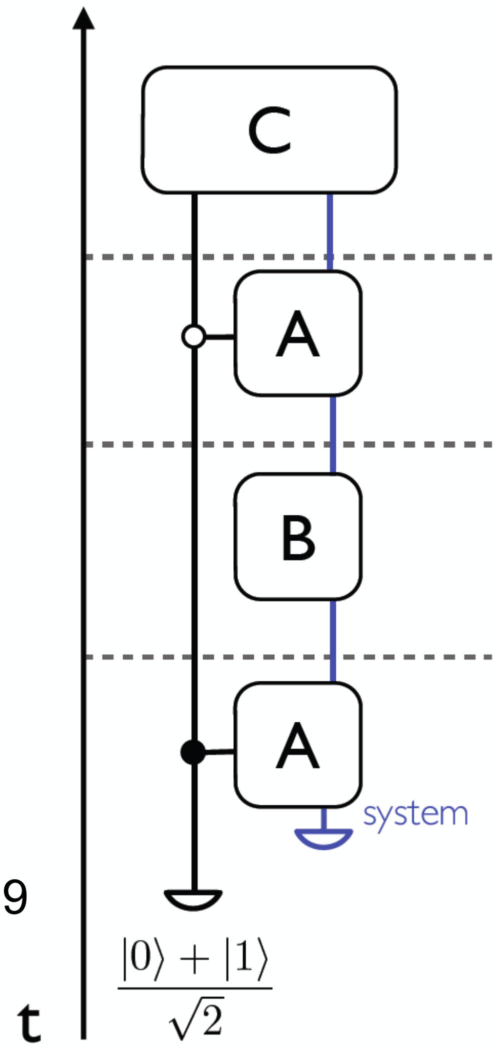
The *tripartite* process is not causally separable!

$$W^{A_1A_2B_1B_2C_1C_2} = |W\rangle\langle W|^{A_1A_2B_1B_2C_1C_2}$$

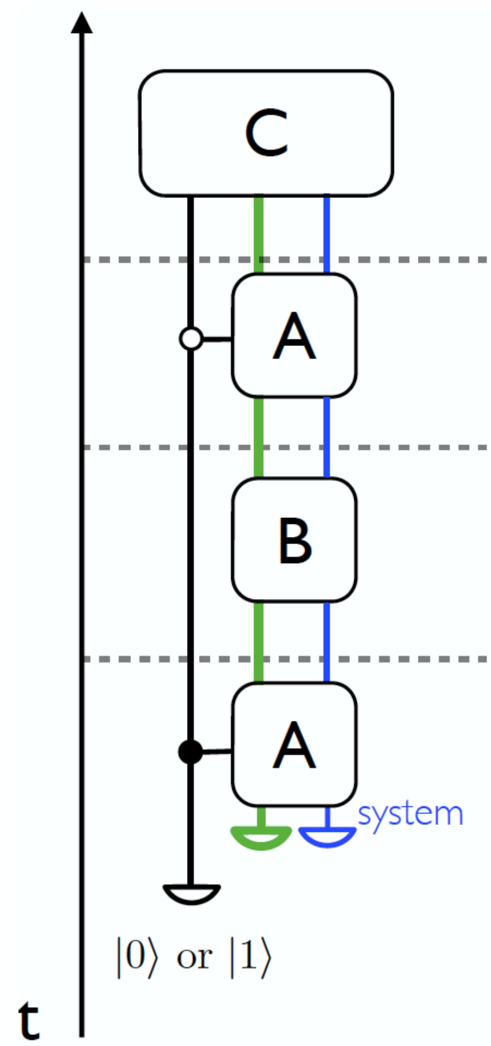
Yet, it cannot violate causal inequalities…

O. Oreshkov and C. Giarmatzi, arXiv:1506.05449

M. Araujo et al., NJP 17, 102001 (2015)



causal simulation



Causality and causal separability are different in the bipartit case too

A. Feix, M. Araujo, and C. Brukner, arXiv:1604.03391.

Example where $W^{A_1 A_2 B_1 B_2}$ is not causally separable,

but $(W^{A_1 A_2 B_1 B_2})^{\mathrm{T}_{B_1 B_2}}$ is causally separable.

(The two have the same statistics on local quantum operations.)

Non-causality can be *activated* by entanglement

O. O. and C. Giarmatzi, arXiv:1506.05449

Example where $W^{A_1A_2B_1B_2C_2}$ is causally separable (and hence causal),

but $W^{A_1A_2B_1B_2C_2} \otimes |\phi^+\rangle\langle\phi^+|^{B_1'C_1'}$ is non-causal.

$$W^{A_1A_2B_1B_2C_2} = \frac{1}{4}(\mathbb{1}^{A_1A_2B_1B_2C_2} + \frac{1}{\sqrt{2}}\sigma_z^{A_1}\sigma_z^{B_1}\sigma_z^{B_2}\sigma_x^{C_2} + \frac{1}{\sqrt{2}}\sigma_z^{A_2}\sigma_z^{B_1}\sigma_z^{C_2})$$

Non-causality can be *activated* by entanglement

O. O. and C. Giarmatzi, arXiv:1506.05449

Example where $W^{A_1A_2B_1B_2C_2}$ is causally separable (and hence causal),

but $W^{A_1A_2B_1B_2C_2} \otimes |\phi^+\rangle\langle\phi^+|^{B_1'C_1'}$ is non-causal.

One may expect that physically relevant processes are *extensible!*

➡ **More natural concepts of interest:**

Extensibly causal (EC)

Extensibly causally separable (ECS)

(the property does not change under extension with ancilla)

Some properties of EC and ECS processes

1) In the bipartite case, ECS = causally separable.

→ ECS is another possible multipartite generalization of the bipartite concept $W^{A_1 A_2 B_1 B_2} = q W^{B \not\preceq A} + (1-q) W^{A \not\preceq B}$.

2) EC ≠ ECS (tripartite example: the quantum switch).

The bipartite case is an open problem.

3) In the bipartite case, C ≠ EC either (Feix et al).

Classically controlled quantum circuits

O. O. and C. Giarmatzi, arXiv:1506.05449

A protocol:

1. Prepare a quantum register in some quantum state.

2. Perform a quantum operation on the register.

3. Depending on the outcome, choose which party is first and which subsystem of the register will be his/her input system.

4. After the first party operates, perform a quantum operation on the transformed register.

5. Depending on the outcome, choose which party is second and which subsystem of the register is his/her input system.

6. Continue analogously until all parties are used.

Classically controlled quantum circuits

O. O. and C. Giarmatzi, arXiv:1506.05449

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4. After the first party operates, perform a quantum operation on the transformed register.
5. Depending on the outcome, choose which party is second and which subsystem of the register is his/her input system.
6. Continue analogously until all parties are used.

Similar to classically controlled quantum Turing machine [Knill (1996), Valiron-Selinger (2005)].

Classically controlled quantum circuits

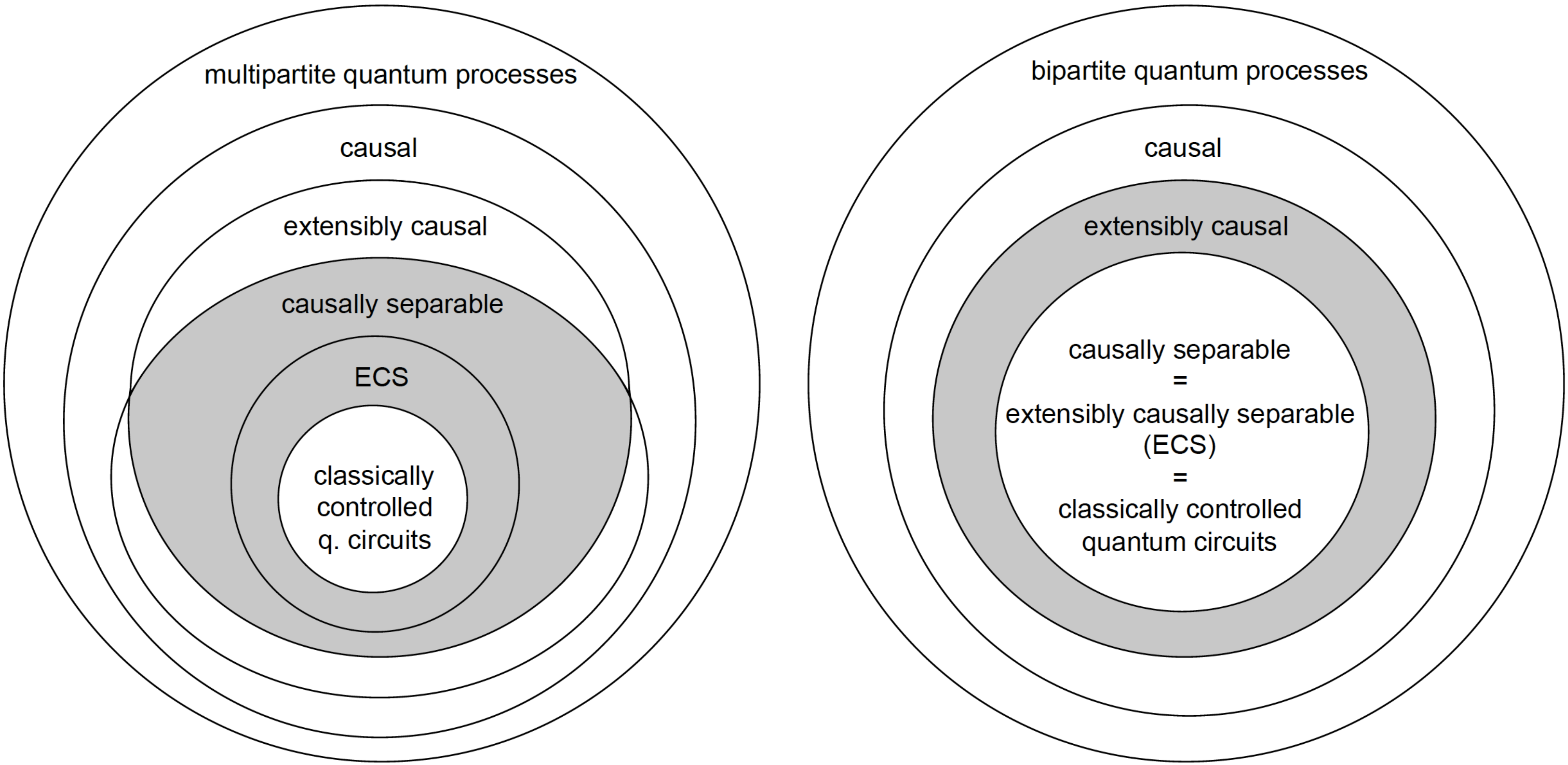
O. O. and C. Giarmatzi, arXiv:1506.05449

The processes realizable within this paradigm are ECS.

Conjecture: The reverse also holds: CCQC = ECS.

(certainly holds in the bipartite case)

What we know at present



a) Multipartite case.

b) Bipartite case.

Outlook

- Two conjectures:

  1) ECS = classically controlled quantum circuits (CCQC)?

  2) EC = quantum controlled quantum circuits (QCQC)?

- What is the structure of CCQC and QCQC process matrices?
- Are there physically admissible processes that are non-causal?
- What are the information processing powers of these classes?
- Causal inference for dynamical and quantum causal relations?

Related work

- Classical causal inference (Pearl, CUP 2009) in the context of quantum theory:

  Wood and Spekkens, New J. Phys. 17, 033002 (2015)
  Ried et al, Nat Phys 11, 414-420 (2015)

- Another notion of 'indefinite causal structures':

  Ried, Spekkens, ... (in preparation)

- Quantum and GPT generalizations of classical causal inference:

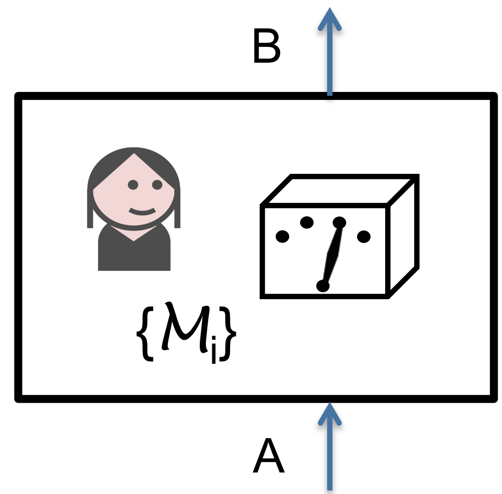
  Fritz, Comm. Math. Phys. 341(2), 391-434 (2016)
  Henson, Lal, Pusey, New J. Phys. 16, 113043 (2014)
  Pienaar and Brukner, New J. Phys. 17 073020 (2015)
  Cavalcanti and Lal, J. Phys. A: Math. Theor. 47, 424018 (2014)
  …

The process framework still assumes time locally, and it is time-asymmetric.

Could we relax the assumption of time also locally?

Recall

Idea 2. No post-selection

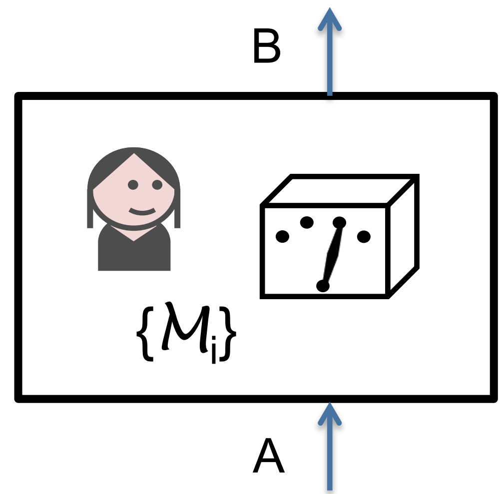


The 'choice' of operation can be known *before* the operation is applied

(Underlies the interpretation that an operation can be 'chosen'.)

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Proposal: drop the ‘no post-selection’ criterion



Operation =

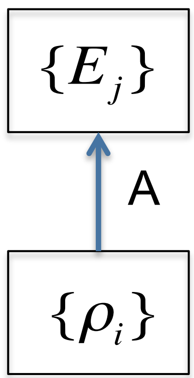
description of the possible events in a box conditional on local information

Time-symmetric quantum theory

O.O. and N. Cerf, Nature Phys. 11, 853 (2015)

Joint probabilities:

The basic probability rule.



$$p(i,j) = \frac{Tr(\rho_i E_j)}{Tr(\bar{\rho}\bar{E})}$$

where

$$\bar{\rho} = \sum_i \rho_i, \quad Tr(\bar{\rho}) = 1$$

$$\bar{E} = \sum_j E_j, \quad Tr(\bar{E}) = d$$

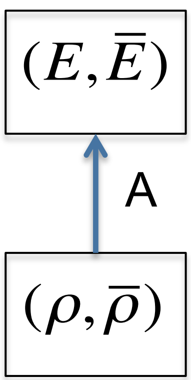
[Pegg, Barnett, Jeffers, J. Mod. Opt. 49, 913 (2002).]

New states and effects

States (equivalent preparation events): $(\rho, \bar{\rho})$, where $0 \le \rho \le \bar{\rho}$, $Tr(\bar{\rho}) = 1$.

Effects (equivalent measurement events): $(E, \bar{E})$, where $0 \le E \le \bar{E}$, $Tr(\bar{E}) = d$.

Joint probabilities:



$$p[(\rho, \bar{\rho}), (E, \bar{E})] = \frac{Tr(\rho E)}{Tr(\bar{\rho}\bar{E})}, \qquad Tr(\bar{\rho}\bar{E}) \neq 0$$

$$= 0, \qquad Tr(\bar{\rho}\bar{E}) = 0$$

States can be thought of as functions on effects and vice versa.

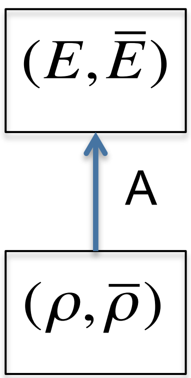
New states and effects

States (equivalent preparation events): $(\rho, \bar{\rho})$, where $0 \le \rho \le \bar{\rho},\ Tr(\bar{\rho}) = 1$.

Effects (equivalent measurement events): $(E, \bar{E})$, where $0 \le E \le \bar{E},\ Tr(\bar{E}) = d$.

Joint probabilities:

The set of states (effects) is not closed under convex combinations!

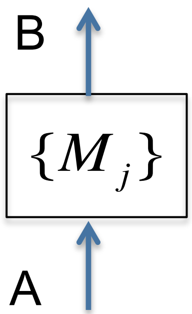


$$
\begin{aligned}
p[(\rho,\bar{\rho}),(E,\bar{E})] &= \frac{Tr(\rho E)}{Tr(\bar{\rho}\bar{E})}, & Tr(\bar{\rho}\bar{E}) \neq 0 \\
&= 0, & Tr(\bar{\rho}\bar{E}) = 0
\end{aligned}
$$

States can be thought of as functions on effects and vice versa.

General operations

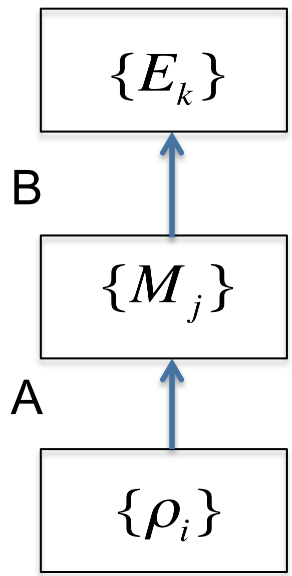
General operations: collections of CP maps $\{M_j\}$, s.t. $Tr(\sum_j M_j(\frac{I}{d_A})) = 1$.



Transformations: $(M, \bar{M})$, where $0 \leq M \leq \bar{M}$, $Tr(\bar{M}(\frac{I}{d_A})) = 1.$

Time reversal symmetry

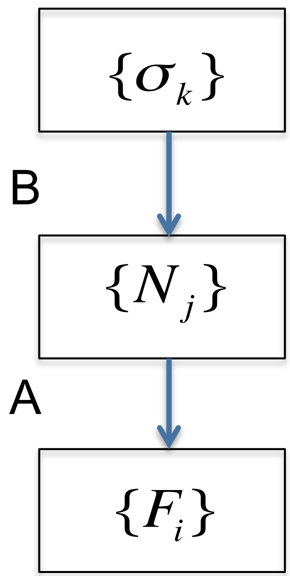
Example:



$$p(i,j,k) = \frac{Tr(E_k^B M_j^{A\to B}(\rho_i^A))}{Tr(\bar{E}^B \bar{M}^{A\to B}(\bar{\rho}^A))}$$

Time reversal symmetry

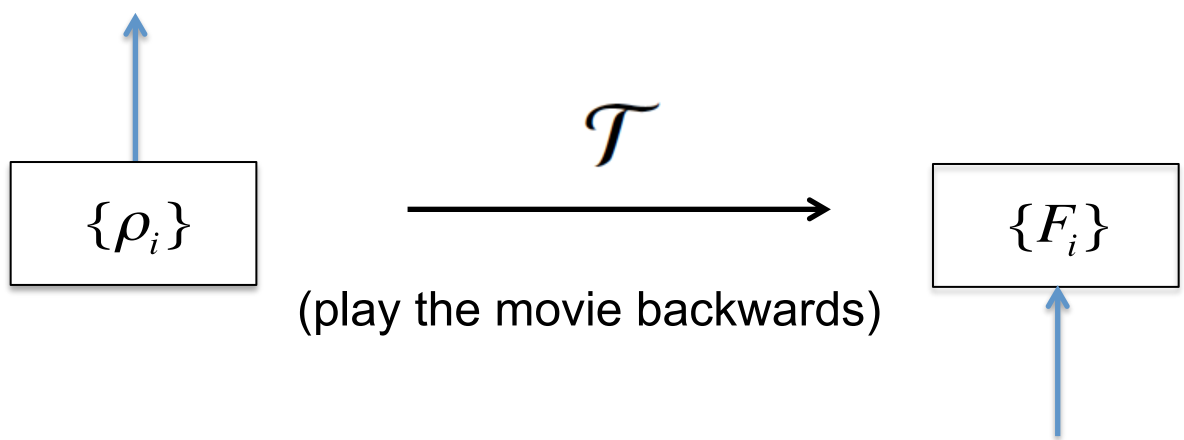
Example:



$$p(i,j,k) = \frac{Tr(F_i^A N_j^{B\to A}(\sigma_k^B))}{Tr(\overline{F}^A \overline{N}^{B\to A}(\overline{\sigma}^B))}$$

Time reversal symmetry

The exact form of time-reversal is not implicit in the formalism!



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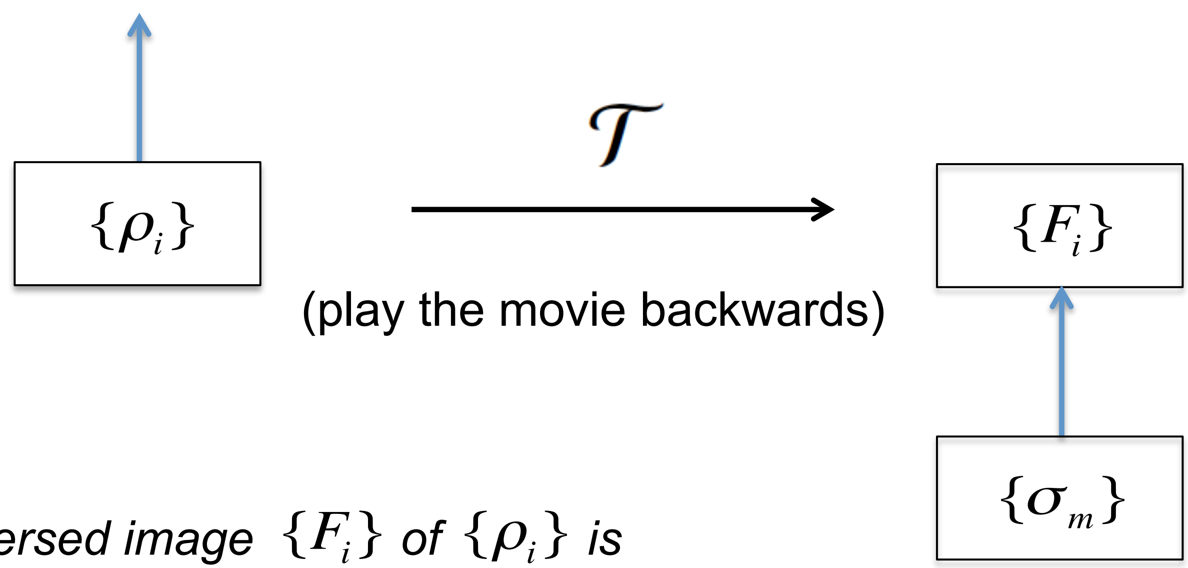


The *time-reversed image* $\{F_i\}$ *of* $\{\rho_i\}$ *is* determined relative to preparations $\{\sigma_m\}$ that have not been time-reversed.

Generalized Wigner's theorem

Important: states and effects are objects that live in *different* spaces.

There is *no natural isomorphism* between the two spaces!

We represent them by operators in the same space based on the bilinear form

$$(E^{A^*}, \rho^A) = \langle \rho^A, E^A \rangle = \mathrm{Tr}[\rho^A E^A] \; ,$$

which defines an isomorphism $E^{A^*} \leftrightarrow E^A$.

This isomorphism has no physical meaning! It is simply based on the choice of bilinear form, and should not be confused with time reversal!

Generalized Wigner's theorem

Two types of symmetry transformation:

Type I - States go to states, and effects go to effects: $(\hat{S}^{A}_{s\to s}, \hat{S}^{A}_{e\to e})$

Type II - States go to effects, and effects go to states: $(\hat{S}^{A}_{s\to e}, \hat{S}^{A}_{e\to s})$

Generalized Wigner's theorem

- Symmetries of type I are described by:

$$\hat{S}_{s\to s}(\rho;\overline{\rho}) = (\sigma;\overline{\sigma}) = \left(\frac{S\rho S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}S^{\dagger})};\frac{S\overline{\rho}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}S^{\dagger})}\right),$$

$$\hat{S}_{e\to e}(E;\overline{E}) = (F;\overline{F}) = \left(d\frac{S^{-1\dagger}ES^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})};d\frac{S^{-1\dagger}\overline{E}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})}\right),$$

or

$$\hat{S}_{s\to s}(\rho;\overline{\rho}) = (\sigma;\overline{\sigma}) = \left(\frac{S\rho^{T}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}^{T}S^{\dagger})};\frac{S\overline{\rho}^{T}S^{\dagger}}{\mathrm{Tr}(S\overline{\rho}^{T}S^{\dagger})}\right),$$

$$\hat{S}_{e\to e}(E;\overline{E}) = (F;\overline{F}) = \left(d\frac{S^{-1\dagger}E^{T}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^{T}S^{-1})};d\frac{S^{-1\dagger}\overline{E}^{T}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^{T}S^{-1})}\right),$$

where S is an invertible operator, and T is a transposition is some basis.

Generalized Wigner's theorem

- Symmetries of type II are described by:

$$\hat{S}_{s\to e}(\rho;\overline{\rho}) = (F;\overline{F}) = \left(d\frac{S\rho S^\dagger}{\mathrm{Tr}(S\overline{\rho}S^\dagger)}; d\frac{S\overline{\rho}S^\dagger}{\mathrm{Tr}(S\overline{\rho}S^\dagger)}\right),$$

$$\hat{S}_{e\to s}(E;\overline{E}) = (\sigma;\overline{\sigma}) = \left(\frac{S^{-1\dagger}ES^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})}; \frac{S^{-1\dagger}\overline{E}S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}S^{-1})}\right),$$

or

$$\hat{S}_{s\to e}(\rho;\overline{\rho}) = (F;\overline{F}) = \left(d\frac{S\rho^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}; d\frac{S\overline{\rho}^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}\right),$$

$$\hat{S}_{e\to s}(E;\overline{E}) = (\sigma;\overline{\sigma}) = \left(\frac{S^{-1\dagger}E^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}; \frac{S^{-1\dagger}\overline{E}^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}\right).$$

where S is an invertible operator, and T is a transposition is some basis.

Generalized Wigner's theorem

If the evolution under time reversal is described by Schrödinger's equation, positivity of energy → **time reversal is in the class**:

$$\hat{S}_{s\to e}(\rho;\overline{\rho}) = (F;\overline{F}) = (d\frac{S\rho^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}; d\frac{S\overline{\rho}^T S^\dagger}{\mathrm{Tr}(S\overline{\rho}^T S^\dagger)}),$$

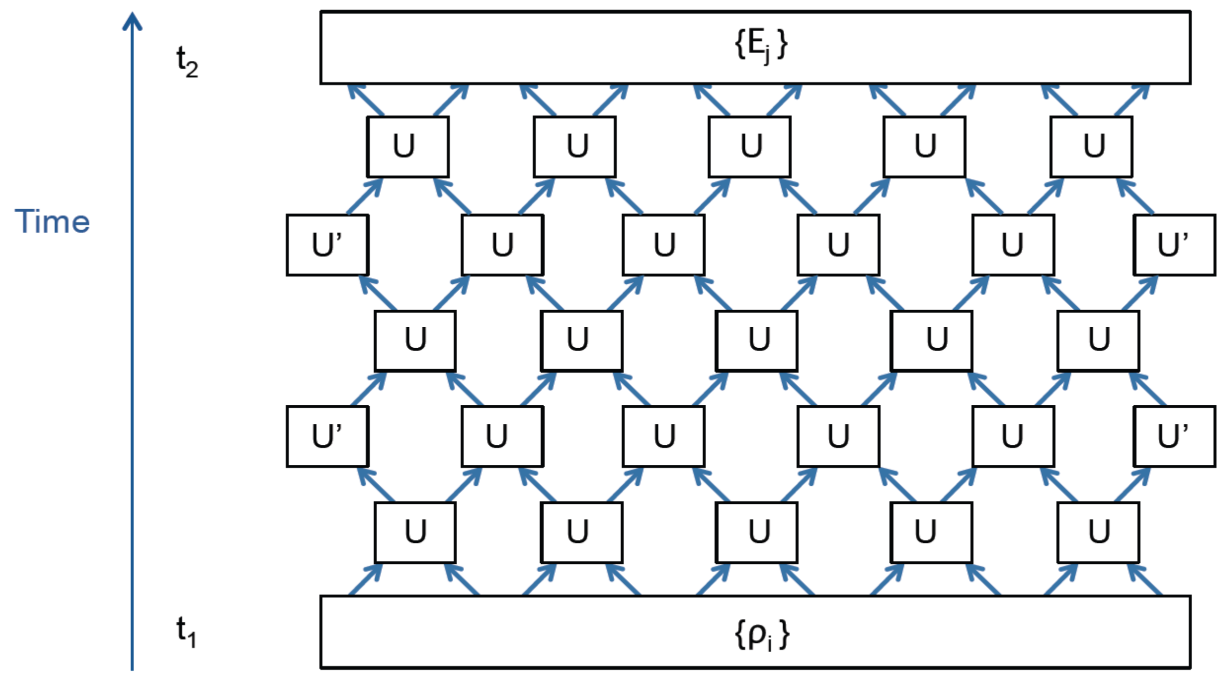
$$\hat{S}_{e\to s}(E;\overline{E}) = (\sigma;\overline{\sigma}) = (\frac{S^{-1\dagger} E^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}; \frac{S^{-1\dagger}\overline{E}^T S^{-1}}{\mathrm{Tr}(S^{-1\dagger}\overline{E}^T S^{-1})}).$$

The standard notion corresponds to unitary S .

Understanding the observed asymmetry

A toy model of the universe:

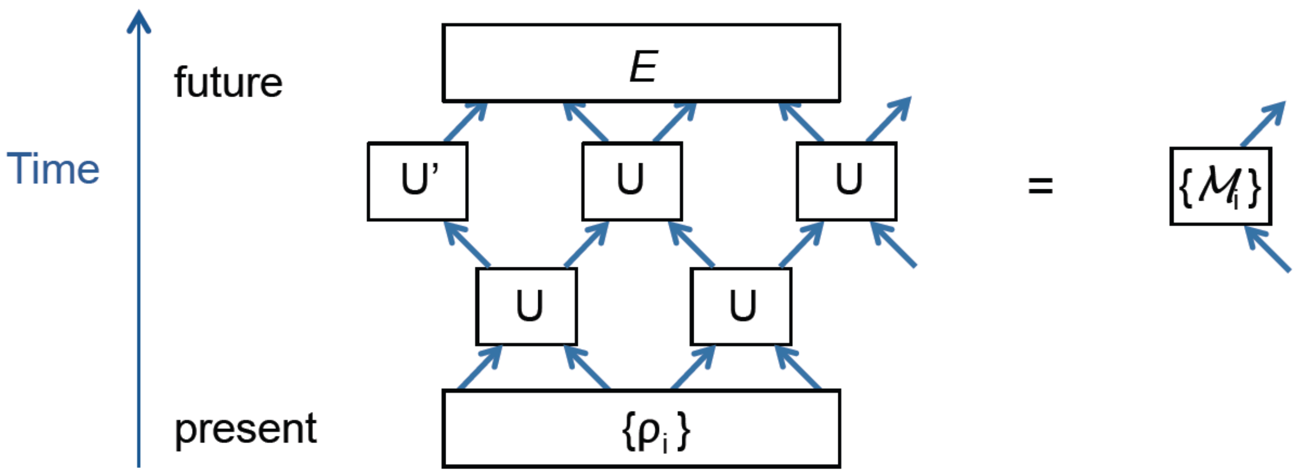
O.O. and N. Cerf, Nature Phys. 11, 853 (2015)



For an observer at t_1, all future circuits contain standard operations iff $\sum_{j \in Q} E_j = \mathbb{1}$.

(linked to the fact that we can remember the past and not the future)

Note: it is logically possible that non-standard operations were obtainable without post-selection



A time-neutral formalism

An isomorphism dependent on time reversal

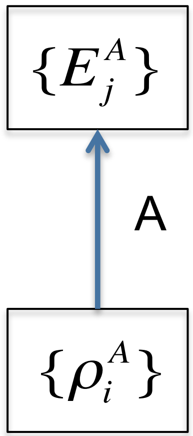
TRANSFORMATIONS EFFECTS ON PAIRS OF SYSTEMS

$$(\mathcal{M}^{A_1 \to B_1}; \overline{\mathcal{M}}^{A_1 \to B_1}) \leftrightarrow (M^{A_1 B_2}; \overline{M}^{A_1 B_2})$$

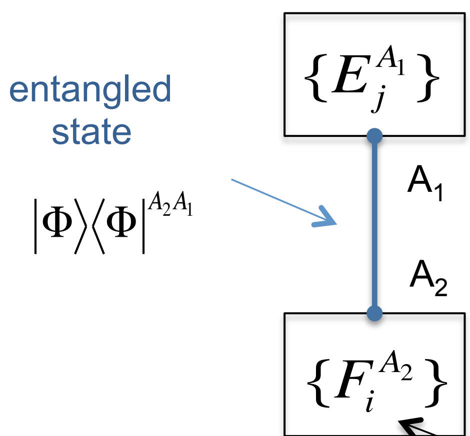
O.O. and N. Cerf, arXiv: 1406.3829

A time-neutral formalism

Example:



$$p(i,j) = \frac{Tr(\rho_i^A E_j^A)}{Tr(\overline{\rho}^A \overline{E}^A)}$$

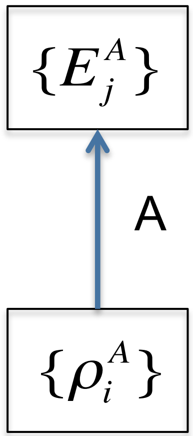


$$p(i,j) = \frac{Tr\left[(F_i^{A_2} \otimes E_j^{A_1})|\Phi\rangle\langle\Phi|^{A_2 A_1}\right]}{Tr\left[(\overline{F}^{A_2} \otimes \overline{E}^{A_1})|\Phi\rangle\langle\Phi|^{A_2 A_1}\right]}$$

F_i is the time-reversed image of ρ_i .

A time-neutral formalism

Example:



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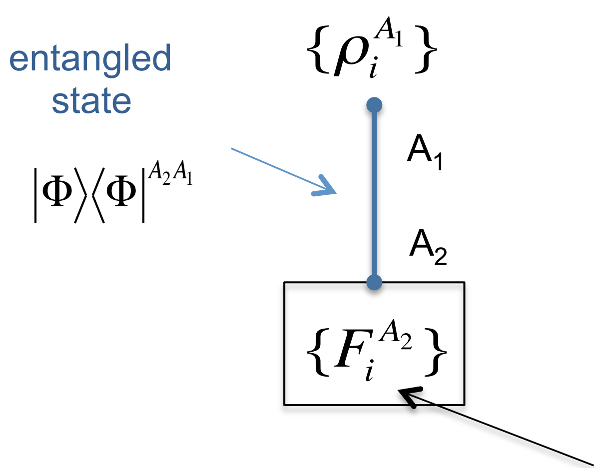
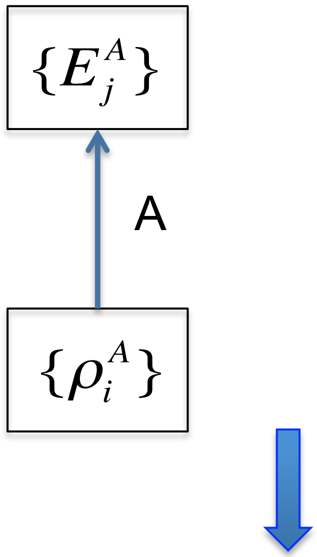


the usual states and effects live on systems of type 1

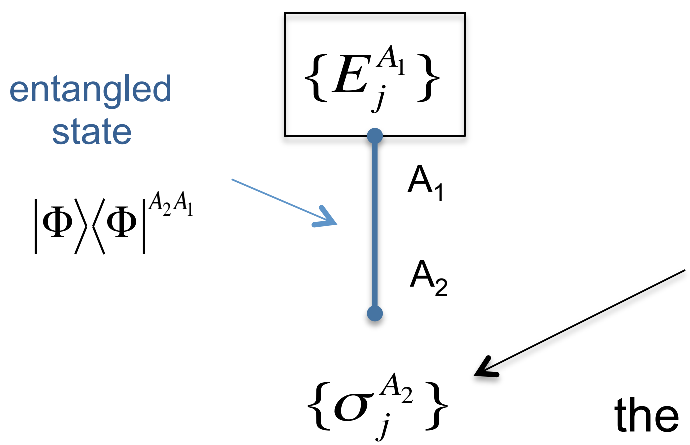
F_i is the time-reversed image of ρ_i .

A time-neutral formalism

Example:



$$p(i,j) = \frac{Tr(\rho_i^A E_j^A)}{Tr(\bar{\rho}^A \bar{E}^A)}$$

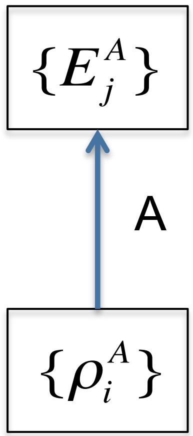


σ_j is the time-reversed image of E_j .

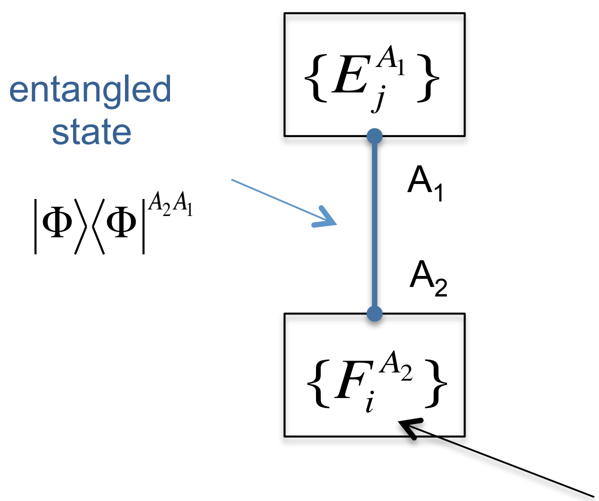
the time-reversed states and effects live on systems of type 2

A time-neutral formalism

Example:



$$p(i,j) = \frac{Tr(\rho_i^A E_j^A)}{Tr(\overline{\rho}^A \overline{E}^A)}$$



$$p(i,j) = \frac{Tr\left[(F_i^{A_2} \otimes E_j^{A_1})|\Phi\rangle\langle\Phi|^{A_2A_1}\right]}{Tr\left[(\overline{F}^{A_2} \otimes \overline{E}^{A_1})|\Phi\rangle\langle\Phi|^{A_2A_1}\right]}$$

F_i is the time-reversed image of ρ_i .

A time-neutral formalism

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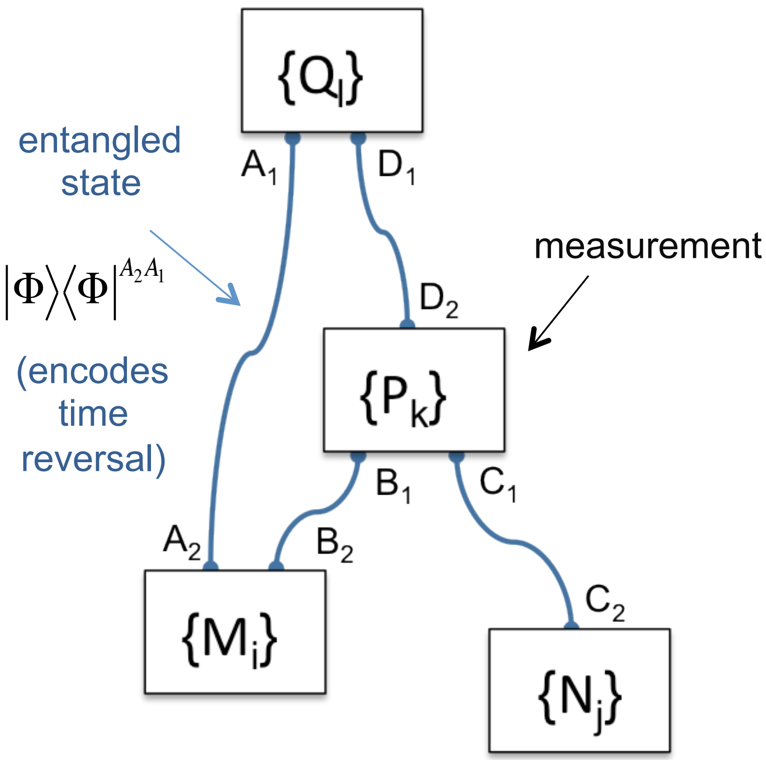


Joint probabilities:

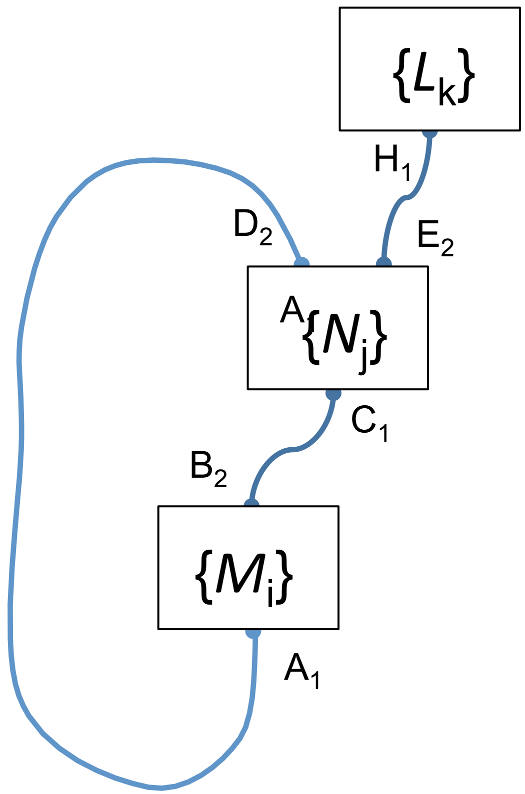
$$p(i,j,k,l\,|\,\{M_i^{A_2B_2}\},\{N_j^{C_2}\},\cdots,W) = \frac{Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}{\sum_{i,j,k,l} Tr[W^{A_1A_2B_1B_2C_1C_2D_1D_2}(M_i^{A_2B_2}\otimes N_j^{C_2}\otimes P_k^{B_1C_1D_2}\otimes Q_l^{A_1D_1})]}$$

'process matrix' (encodes the connections)

$$= |\Phi\rangle\langle\Phi|^{A_1A_2}\otimes|\Phi\rangle\langle\Phi|^{B_1B_2}\otimes|\Phi\rangle\langle\Phi|^{C_1C_2}\otimes|\Phi\rangle\langle\Phi|^{D_1D_2}$$

A time-neutral formalism

Can describe circuits with *cycles*:



All such circuits can be realized using post-selection.

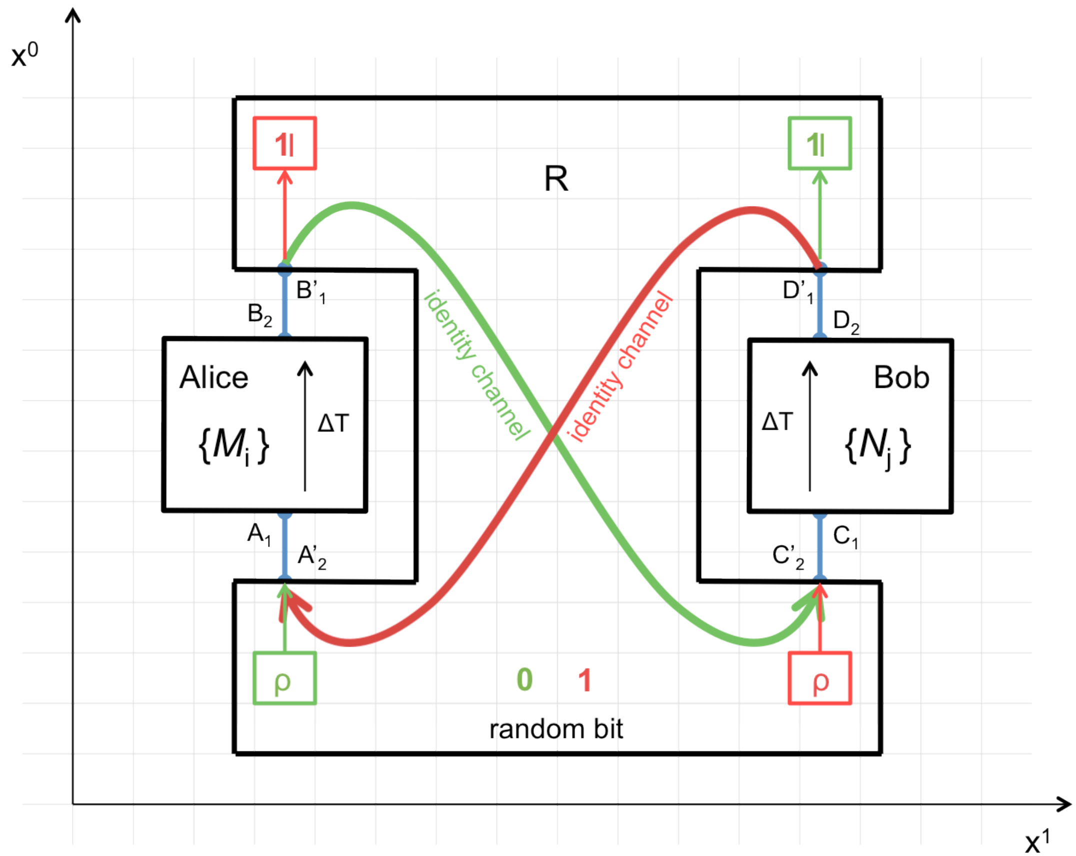
Compatible with closed timelike curves (P-CTC):

Bennett and Schumacher, talk at QUPON (2005);   Svetlichny, arXiv:0902.4898 (2009); Lloyd et al., Phys. Rev. Lett 106, 040403 (2011); …

A time-neutral formalism

There exist circuits with cycles that can be obtained without post-selection!

(classical switch)

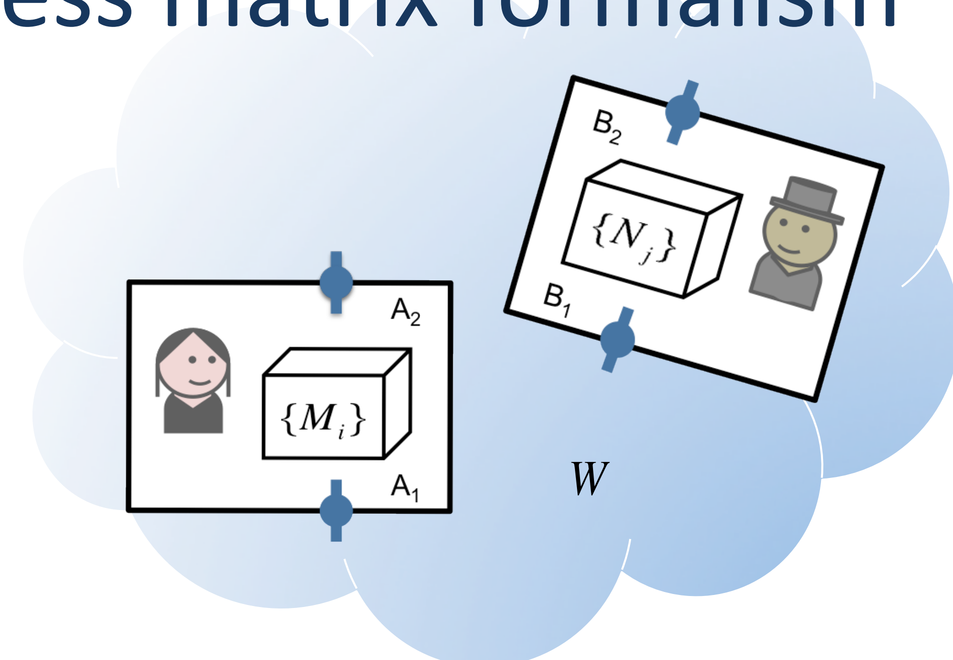


the idea of *background independence* extended to random events

(provides a basis for understanding experiments with the quantum switch)

Time-symmetric process matrix formalism

Equivalently:



external variables

$$p(i, j, \cdots | \{M_i^{A_1A_2}\}, \{M_j^{B_1B_2}\}, \cdots, W) = \frac{Tr[W^{A_1A_2B_1B_2\cdots}(M_i^{A_1A_2} \otimes M_j^{B_1B_2} \otimes \cdots)]}{Tr[W^{A_1A_2B_1B_2\cdots}(\bar{M}^{A_1A_2} \otimes \bar{M}^{B_1B_2} \otimes \cdots)]}$$

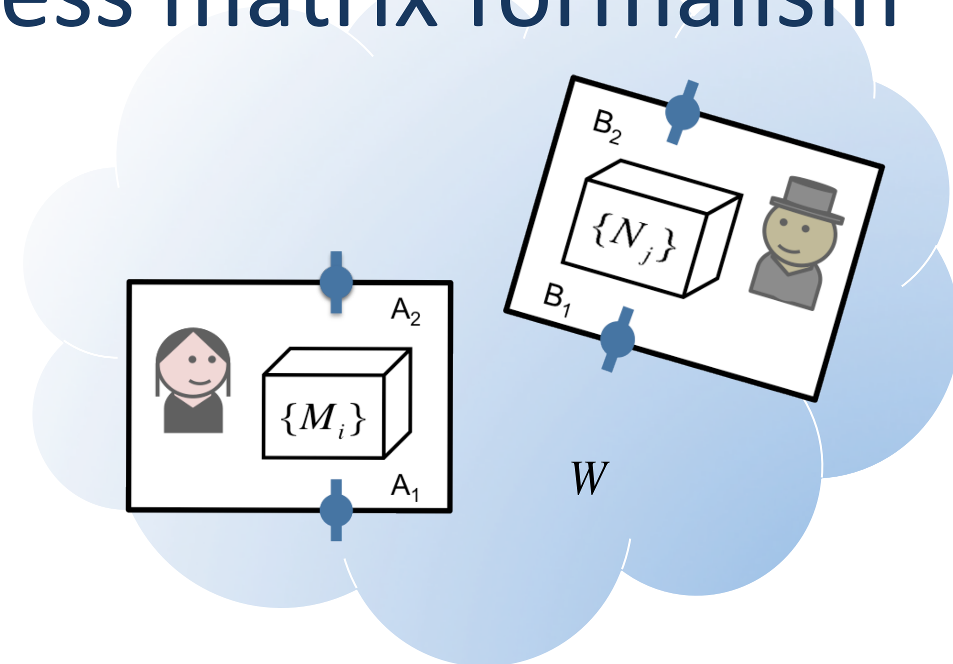
The 'process matrix':

$$W^{A_1A_2B_1B_2\cdots} \geq 0, \quad Tr(W^{A_1A_2B_1B_2\cdots}) = 1$$

Note: Any process matrix is allowed.

O.O. and N. Cerf, arXiv: 1406.3829

Time-symmetric process matrix formalism



Equivalently:

external variables

$$p(i, j, \cdots | \{M_i^{A_1A_2}\}, \{M_j^{B_1B_2}\}, \cdots, W) = \frac{Tr[W^{A_1A_2B_1B_2\cdots}(M_i^{A_1A_2} \otimes M_j^{B_1B_2} \otimes \cdots)]}{Tr[W^{A_1A_2B_1B_2\cdots}(\bar{M}^{A_1A_2} \otimes \bar{M}^{B_1B_2} \otimes \cdots)]}$$

Linked to two-time and multi-time state vector formalism:

Aharonov, Bergmann, Lebowitz, PRB 134, 1410 (1964)
Aharonov, Popescu, Tollaksen, Vaidman, arXiv:0712.0320 (2007)

Dropping the assumption of local time

Observation: The predictions are the same whether the systems are of type 1 or type 2.

Proposal: There is no a priori distinction between systems of type 1 and 2.

The concept of time should come out from properties of the dynamics!

Dropping the assumption of local time

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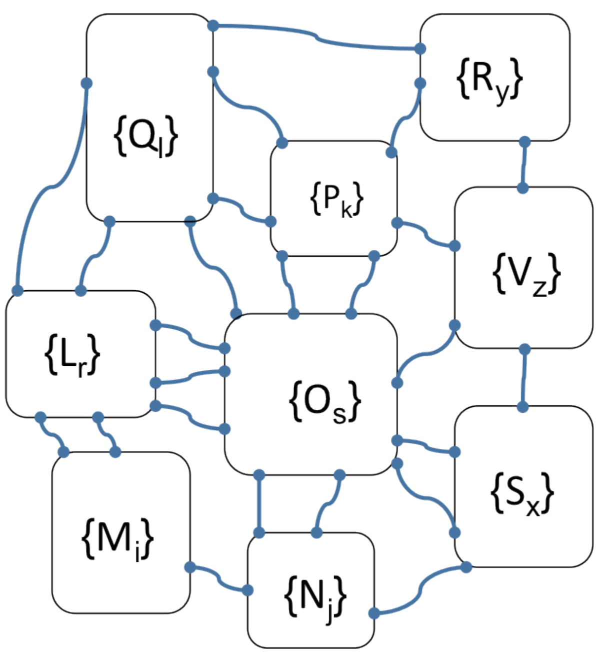
The general picture:

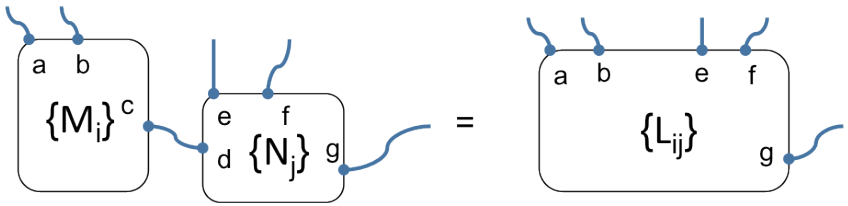


Main probability rule

$$p(i,j,\cdots|\{M_i^{\cdots}\},\{N_j^{\cdots}\},\cdots) = \frac{Tr[W_{wires}^{\cdots\cdots}(M_i^{\cdots} \otimes N_j^{\cdots} \otimes \cdots)]}{Tr[W_{wires}^{\cdots\cdots}(\bar{M}^{\cdots} \otimes \bar{N}^{\cdots} \otimes \cdots)]}$$

O.O. and N. Cerf, arXiv: 1406.3829

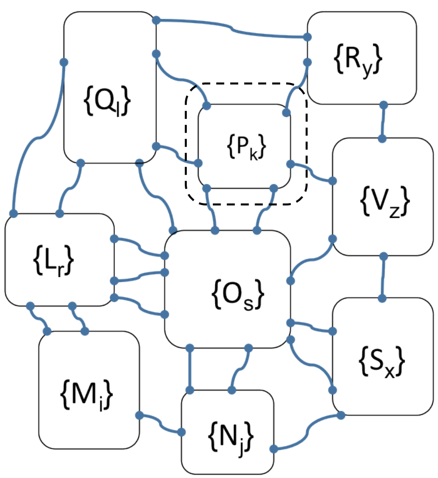
- Connecting operations amounts to new operations.



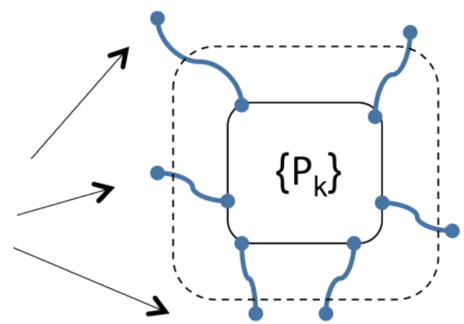
$$L_{ij}^{abefg} = \frac{Tr_{cd}[|\Phi\rangle\langle\Phi|^{cd}(M_i^{abc} \otimes N_j^{defg})]}{Tr[|\Phi\rangle\langle\Phi|^{cd}(\bar{M}_i^{abc} \otimes \bar{N}_j^{defg})]}$$

(In some cases this may be the *null* operation.)

- Every region performs a 'measurement' on the state prepared by its complement.



A region 'outputs' states

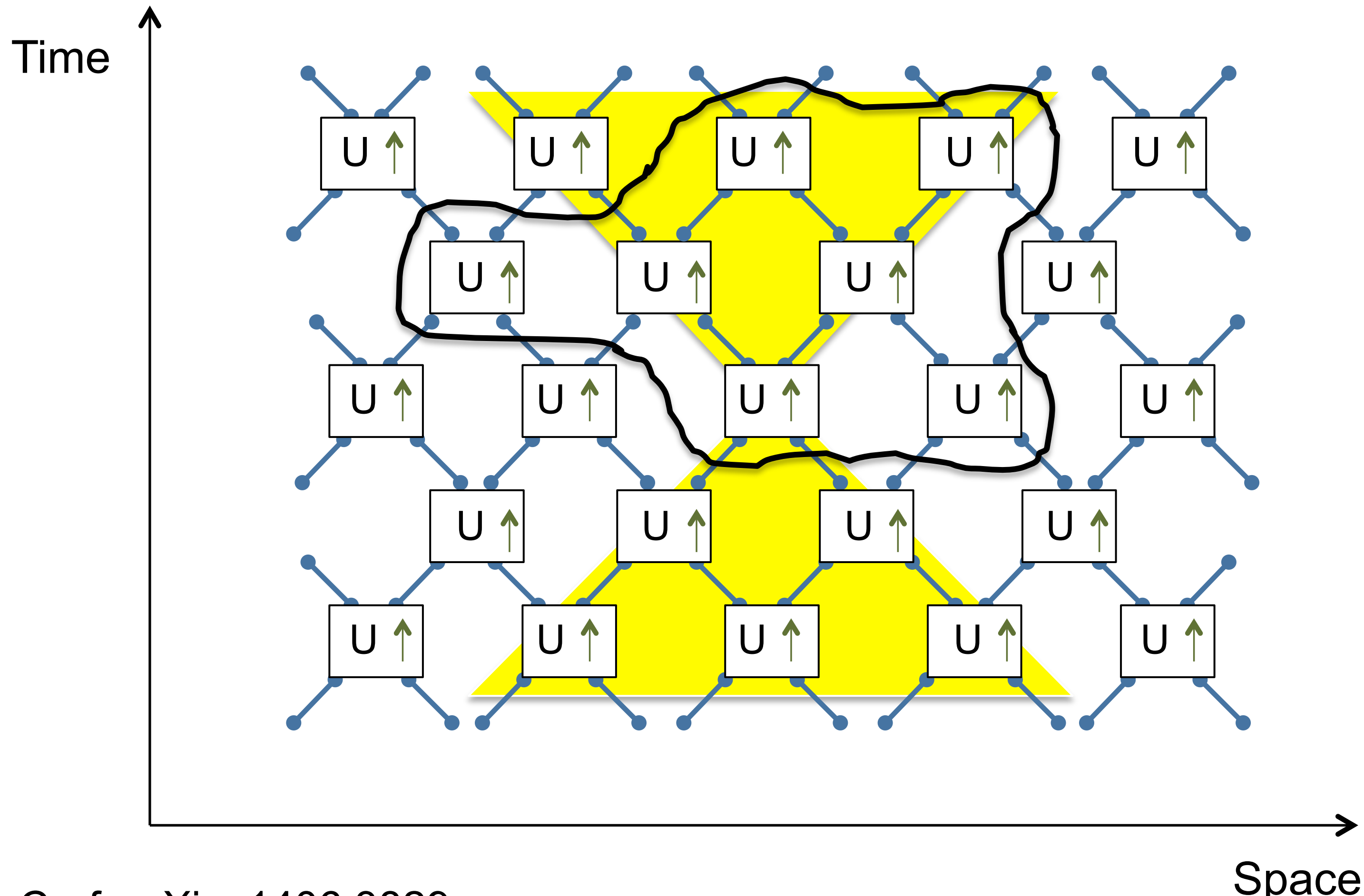


- There is an update rule for states and operations upon learning of information (not shown here).

Limit of quantum field theory

R. Oeckl, Phys. Lett. B 575, 318 (2003), ... , Found. Phys. 43, 1206 (2013)

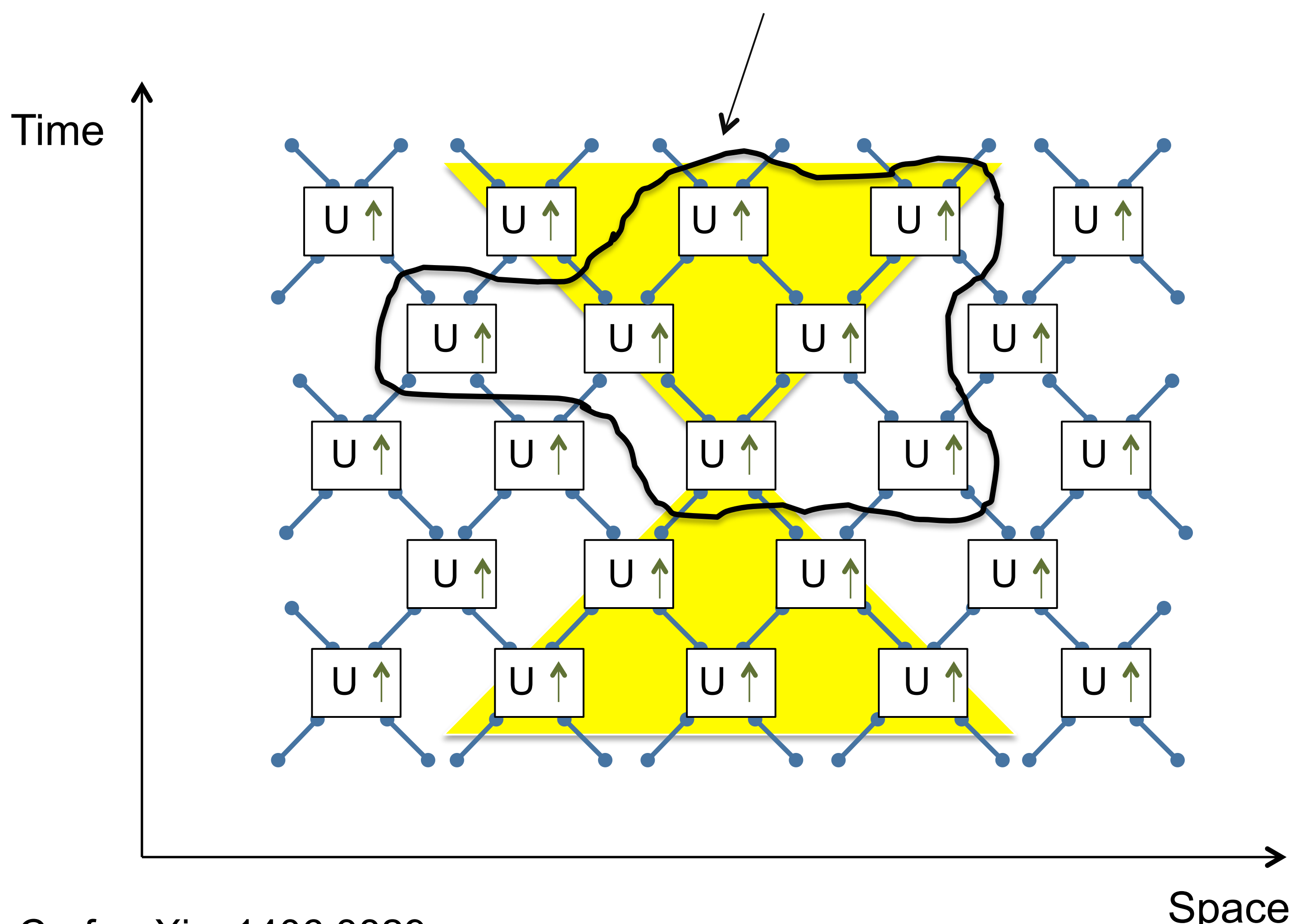
(the 'general boundary' approach with a few generalization)



O.O. and N. Cerf, arXiv: 1406.3829

Proposal: causal structure from correlations

The causal structure underlying the dynamics in the region is reflected in correlation properties of the state on the boundary.



O.O. and N. Cerf, arXiv: 1406.3829

Conclusion on the last part

It is possible to formulate a QT without any predefined time, which

- agrees with experiment

- has a physical and informational interpretation

- opens up the possibility to understand time and causal structure as dynamical and explore new forms of dynamics

- Is the metric/causal structure emergent, or do we need to postulate it as another field?
- What processes/networks can be realized without post-selection (e.g., can we violate causal inequalities?)
- How can we formulate general covariant laws of dynamics in this framework?
- What does it imply for the foundations of information processing?