Indefinite causal structures using diagrammatic methods

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THERE ARE NO MISTAKES
JUST HAPPY LITTLE ACCIDENTS
quickmeme.com

The setting

- They own a lab,

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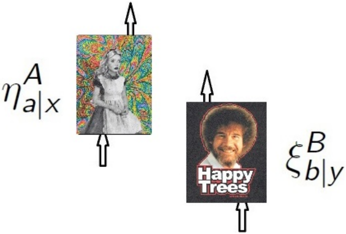

Note:

We do not assume a fixed causal order between Alice and Bob. Only "local quantum mechanics".

- Processes

$$\eta^A_{a|x} : B(H_{A_I}) \to B(H_{A_O})$$
$$\xi^B_{b|y} : B(H_{B_I}) \to (H_{B_O})$$

- Completely positive trace non-increasing

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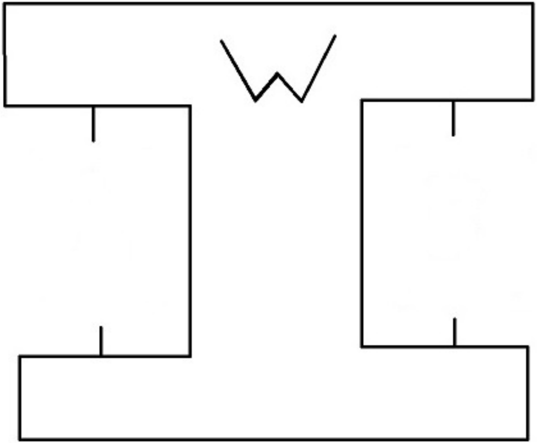
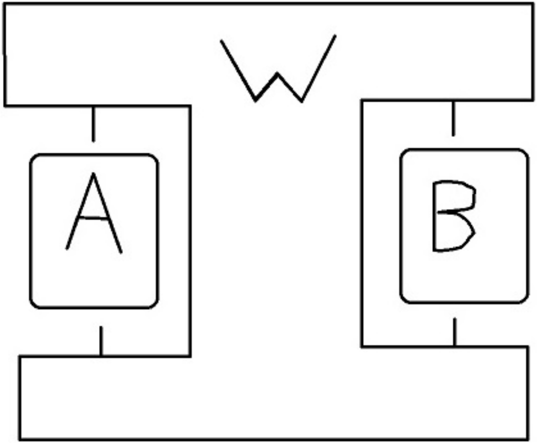
- W can be seen as a "generalized state"

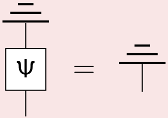

W


W
A
B

causality

Definition

A process $\Psi : X \to Y$ *is called first order causal if it is trace preserving. That is:*



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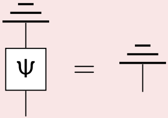
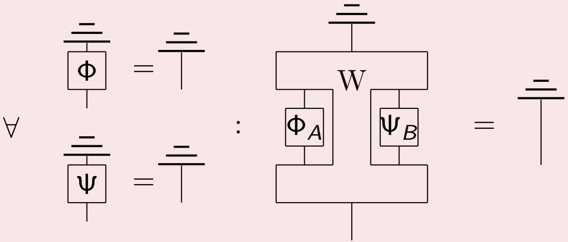
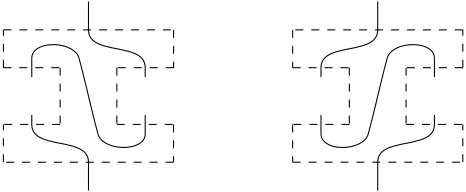


"If we discard the output of a causal process we might as well have discarded the input right away"

Definition

A process $W : (A_I \otimes B_I \to A_O \otimes B_O) \to (C_I \to C_O)$ is called bipartite second-order causal (SOC_2) if it sends causal processes to causal processes



- Example: connect Alice to Bob or vice versa
- 

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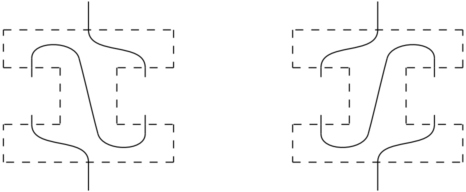
- 
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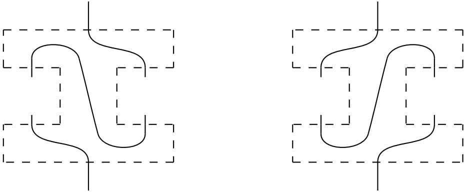
- 
- Fixed causal order
- There are W process matrices which do not admit a causal order!

- Note SOC_2 only has to hold for separable processes

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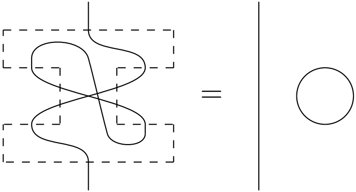
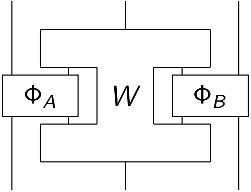
- Example: swap



- (This implies it holds for non-signalling processes)

- What if A and B have ancilla systems



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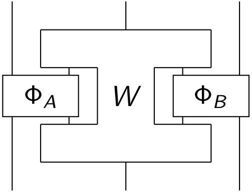
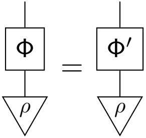


Theorem

$$SOC_2 \Rightarrow Completely\, SOC_2$$

- We say a process theory has *enough causal states* if

$\Big(\forall \rho \text{ causal } .$



$\Big) \implies \Phi = \Phi'$

proof

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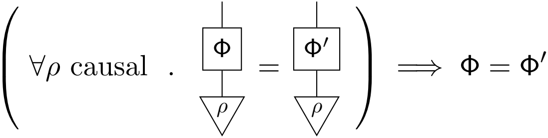
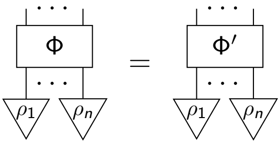


- This is the case for operator algebras with cp maps

Enough *separable* causal states if

$\left(\forall \rho_1, \ldots, \rho_n \text{ causal} \ . \right.$



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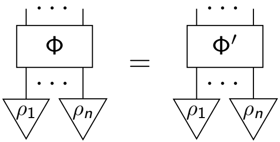

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Compactness gives: enough causal states ⇒ enough separable causal states

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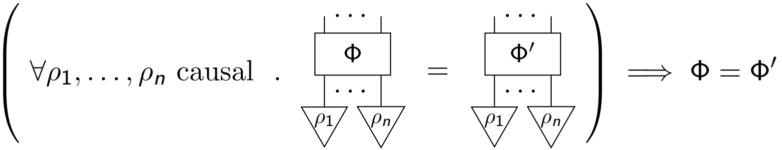
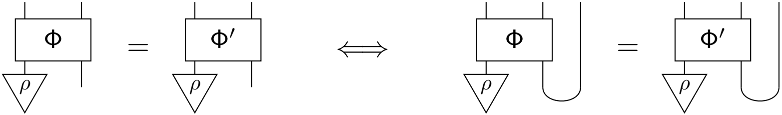


Compactness gives: enough causal states $\Rightarrow$ enough separable causal states



- For causal states ρ_A, ρ_B

are causal

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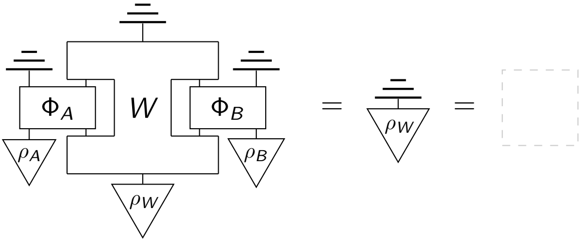
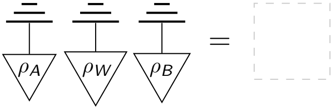
- So



- but also



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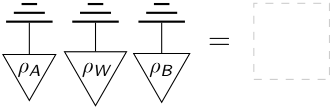
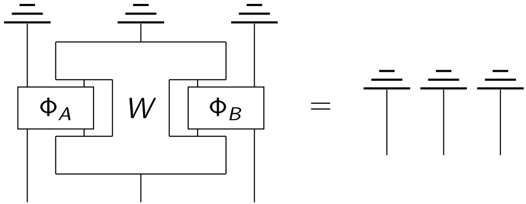


- Hence (enough separable causal states)



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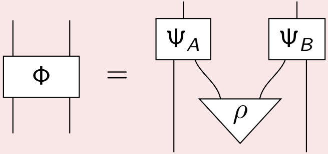
A process is called strongly non-signalling if it is of the form



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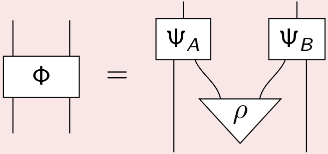
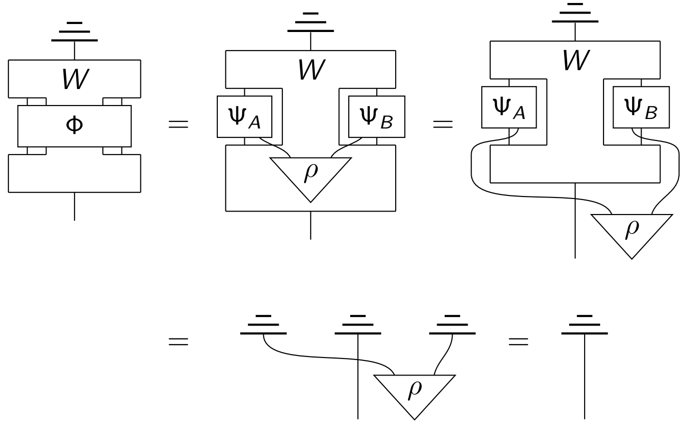
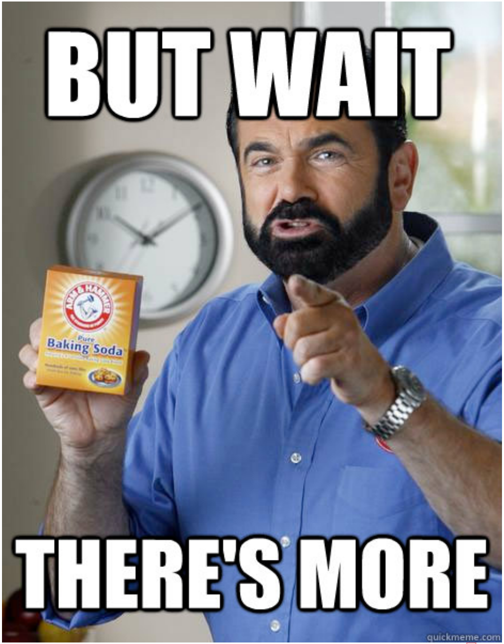


I.e., Alice and Bob share a state, but no other communication.

corollary

If Φ is strongly non-signalling and W is SOC_2 then




BUT WAIT
Pure
Baking Soda
THERE'S MORE
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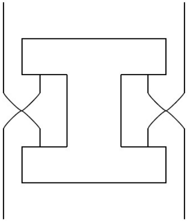
Application

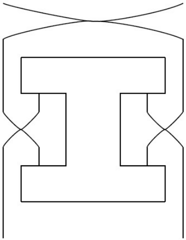
Swap maps are causal.

Plug them into W.



Then this is causal again.

And in fact non-signalling!



Bijection between Ws and certain non-signaling boxes