Noncontextuality inequalities for Specker's compatibility scenario, and beyond

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(based on joint work with Rob Spekkens.)

Motivation

To go beyond "state-dependent" proofs of KS-contextuality for quantum theory to tests of contextuality for *arbitrary* operational theories.[1] Part of the larger project of making noncontextuality an experimentally testable hypothesis that doesn't presume determinism, much like local causality.[2]

[1] To see how to do this for "state-independent" *quantum* contextuality based on Kochen-Specker uncolourability, see R. Kunjwal and R. W. Spekkens, Phys. Rev. Lett. 115, 110403 (2015).

[2] See R. W. Spekkens, Phys. Rev. A 71, 052108 (2005), for a detailed account of the motivations and proposed formalism for this approach.

Previously...

We have obtained noncontextuality inequalities as constraints on the predictability of measurements with respect to corresponding preparations under the assumption of noncontextuality for preparations and/or measurements:

- Noncontextuality inequalities based on KS-uncolourability: R. Kunjwal and R. W. Spekkens, Phys. Rev. Lett. 115, 110403 (2015).[3]
- Noncontextuality inequality going beyond the traditional Kochen-Specker notion of a 'context'[4]: Mazurek et al., arXiv:1505.06244.

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[3] A more detailed follow-up to this is in the works.

[4] Namely, in quantum theory, "commutative contexts": if $[A, B] = 0$ and $[A, C] = 0$, then B and C provide two different (commutative) contexts for the measurement of A.

Quick intro to the basic notions that we need...

Operational theory

$(\mathcal{P}, \mathcal{M}, p)$, where $p : (\mathcal{M}, \mathcal{P}) \to [0,1]$ is the probability $p(k|M,P)$ that $k \in \mathcal{K}_M$ occurs when $M \in \mathcal{M}$ is implemented following $P \in \mathcal{P}$. For each M:

$$\sum_{k \in \mathcal{K}_M} p(k|M,P) = 1 \quad \forall P \in \mathcal{P}. \tag{1}$$

$[k|M]$ denotes the event: outcome k occurs for measurement M.

Ontological model of an Operational theory

(Λ, μ, ξ), where each preparation $P \in \mathcal{P}$ is associated with a distribution $\mu(\lambda|P) \in [0,1]$ such that $\sum_{\lambda\in\Lambda} \mu(\lambda|P) = 1$ for all $P \in \mathcal{P}$, each $[k|M]$ with the probability $\xi(k|M,\lambda) \in [0,1]$ that $[k|M]$ occurs when the ontic state of the system is λ, and for each $M \in \mathcal{M}$:

$$\sum_{k\in\mathcal{K}_M} \xi(k|M,\lambda) = 1 \quad \forall\lambda \in \Lambda. \tag{2}$$

Assumption of outcome determinism: for any $[k|M]$, $\xi(k|M,\lambda) \in \{0,1\} \forall\lambda \in \Lambda$.

An ontological model of an operational theory must be empirically adequate, that is:

$$p(k|M,P) = \sum_{\lambda \in \Lambda} \xi(k|M,\lambda)\mu(\lambda|P) \tag{3}$$

for all $P \in \mathcal{P}, M \in \mathcal{M}$. This is how an operational theory and its ontological model fit together.

Operational equivalence of experimental procedures

- $[k|M]$ and $[k'|M']$ operationally equivalent ($[k|M] \simeq [k'|M']$) if no preparation procedure yields differing outcome probabilities for them, i.e.,

$$\forall P \in \mathcal{P} : p(k|M, P) = p(k'|M', P). \tag{4}$$

  Two measurement procedures M and M' are operationally equivalent, i.e., $M \simeq M'$, if each effect belonging to M is operationally equivalent to an effect belonging to M' and vice versa.

- P and P' operationally equivalent ($P \simeq P'$) if no measurement event $[k|M]$ yields differing outcome probabilities for them, i.e.,

$$p(k|M, P) = p(k|M, P') \quad \forall k \in \mathcal{K}_M, (M, \mathcal{K}_M) \in \mathcal{M}. \tag{5}$$

What is a 'context'?

- Any distinction between two operationally equivalent experimental procedures.[5]
- Measurement contexts: (a) whether M_1 is jointly measured with M_2 (M_{12}) or with M_3 (M_{13}), where $M_1^{(2)} \simeq M_1^{(3)} \simeq M_1$, (b) different operationally equivalent ways of implementing a fair coin flip measurement.[6]
- Preparation contexts: (a) different convex decompositions: $\frac{I}{2} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$, (b) different purifications: $\rho_A = \mathrm{Tr}_B|\psi\rangle\langle\psi|_{AB} = \mathrm{Tr}_C|\phi\rangle\langle\phi|_{AC}$.

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[5] A distinction that doesn't make a difference, operationally. 'Contextuality': this distinction sometimes *necessarily* makes a difference in *any* ontological model underlying the operational statistics.

[6] M. D. Mazurek, M. F. Pusey, R. Kunjwal, K. J. Resch, and R. W. Spekkens, An experimental test of noncontextuality without unwarranted idealizations, arXiv:1505.06244 [quant-ph] (2015).

Noncontextuality (or Leibnizianity): identity of indiscernibles

If there exists no operational way to distinguish two things, then they are physically identical.[7]

- Measurement noncontextuality:

$$[k|M] \simeq [k'|M'] \Rightarrow \xi(k|M,\lambda) = \xi(k'|M',\lambda) \quad \forall \lambda \in \Lambda$$

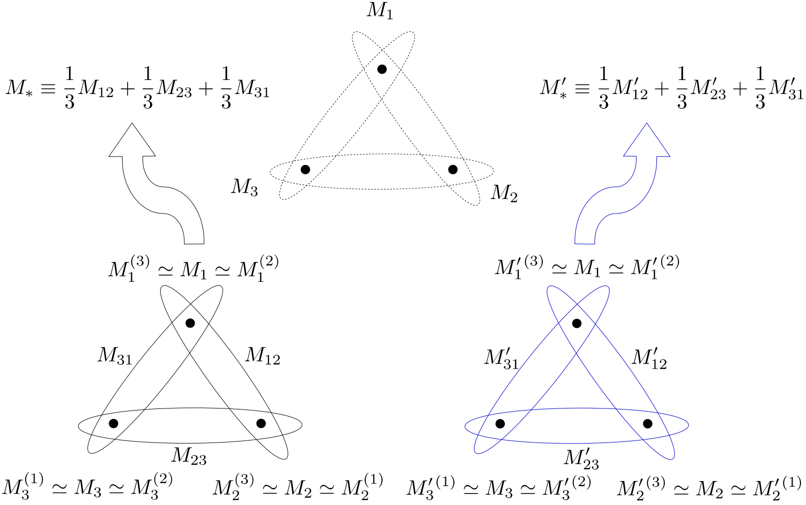
- Preparation noncontextuality:

$$P \simeq P' \Rightarrow \mu(\lambda|P) = \mu(\lambda|P') \quad \forall \lambda \in \Lambda$$

[7]Contrapositively: if two things are non-identical, or physically distinct, then there exists an operational way to distinguish them.

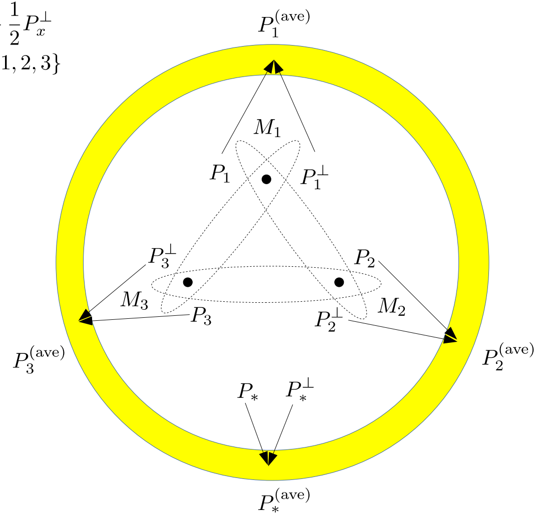
Questions?

Specker's compatibility scenario: measurement procedures



Preparation procedures

$$P_x^{(\mathrm{ave})} = \frac{1}{2}P_x + \frac{1}{2}P_x^{\perp}$$
$$\forall x \in \{*, 1, 2, 3\}$$



Operational equivalences: $P_1^{(\mathrm{ave})} \simeq P_2^{(\mathrm{ave})} \simeq P_3^{(\mathrm{ave})} \simeq P_*^{(\mathrm{ave})}$

Quantities of interest

$$p(\text{anti}|M_*, P) \equiv \frac{1}{3}\sum_{(ij)} p(X_i \neq X_j | M_{ij}, P)$$

$$p(\text{anti}|M'_*, P) \equiv \frac{1}{3}\sum_{(ij)} p(X_i \neq X_j | M'_{ij}, P)$$

$$\text{Predictability of } (M, P) : \eta(M, P) \equiv 2 \max_{X \in \{0,1\}} p(X|M, P) - 1$$

$$\eta_{\text{ave}} \equiv \frac{1}{6}\sum_{i=1}^{3}((\eta(M_i, P_i) + \eta(M_i, P_i^{\perp}))$$

Noncontextuality inequalities

$$p(\text{anti}|M_*, P_*) + p(\text{anti}|M'_*, P_*^{\perp}) \leq 2(1 - \frac{1}{3}\eta_{\text{ave}})$$

$$p(\text{anti}|M_*, P_*) + p(\text{anti}|M_*, P_*^{\perp}) \leq 2(1 - \frac{1}{3}\eta_{\text{ave}})$$

$$p(\text{anti}|M_*, P_*) \leq \frac{2}{3}(2 - \eta_{\text{ave}})$$

Only for $\eta_{\text{ave}} = 1$ do these inequalities resemble Kochen-Specker inequalities.

Deterministic vertices

- 8 deterministic vertices, $\{000, 001, 010, 011, 100, 101, 110, 111\}$, characterized by 4 KS inequalities: no more than 2 anticorrelated pairs (one inequality) and anticorrelation of any pair is less than the sum of anticorrelations of the remaining pairs (three inequalities).

$$\sum_{(ij)} \xi(X_i \neq X_j | M_{ij}, \lambda) \leq 2 \tag{6}$$

$$\xi(X_i \neq X_j | M_{ij}, \lambda) \leq \xi(X_j \neq X_k | M_{jk}, \lambda) + \xi(X_k \neq X_i | M_{ki}, \lambda)$$
$$\forall (ijk) \in \{(123), (231), (312)\} \tag{7}$$

(Relabelling X_k shows equivalence with the first inequality.)

Indeterministic vertices

$$\xi(X_i = 0, X_j = 1 | M_{ij}, \lambda) = \xi(X_i = 1, X_j = 0 | M_{ij}, \lambda) = \frac{1}{2}$$
$$\forall (ij) \in \{(12), (23), (31)\}, \tag{8}$$

$$\xi(X_i = 0, X_j = 1 | M_{ij}, \lambda) = \xi(X_i = 1, X_j = 0 | M_{ij}, \lambda) = \frac{1}{2}$$
$$\xi(X_j = 0, X_k = 0 | M_{jk}, P) = \xi(X_j = 1, X_k = 1 | M_{jk}, \lambda) = \frac{1}{2}$$
$$\xi(X_k = 0, X_i = 0 | M_{ki}, P) = \xi(X_k = 1, X_i = 1 | M_{ki}, \lambda) = \frac{1}{2}$$
$$\text{for } (ijk) \in \{(123), (231), (312)\}. \tag{9}$$

We allow preparations to sample from λ corresponding to indeterministic vertices. Traditionally, you restrict yourself to deterministic vertices.

In terms of predictability:

$$\eta_{\mathrm{ave}} \leq 3 - \frac{3}{2}(p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M'_*, P_*^{\perp}))$$

$$\eta_{\mathrm{ave}} \leq 3 - \frac{3}{2}(p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M_*, P_*^{\perp}))$$

$$\eta_{\mathrm{ave}} \leq 2 - \frac{3}{2}p(\mathrm{anti}|M_*, P_*)$$

Is there a quantum realization?

- If there is, it cannot be in terms of sharp/projective measurements. Firstly, three pairwise commuting projective measurements admit simultaneous value assignments and therefore can't even violate the 2/3 Kochen-Specker bound on anticorrelation. Secondly, the measurements $\{M_1, M_2, M_3\}$ necessarily have to be unsharp/nonprojective in order to admit non-unique joint measurements M_{ij} and M'_{ij} for pairs $\{M_i, M_j\}$.
- Since the measurements are nonprojective, we are not guaranteed the existence of preparations with respect to which they perfectly predictable. Hence, it's not clear $\eta_{\text{ave}} = 1$ is possible with nonprojective measurements.
- We construct a qubit realization with unsharp measurements and imperfect predictability.

Quantum violation

$$p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M'_*, P_*^{\perp}) \leq 2(1 - \frac{1}{3}\eta_{\mathrm{ave}})$$

- Measurements: M_i $(i = 1, 2, 3)$ is the qubit POVM $\{E_0^{(i)}, E_1^{(i)}\}$, given by

$$E_{X_i}^{(i)} \equiv \frac{1}{2} I + (-1)^{X_i} \frac{1}{2} \eta_0 \vec{\sigma} \cdot \hat{n}_i, \tag{10}$$

- Preparations: P_i is the qubit state given by rank 1 projector $| + \hat{n}_i\rangle\langle + \hat{n}_i|$ and $P_i^{\perp}$ by $| - \hat{n}_i\rangle\langle - \hat{n}_i|$, for $i = 1, 2, 3$. P_* is a qubit state given by $| + \hat{n}_*\rangle\langle + \hat{n}_*|$ and $P_*^{\perp}$ by $| - \hat{n}_*\rangle\langle - \hat{n}_*|$.

Pairwise joint measurability of $\{M_1, M_2, M_3\}$:

$$\eta_0 \leq \min_{(i,j)} \frac{1}{\sqrt{1 + \sqrt{1 - (\hat{n}_i \cdot \hat{n}_j)^2}}}. \tag{11}$$

Joint measurements

$$M_{ij} : E^{(ij)}_{X_i X_j} \equiv \frac{1}{2}\left(1 + (-1)^{X_i + X_j}\eta_0^2 \hat{n}_i.\hat{n}_j\right)\Pi_{\hat{n}^{ij}_{X_i X_j}} \tag{12}$$

where

$$\Pi_{\hat{n}^{ij}_{X_i X_j}} \equiv \frac{1}{2}(I + \vec{\sigma}.\hat{n}^{ij}_{X_i X_j}),$$

$$\hat{n}^{ij}_{X_i X_j} \equiv \frac{\eta_0((-1)^{X_i}\hat{n}_i + (-1)^{X_j}\hat{n}_j) - (-1)^{X_i + X_j}\vec{a}_{ij}}{1 + (-1)^{X_i + X_j}\eta_0^2 \hat{n}_i.\hat{n}_j},$$

$$M'_{ij} : E'^{(ij)}_{X_i X_j} \equiv \frac{1}{2}\left(1 + (-1)^{X_i + X_j}\eta_0^2 \hat{n}_i.\hat{n}_j\right)\Pi_{\hat{n}'^{ij}_{X_i X_j}} \tag{13}$$

where

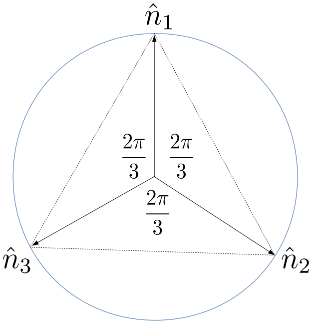
$$\Pi_{\hat{n}'^{ij}_{X_i X_j}} \equiv \frac{1}{2}(I + \vec{\sigma}.\hat{n}'^{ij}_{X_i X_j}),$$

$$\hat{n}'^{ij}_{X_i X_j} \equiv \frac{\eta_0((-1)^{X_i}\hat{n}_i + (-1)^{X_j}\hat{n}_j) + (-1)^{X_i + X_j}\vec{a}_{ij}}{1 + (-1)^{X_i + X_j}\eta_0^2 \hat{n}_i.\hat{n}_j},$$

and

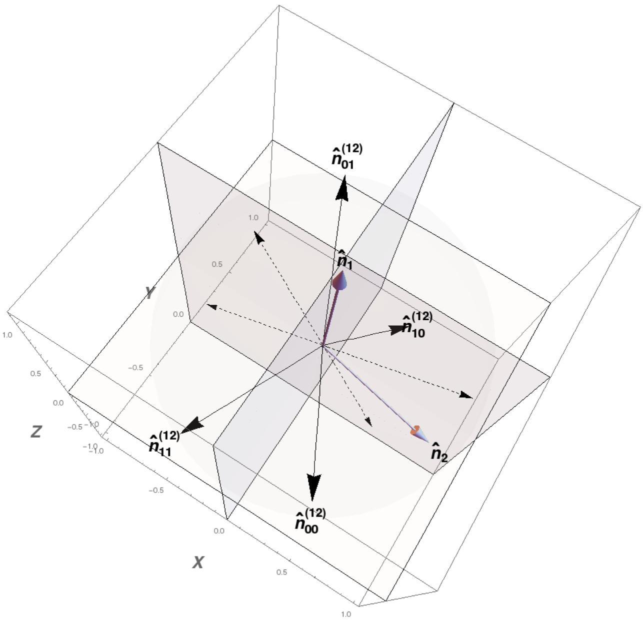
$$\vec{a}_{ij} \equiv (0, \sqrt{1 + \eta_0^4(\hat{n}_i.\hat{n}_j)^2 - 2\eta_0^2}, 0).$$

Choice of measurement/preparation directions

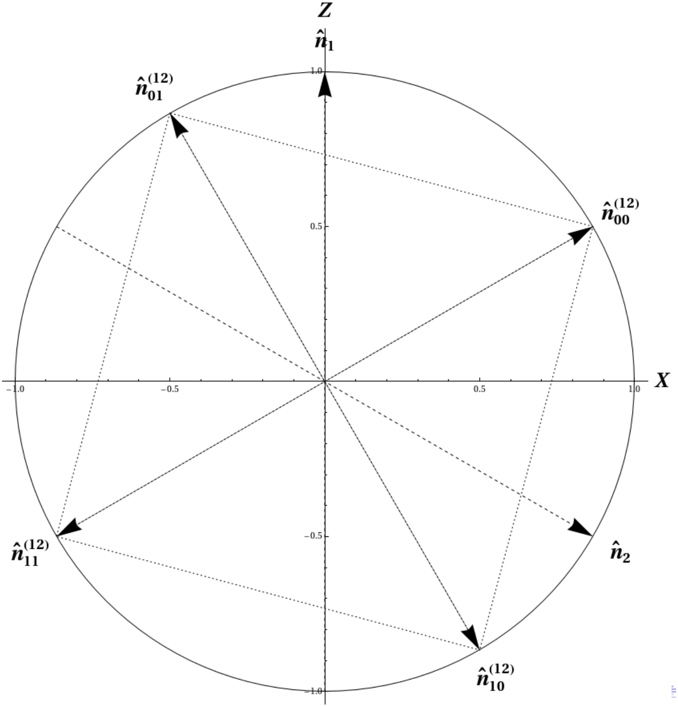


$$\hat{n}_1 \equiv (0,0,1), \hat{n}_2 \equiv (\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}), \hat{n}_3 \equiv (-\frac{\sqrt{3}}{2}, 0, -\frac{1}{2}), \hat{n}_* \equiv (0,1,0),$$

$$\eta_0 \in [0, \sqrt{3}-1] \text{ or } 0 \leq \eta_0 \leq 0.732$$



$\hat{n}_{01}^{(12)}$
$\hat{n}_1$
$\hat{n}_{10}^{(12)}$
$\hat{n}_{11}^{(12)}$
$\hat{n}_2$
$\hat{n}_{00}^{(12)}$
X
Y
Z
1.0
0.5
0.0
-0.5
-1.0



Z
$\hat{n}_1$
$\hat{n}_{01}^{(12)}$
$\hat{n}_{00}^{(12)}$
$\hat{n}_{11}^{(12)}$
$\hat{n}_2$
$\hat{n}_{10}^{(12)}$
X
1.0
0.5
−0.5
−1.0

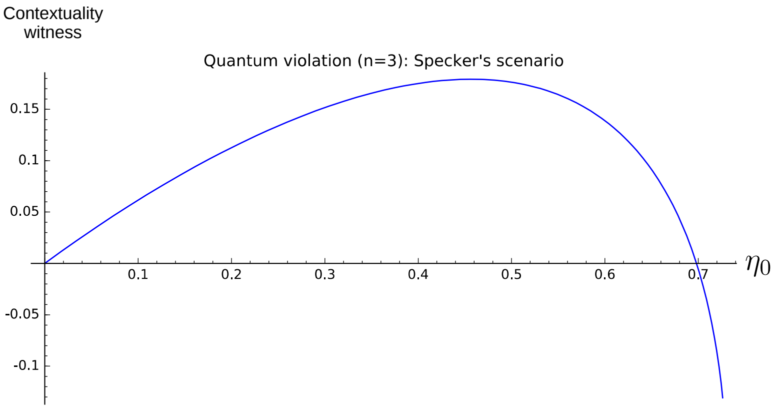
Quantum value:

$$\begin{aligned} & p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M'_*, P_*^{\perp}) \\ = \quad & 1 + \eta_0^2 \cos\frac{\pi}{3} + \sqrt{1 + \eta_0^4 \left(\cos\frac{\pi}{3}\right)^2 - 2\eta_0^2}. \end{aligned} \tag{14}$$

Noncontextuality bound:

$$2\left(1 - \frac{\eta_0}{3}\right) \tag{15}$$

The largest violation of the inequality for our choice of preparations and measurements occurs when $\eta_{\mathrm{ave}} = \eta_0 \approx 0.4566$ so that the violation is 0.1793: in this case the noncontextual bound on the anticorrelation is 1.6956 and the quantum value is 1.8749.


Contextuality witness
Quantum violation (n=3): Specker's scenario
0.15
0.1
0.05
-0.05
-0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
η_0

Generalization to arbitrary $n > 3$

- *Odd* $n \geq 3$: We compute

$$p(\text{anti}|M_*, P) \equiv \frac{1}{n} \sum_{i=1}^{n} p(X_i \neq X_j | M_{ij}, P) \qquad (16)$$

  where $j = i + 1 \mod n$ for a given i. We are also interested in $p(\text{anti}|M'_*, P)$.

- *Even* $n \geq 4$: We compute

$$p(\text{chained}|M_*, P) \equiv \frac{1}{n} \sum_{i=1}^{n-1} p(X_i = X_j | M_{ij}, P) + \frac{1}{n} p(X_n \neq X_1 | M_{n1}, P). \qquad (17)$$

  We are also interested in $p(\text{chained}|M'_*, P)$.

Operational equivalences

$$\begin{aligned} M_1^{(2)} &\simeq M_1^{(n)} \simeq M_1, \\ M_2^{(1)} &\simeq M_2^{(3)} \simeq M_2, \\ &\vdots \\ M_n^{(n-1)} &\simeq M_n^{(1)} \simeq M_n. \end{aligned} \tag{18}$$

$$\begin{aligned} M_1'^{(2)} &\simeq M_1'^{(n)} \simeq M_1, \\ M_2'^{(1)} &\simeq M_2'^{(3)} \simeq M_2, \\ &\vdots \\ M_n'^{(n-1)} &\simeq M_n'^{(1)} \simeq M_n. \end{aligned} \tag{19}$$

$$P_1^{(\mathrm{ave})} \simeq P_2^{(\mathrm{ave})} \simeq \cdots \simeq P_n^{(\mathrm{ave})} \simeq P_*^{(\mathrm{ave})}. \tag{20}$$

Noncontextuality inequalities

$$p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M'_*, P_*^{\perp}) \leq 2\left(1 - \frac{1}{n}\eta_{\mathrm{ave}}\right), \qquad (21)$$

for odd $n \geq 3$, and

$$p(\mathrm{chained}|M_*, P_*) + p(\mathrm{chained}|M'_*, P_*^{\perp}) \leq 2\left(1 - \frac{1}{n}\eta_{\mathrm{ave}}\right), \qquad (22)$$

for even $n \geq 4$, where

$$\eta_{\mathrm{ave}} \equiv \frac{1}{2n}\sum_{i=1}^{n}\left(\eta(M_i, P_i) + \eta(M_i, P_i^{\perp})\right). \qquad (23)$$

$$p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M_*, P_*^{\perp}) \leq 2\left(1 - \frac{1}{n}\eta_{\mathrm{ave}}\right), \tag{24}$$

$$\begin{aligned} &p(\mathrm{anti}|M_*, P_*) \\ &\leq \frac{n-1}{n} + 2\frac{(1-\eta_{\mathrm{ave}})}{n}, \end{aligned} \tag{25}$$

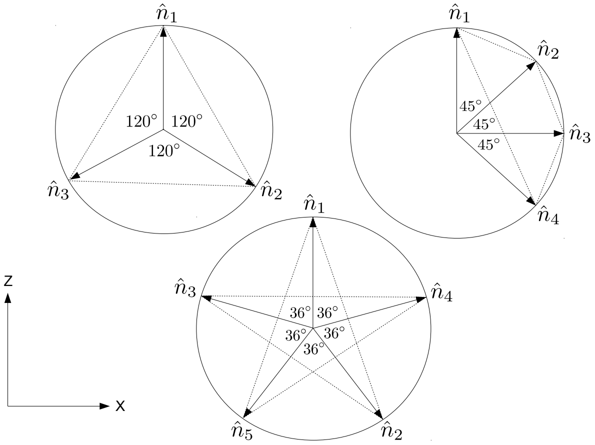
for odd n-cycle scenarios, and

$$p(\mathrm{chained}|M_*, P_*) + p(\mathrm{chained}|M_*, P_*^{\perp}) \leq 2\left(1 - \frac{1}{n}\eta_{\mathrm{ave}}\right), \tag{26}$$

$$\begin{aligned} &p(\mathrm{chained}|M_*, P_*) \\ &\leq \frac{n-1}{n} + 2\frac{(1-\eta_{\mathrm{ave}})}{n}, \end{aligned} \tag{27}$$

for even n-cycle scenarios.

Quantum violation



Odd n

- $\hat{n}_i.\hat{n}_j = \cos\frac{n-1}{n}\pi$, where $i \in \{1, \ldots, n\}$ and $j = (i+1)$ mod n.
- That is, our measurements are in an equatorial plane of the Bloch sphere, say the ZX plane, such that $\hat{n}_i$ and $\hat{n}_j$ are at an angle of $\frac{n-1}{n}\pi$ relative to each other: $\hat{n}_k \equiv \left(\sin\frac{(k-1)(n-1)}{n}\pi, 0, \cos\frac{(k-1)(n-1)}{n}\pi\right)$, for all $k \in \{1, 2, \ldots, n\}$, and, as before, $\hat{n}_* \equiv (0, 1, 0)$.
- Our construction of the pairwise joint measurements proceeds exactly as in the $n = 3$ case described earlier, the joint POVMs given by $M_{ij} = \{E^{(ij)}_{X_iX_j}\}$ and $M'_{ij} = \{E'^{(ij)}_{X_iX_j}\}$.

Even n

- $\hat{n}_i.\hat{n}_j = \cos\frac{\pi}{n}$, where $i \in \{1,\ldots,n-1\}$ and $j = i+1$, and $\hat{n}_n.\hat{n}_1 = \cos\frac{(n-1)\pi}{n}$: $\hat{n}_k \equiv (\sin\frac{(k-1)\pi}{n}, 0, \cos\frac{(k-1)\pi}{n})$ for all $k \in \{1,2,\ldots,n\}$. Also, $\hat{n}_* \equiv (0,1,0)$.
- The joint POVMs are given by $M_{ij} = \{F^{(ij)}_{X_iX_j}\}$ and $M'_{ij} = \{F'^{(ij)}_{X_iX_j}\}$, where $F^{(n1)}_{X_nX_1} = E^{(n1)}_{X_nX_1}$ and $F'^{(n1)}_{X_nX_1} = E'^{(n1)}_{X_nX_1}$, while for $i \in \{1,\ldots,n-1\}, j = i+1$:

$$F^{(ij)}_{X_iX_j} \equiv \frac{1}{2}\left(1 - (-1)^{X_i+X_j}\eta_0^2\hat{n}_i.\hat{n}_j\right)\Pi_{\hat{n}^{ij}_{X_iX_j}} \tag{28}$$

where

$$\Pi_{\hat{n}^{ij}_{X_iX_j}} \equiv \frac{1}{2}(I + \vec{\sigma}.\hat{n}^{ij}_{X_iX_j}),$$

$$\hat{n}^{ij}_{X_iX_j} \equiv \frac{\eta_0((-1)^{X_i}\hat{n}_i + (-1)^{X_j}\hat{n}_j) + (-1)^{X_i+X_j}\vec{a}_{ij}}{1 - (-1)^{X_i+X_j}\eta_0^2\hat{n}_i.\hat{n}_j},$$

$$F'^{(ij)}_{X_i X_j} \equiv \frac{1}{2}\left(1-(-1)^{X_i+X_j}\eta_0^2\hat{n}_i.\hat{n}_j\right)\Pi_{\hat{n}'^{ij}_{X_i X_j}} \tag{29}$$

where

$$\Pi_{\hat{n}'^{ij}_{X_i X_j}} \equiv \frac{1}{2}(I+\vec{\sigma}.\hat{n}'^{ij}_{X_i X_j}),$$

$$\hat{n}'^{ij}_{X_i X_j} \equiv \frac{\eta_0((-1)^{X_i}\hat{n}_i+(-1)^{X_j}\hat{n}_j)-(-1)^{X_i+X_j}\vec{a}_{ij}}{1-(-1)^{X_i+X_j}\eta_0^2\hat{n}_i.\hat{n}_j},$$

and

$$\vec{a}_{ij} \equiv (0,\sqrt{1+\eta_0^4(\hat{n}_i.\hat{n}_j)^2-2\eta_0^2},0).$$

The quantum value for both odd and even n takes the same form given our construction. For odd $n \geq 3$:

$$\begin{aligned} & p(\mathrm{anti}|M_*, P_*) + p(\mathrm{anti}|M'_*, P_*^{\perp}) \\ = \quad & 1 + \eta_0^2 \cos\frac{\pi}{n} + \sqrt{1 + \eta_0^4 \left(\cos\frac{\pi}{n}\right)^2 - 2\eta_0^2}. \end{aligned} \tag{30}$$

For even $n \geq 4$:

$$\begin{aligned} & p(\mathrm{chained}|M_*, P_*) + p(\mathrm{chained}|M'_*, P_*^{\perp}) \\ = \quad & 1 + \eta_0^2 \cos\frac{\pi}{n} + \sqrt{1 + \eta_0^4 \left(\cos\frac{\pi}{n}\right)^2 - 2\eta_0^2}. \end{aligned} \tag{31}$$

The noncontextuality bound is $2\left(1 - \frac{\eta_0}{n}\right)$. The quantum violation is therefore given by

$$Q_{viol} \equiv \sqrt{1 + \eta_0^4 \left(\cos\frac{\pi}{n}\right)^2 - 2\eta_0^2} + \eta_0^2 \cos\frac{\pi}{n} + 2\frac{\eta_0}{n} - 1 \tag{32}$$

n	Q_{viol}	Optimal η_0
3	0.1793	0.4566
4	0.1557	0.5029
5	0.1393	0.5412
6	0.1266	0.5727
7	0.1164	0.5990
8	0.1079	0.6213
9	0.1007	0.6403
10	0.0944	0.6569
11	0.0889	0.6715
12	0.0841	0.6822
13	0.0798	0.6960
14	0.0759	0.7064
99	0.0160	0.8881
100	0.0159	0.8887
199	0.0086	0.9211
200	0.0085	0.9213

Takeaway

- Noncontextuality inequalities can be derived for Specker's compatibility scenario without presuming outcome determinism or the validity of quantum theory. At least one of these even admits a quantum violation on a qubit.
- Unsharp measurements are fundamental to testing for noncontextuality in Specker's compatibility scenario.
- Our construction – of the noncontextuality inequalities and their quantum violation – for $n = 3$ can be generalized to arbitrary $n > 3$, both odd and even.
- Work-in-progress: It remains to show how all known examples of "state-dependent" KS-contextuality can be made robust in this operational approach. This will complement our work on noncontextuality inequalities from KS-uncolourability proofs. It also remains to tackle the case "state-independent" KS-contextuality in quantum theory that does not arise from KS-uncolourability (Yu-Oh 13 ray example).

Questions?[8]

[8] If the answers don't satisfy you, go see this guy.