Grover's search and higher-order interference

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Abstract

The following is an extended abstract of the paper [Lee, C.M., & Selby, J.H. Generalised phase kick-back: the structure of computational algorithms from physical principles, New J. Phys, Vol. 18, 2016 (arXiv preprint at arXiv:1510.04699)] together with subsequent work by the same authors.

1 Introduction

One of the major conceptual breakthroughs in physics over the past thirty years was the realisation that quantum theory offers dramatic advantages [12] for various information-processing tasks—computation in particular [12]. This raises the general question of how physical principles bound computational power. Moreover, what broad relationships exist between such principles and computation? A major roadblock to such an investigation is that quantum computation is phrased in the language of Hilbert spaces, which lacks direct physical or operational significance.

In contrast, the framework of operationally-defined theories [3, 4, 7, 10, 2] provides a clear-cut operational language in which to investigate this problem. Theories within this framework can differ [2] from classical and quantum theories. For example, classical probability theory, quantum theory, Spekken's toy model [9, 19], and the theory of PR boxes [15] can all be described in this framework. Whilst many of them may not correspond to descriptions of our physical world, they make good operational sense and allow one to assess how computational power depends on the physical principles underlying them in a systematic manner.

Previous investigations into computation within this framework have taken a high-level approach using the language of complexity classes to derive general bounds on the power of computation [10]. However, much of quantum computing is concerned not so much with this high-level view, but instead with the construction of concrete algorithms to solve specific problems. A deeper understanding of the general structure of computational algorithms in this framework has so far remained elusive. Here we take this low-level algorithmic view and ask which physical principles are required to allow for some of the common machinery of quantum computation in this context.

In our work we show that three physical principles, *causality* (which roughly states that information propagates from present to future), *purification* (roughly, that information is fundamentally conserved) and *strong symmetry* (all information carriers of the same size are equivalent)—which are necessary for a well defined notion of information—are sufficient for the existence of reversible controlled transformations and a generalised *phase kick-back mechanism*. In the quantum case, the phase kick-back mechanism plays a vital role in almost all algorithms—notably the Deutsch-Jozsa algorithm, Grover's search algorithm and Simon's algorithm—whilst reversible controlled transformations are central components of most well-studied universal gate sets and fundamental for the definition of computational oracles.

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One might ask how the computational power of theories with these crucial algorithmic components depends on their underlying physical properties. One such property—currently under both theoretical [1, 13, 8, 21] and experimental [16, 14] investigation—is the existence of *higher-order interference.*

Sorkin [17, 18] has introduced a hierarchy of mathematically conceivable *higher-order* interference behaviours, where classical theory is at the first level of the hierarchy and quantum theory belongs to the second. Informally, the order in the hierarchy corresponds to the number of paths that have an irreducible interaction in a multi-slit experiment. In the quantum case, this corresponds to the fact that interference patterns created in a three—or more—slit experiment can be written in terms of the two and one slit patterns obtained by blocking some of the slits; no genuinely new features result from considering three slits instead of two.

Quantum interference between computational paths has been posited [20] as a key resource behind the quantum "speed-up" over classical computation. However, as discussed above, there is a limit to this interference—at most pairs of paths can ever interact in a fundamental way. To get a greater understanding of the role of interference in computation, we consider how Grover's speed-up depends on the order of interference of a theory.

Grover's algorithm [6], which provides the optimal quantum solution to the search problem, is one of the most versatile and influential quantum algorithms. The search problem—in its simplest form—asks one to find a single "marked" item from an unstructured list of N elements by querying an oracle which recognises the marked item. The importance of Grover's algorithm stems from the ubiquitous nature of the search problem, an efficient solution to which would provide a method to efficiently solve **NP**-complete problems. Classical computers require $O(N)$ queries to solve this problem, but quantum computers—using Grover's algorithm—only require $O(\sqrt{N})$ queries.

Restriction to the second level of Sorkin's hierarchy implies many 'quantum-like' features, which, at first glance, appear to be unrelated to interference. For example, such interference behaviour restricts correlations [5] to the 'almost quantum correlations' discussed in [11], and bounds contextuality in a manner similar to quantum theory [8, 13]. Based on this, one might expect post-quantum interference to allow for a speed-up over quantum computation—similar to the quantum speed-up over classical computation.

Surprisingly, we show that this is not the case—at least from the point of view of the search problem. We consider theories satisfying two further physical principles, *purity preservation* (which, like purification, roughly says that information is fundamentally conserved) and *pure sharp effect* (information can be reliably encoded in physical media) and show that a theory at level h in Sorkin's hierarchy requires $\Omega(\sqrt{N/h})$ queries to solve the search problem. Thus, post-quantum interference does not imply a computational speed-up over quantum theory. Moreover, from the point of view of the search problem, all (finite) orders of interference are asymptotically equivalent.

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