

### Introduction, 1 sample t-test and t-interval

87 individuals were given a flu vaccination. After 28 days, blood samples were taken to assess the concentration of antibody ( $X$ ) in their serum.

Some summary statistics for the sample are as follows.

$$n = 87, \bar{X} = 1.689, s = 1.549.$$

Assume that the sample is from a normal population with mean  $\mu$  and variance  $\sigma^2$ .

### Confidence interval for the mean of a normal population

The form of the  $100(1 - \alpha)$  % confidence interval for  $\mu$  is

$$\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

where  $t_{\alpha/2, n-1}$  is the upper  $\alpha/2$  percentage point of the  $t$  distribution with  $n - 1$  degrees of freedom.

- To find a 99% CI,  $\alpha = .01$ , so  $\alpha/2 = .005$
- $n = 87$ , so there are 86 degrees of freedom
- $t_{.005, 86} \approx t_{.005, 75} = 2.643$   
(using 75 degrees of freedom, which is the df nearest 86 from t table in Deveaux et al). If you are using Devore, the closest df is 60, which gives  $t_{.005, 60} = 2.660$ .
- lower confidence limit =  $1.689 - 2.643(1.549)/\sqrt{(87)} = 1.25$ .
- upper confidence limit =  $1.689 + 2.643(1.549)/\sqrt{(87)} = 2.13$ .
- The 99% CI for  $\mu$  is (1.25, 2.13).

The **confidence coefficient** is the number  $1 - \alpha$  (or the percentage  $100(1 - \alpha)$ ).

**The statement that “the probability that  $\mu$  lies in (1.25, 2.13)” is .99 is INCORRECT**

The only **interpretation of the confidence coefficient** is that among the collection of all such intervals,  $100(1 - \alpha)$  % of them will contain the true but unknown mean  $\mu$ .

### Hypothesis test for the mean of a normal population

In general, we test the null hypothesis  $H_0 : \mu = \mu_0$ , where  $\mu_0$  is some specified value.

The test statistic used is a standardized form of  $\bar{X}$ ,

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

This has a t-distribution with  $n - 1$  degrees of freedom if the population sampled is normal with mean  $\mu_0$  and unknown variance.

Let  $t_{obs}$  be the observed value of  $t$ . The p-value calculations for the test of  $H_0 : \mu = \mu_0$  against the three possible alternative hypotheses  $H_A$  are given in the following table.

$H_A$	p-value
$\mu > \mu_0$	$P(t_{n-1} > t_{obs})$
$\mu < \mu_0$	$P(t_{n-1} < t_{obs})$
$\mu \neq \mu_0$	$2P(t_{n-1} >  t_{obs} )$

where  $t_{n-1}$  denotes a  $t$  random variable with  $n - 1$  degrees of freedom. The probabilities are approximated using the t-table.

eg. test  $H_0 : \mu_1 = 2$  against the alternative  $H_A : \mu_1 < 2$ . Find the p-value and report your conclusion when testing at level of significance  $\alpha = .05$ .

(Recall that the **level of significance** is the probability that the test will lead to a type 1 error - that the null hypothesis is incorrectly rejected).

$n = 87$ ,  $\bar{X} = 1.689$ ,  $s = 1.549$ , so

$$t_{obs} = (1.689 - 2)/(1.549/\sqrt{87}) = -1.87$$

The p-value is the probability that a  $t$  variable with 86 degrees of freedom is less than -1.87.

- Go to the t-table. Approximate using 75 degrees of freedom, which is the df closest to 86.
- The t-distribution is symmetric about 0. Therefore the probability that  $t < -1.87$  is the same as the probability that  $t > 1.87$ .
- From the table  $P(t_{75} > 1.665) = .05$  and  $P(t_{75} > 1.992) = .025$ . Therefore the p-value is between .025 and .05.
- If p-value  $< \alpha$ , reject the null hypothesis in favour of  $H_A$  at level of significance  $\alpha$ , otherwise do not reject  $H_0$ . Conclusion: reject  $H_0$  at level .05.

What is the p-value if we test  $H_0 : \mu_1 = 2$  against the two sided alternative  $H_A : \mu_1 \neq 2$ ?

$t_{obs} = -1.87$  as before.

The p-value for the two sided alternative is  $2P(t_{86} > |-1.87|) \approx 2P(t_{90} > 1.87)$ .

We just saw that  $.025 < P(t_{90} > 1.87) < .05$ , so  $.05 < p - value < .1$ . Do not reject  $H_0$  in favour of the two sided alternative at level .05.

Recall that in general, **the p-value is the probability of obtaining a sample presenting at least as much evidence against the null hypothesis as does the observed data, when in fact, the null hypothesis is true.**

Thus a small p-value is taken as evidence against the null hypothesis.

Here's a minitab computer output. The 87 observations are in column C9.

```
MTB > onet c9;
SUBC> confidence .99;
SUBC> test 2.
```

One-Sample T: C9

Test of mu = 2 vs not = 2

Variable	N	Mean	StDev	SE Mean	99% CI	T	P
C9	87	1.689	1.549	0.166	(1.252, 2.126)	-1.87	0.065

**Relationship of p-values to significance tests:** The null hypothesis is rejected in favour of the alternative at level  $\alpha$  if and only if the p-value is less than  $\alpha$ .

Another confidence interval example: Suppose we evaluate vitamin C levels (mg/100 gm) in 8 batches of corn soy blend (CSB) from a production run and get:

26 31 23 22 11 22 14 31

Find a 95% confidence interval for the mean vitamin C content of CSB produced during this run.

$$\bar{X} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

Summary statistics are:

$$n = 8$$

$$\bar{x} = \frac{\sum x_i}{n} = 22.50$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 51.714, \text{ so } s = 7.19$$

- $(1 - \alpha) = .95$ ,  $\alpha/2 = .025$ ,  $\nu = n - 1 = 7$  and  
 $t_{\alpha/2, \nu} = t_{.025, 7} = 2.365$  from the table.
- $U = 22.50 + 2.365\left(\frac{7.19}{\sqrt{8}}\right) = 22.50 + 6.012 = 28.5$
- $L = 22.50 - 2.365\left(\frac{7.19}{\sqrt{8}}\right) = 22.50 - 6.012 = 16.5$
- 95% Confidence Interval for  $\mu$ :  $(L, U) = (16.5, 28.5)$