TPS - 15'
Triangular matrices - 30'
diagonal matrices - 30'

TPS In some basis e_1, e_2, e_3, e_4, an operator T has the matrix

$$[T] = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Two of the basis vectors are eigenvectors. Which ones?

TRIANGULAR MATRICES (cont.)

Last time:

1) $[T]$ is upper triangular $\Leftrightarrow$ $Te_k \in \text{span}(e_1, \dots, e_k)$

2) The eigenvalues of T are the diagonal of $[T]$ if it's upper triangular.

PD

$$[T] = \begin{bmatrix} \lambda_1 & & * \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

Lemma $(T-\lambda_1 I)(T-\lambda_2 I)\cdots(T-\lambda_n I) = 0.$

PB $(T-\lambda_1 I)e_1 = 0$

$(T-\lambda_2 I)e_2 \in \mathrm{span}(e_1) \to (T-\lambda_1)(T-\lambda_2)e_2 = 0$

$\vdots$

$(T-\lambda_k I)e_k \in \mathrm{span}(e_1,\dots,e_{k-1}) \Rightarrow (T-\lambda_1)\cdots(T-\lambda_k)e_k = 0$

imply that $(T-\lambda_1)\cdots(T-\lambda_n)e_k = 0 \ \forall k.$

$\Rightarrow (T-\lambda_1)\cdots(T-\lambda_n)v = 0 \ \forall v.$

$\boxtimes$

PB Proof of thm

e_1 is an eigvalue $[Te_1 = \lambda_1 e_1]$

~~e_2 is not, but $Te_2 = a_{21}e_1 + \lambda_2 e_2$~~

~~so $T\left(e_2 - \frac{a_{21}}{\lambda_1}e_1\right) = (a_{21}e_1 + \lambda_2 e_2)$~~

e_2 is not, but $(T-\lambda_2 I)e_2 \in \mathrm{span}(e_1)$; so $T-\lambda_2 I$ isn't injective on $\mathrm{span}(e_1,e_2) \to (T-\lambda_2 I)v = 0$ for some $v \in \mathrm{span}(e_1,e_2) \to Tv = \lambda_2 v$. $\Rightarrow$

(and so on) $\boxtimes$

(cont.) it remain to show that all eigenvalues appear. This is because of the lemma, as the minimal pol. will have to divide

$$(T-\lambda_1 I) \cdots (T-\lambda_n I)=0.$$

[alternative: determinants].

Thm Every transformation is triangularizable.

Pf Book. Go over it; it's neat.

DIAGONAL MATRICES

A matrix is diagonalizable iff it has a basis of eigenvectors. It will fail to be diagonalizable if some eigenspaces "don't have enough eigenvalues". (e.g. $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$)

(write more later