The category theory you know

Daniel Teixeira

Supervisor: Theo Johnson-Freyd

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DALHOUSIE UNIVERSITY

This presenter is in debt with Ross Street for the presentation.
How?

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Categories List

How to use the list
Archives
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Conferences of interest
Seminar-related, and Local Sites
Theory and Applications of Categories - refereed electronic journal.

Using the list:

Submissions should be sent to
categories@mta.ca

Theory and Applications of Categories

TAC

Category theory at Dalhousie

Atlantic Category Theory Group
Halifax, Canada

Faculty:

- Geoff Cruttwell (Mount Allison)
- Robert Dawson (Saint Mary's)
- Darien DeWolf (St. Francis Xavier)
- Toby Kenney (Dalhousie)
- Theo Johnson-Freyd (Dalhousie)
- Mitja Mastnak (Saint Mary's)
- Bob Paré (Dalhousie)
- Dorette Pronk (Dalhousie)
- Bob Rosebrugh (Mount Allison)
- Julien Ross (Dalhousie)
- Peter Selinger (Dalhousie)
- Richard Wood (Dalhousie)

Postdoctoral Researchers:

- **Christopher Dean** (Dalhousie)
- **Jonathan Gallagher** (Dalhousie)
- Andre Kornell (Dalhousie)
- **Dongho Lee** (Dalhousie)

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"Our research interests focus on pure and applied category theory, including double categories, bicategories, enriched categories, higher dimensional category theory, adjunctions, homotopy theory, applications to quantum field theory, categorical lattice theory, complete distributivity, Hopf algebras, homological algebra, operator theory, categorical logic, the mathematical foundations of computer science, the semantics of programming languages, models of quantum computing, computational category theory, constructive mathematics, topos theory, and the categorical theory of database systems."

What is the idea of category theory?

"the essential categorical constraint is: knowing an object, not from inside via its elements, but from outside via its relations with its environment."

Borceux , Bourn

"You work at a particle accelerator. You want to understand some particle. All you can do are throw other particles at it and see what happens. If you understand how your mystery particle responds to all possible test particles at all possible test energies, then you know everything there is to know about your mystery particle."

the person at room 214

Why Dal??

William Lawvere

Why Dal??

William Lawvere

Dalhousie Gazette

Vol. 103 January 22, 1971 Number 13

Le Chateau Men's Wear Ltd. SCOTIA SQUARE 2179 GOTTINGEN ST. 5520 SPRING GARDEN RD. HALIFAX, N.S.

Classroom Clamp-down

Quebec purges

On Friday, January 8, the Quebec Teachers Corporation has criticized the provincial education department for its handling of an investigation into alleged political indoctrination in classrooms.

In a statement on Friday, January 8, the 70,000-member teachers' union called the issue a "leaking political football," and said: "An administration which is not even capable of dealing with a few complaints in an organization of 100,000 teachers without resorting to publicity tricks is obviously in an alarming situation."

The day before, Education minister Guy St. Pierre was quoted as saying 50 teachers would be brought before a special committee investigating the political activity of teachers in classrooms. The setting up of this committee

Math dept. moves to remove three

by Lloyd Buchinski

An internationally known researcher and professor in the field of mathematics, Dr. F. W. Lawvere, is being dismissed from Dalhousie "in a move which rings of McCarthyism"

Tingley denied that this was the reason for the decision of the Appointments Committee.

Since no real reasons, academic or otherwise, havw been given for Lawvere's dismissal, it seems to be based

vague. "The presence of Dr. Lawvere in the department causes stresses and strains to the detriment of its proper functioning." and "he has to some extent used a class for a political forum." The class

full article at tinyurl.com/mtnau3jp

Sets

Early in the 20th century, Math was expressed in terms of **sets**.

- the integers

$$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$$

- the line $\mathbb{R}$
- the Euclidean space $\mathbb{R}^n$
- the function $\cos : \mathbb{R} \to \mathbb{R}$ is the set

$$\{(x, \cos(x) \mid x \in \mathbb{R})\} \subseteq \mathbb{R}$$

- the torus

areas of Mathematics = sets with structure

areas of Mathematics = sets with structure

All of your classes fall in this framework:

- MATH 1000/1010 are about intervals of $\mathbb{R}$.
- MATH 2001/2002/2505/3501/3502: ehh
- MATH1030/2135 are about vector spaces.
- MATH3031/32 are about groups and rings.
- MATH3080 is about open sets of $\mathbb{C}$.
- MATH3045 is about surfaces in $\mathbb{R}^n$.

Telling things apart: invariants

In many cases there is a notion of equivalence.

A strategy to tell things apart is to define a number - an **invariant** - and show that equivalent things have the same number.

- Vector spaces are isomorphic if they have basis with the same number of elements $\rightsquigarrow$ dimension
- Topological spaces are not isomorphic if they have different Betti numbers, Euler characteristic...

Arrows

Between 1930 and 1940,* functions started being denoted as **arrows**

$$f : X \to Y,$$

meaning that for each $x \in X$ we assign an element $f(x) \in Y$.

In each course you would study functions that "preserve" the special structure of the sets.†
Two features: compositions and identities.

* tinyurl.com/3xkuyswj

† or is it the other way around?

Categories

A category consists of

- a collection $x, y, z, \ldots$ of *objects*
- for any each pair of objects, a set of $\mathcal{C}x, y$ of *morphisms* $f : x \to y$

and an associative composition rule with identities $1_x : x \to x$

category	**objects**	**morphisms**
Set	sets	functions
Calc1	intervals	differentiable functions
LinAlg	vector spaces	linear maps
Top	topological spaces	continuous functions

category	**objects**	**morphisms**
Set	sets	functions
Calc1	intervals	differentiable functions
LinAlg	vector spaces	linear maps
Top	topological spaces	continuous functions
$B\mathbb{Z}$	$*$	integers
Mat	integers	$n \times m$ matrices
$\mathbb{N}$	$0, 1, 2, \ldots$	just identities!
$\mathcal{O}(\mathbb{R})$	intervals	inclusion of intervals

Isomorphisms

A morphism $f : x \to y$ in a category $\mathcal{C}$ is **invertible** if there is a morphism $g : y \to x$ such that

$$g \circ f = 1_x \quad \text{and} \quad f \circ g = 1_y$$

So there is a *canonical* notion of isomorphic objects.

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So there is a *canonical* notion of isomorphic objects.
What is an invariant?

Functors

What is a morphism between categories?

Functors

What is a morphism between categories?

A **functor** $F : \mathcal{C} \to \mathcal{D}$ sends

- an object $c \in \mathcal{C}$ to an object $d \in \mathcal{D}$
- a morphism $f : c \to c'$ to a morphism $Ff : Fc \to Fc'$

such that

$$F(1_c) = 1_{Fc} \quad \text{and} \quad F(g \circ f) = Fg \circ Ff$$

- There is a functor $\dim : \mathbf{LinAlgMat} \to$.

- Can you make the derivative a a functor $d : \mathbf{Calc1} \to \mathbf{Calc1}$?

Proposition: functors preserve isomorphisms.

There is a homology functor

$$H_n : \mathbf{Top} \to \mathbf{LinAlg}.$$

The Betti number b_n of X is $\dim H_n(X)$.
This is also a functor

$$\mathbf{Top} \to \mathbf{LinAlg} \xrightarrow{\dim} \mathbb{N},$$

but is it the natural thing to do?

Look to the frame at your right.

Look to the frame at your right.

That lady is the person responsible for the shift
numbers $b_n \to$ *groups* H_n.

read more here https://hirzebruch.mpim-bonn.mpg.de/id/eprint/98/6/preprint_1997_34.pdf

But category theory hadn't been created at the time of Noether. When did that happen and why?

GENERAL THEORY OF NATURAL EQUIVALENCES

BY

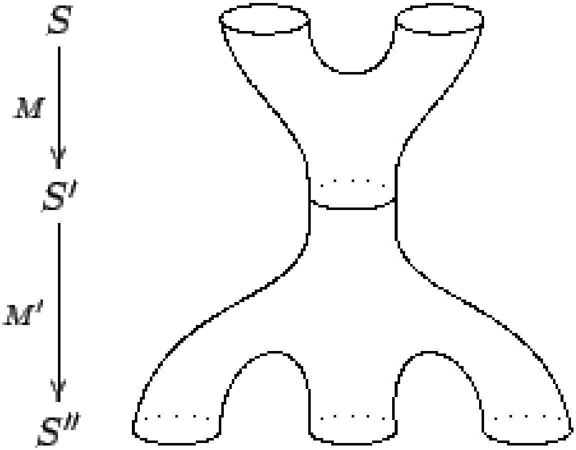
SAMUEL EILENBERG AND SAUNDERS MACLANE

CONTENTS

"What precisely is NATURALITY in Mathematics?
For that they needed FUNCTORS.
For that they defined CATEGORIES."

Ross Street

A little about research...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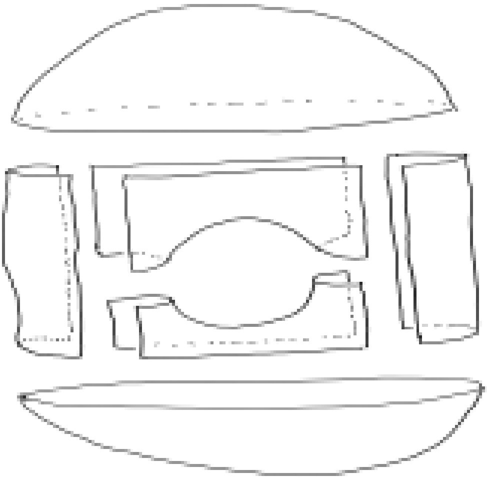

Thank you!