# Climbing the categorical ladder (of 2-categories)

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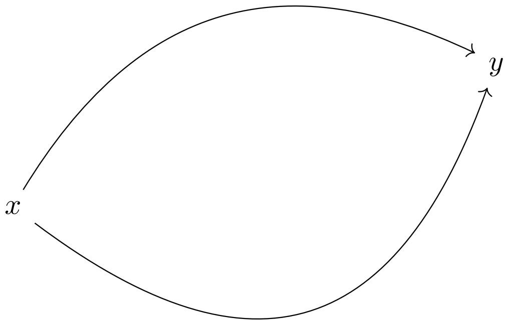
# Climbing the categorical ladder

# The idea of higher categories

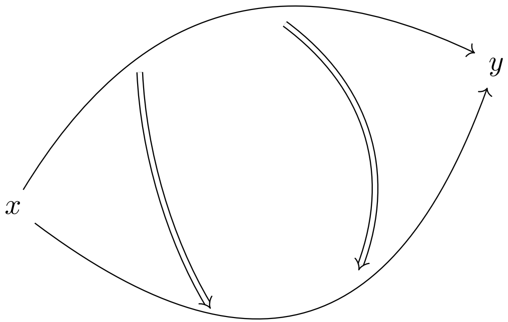
$y$

$x$

# The idea of higher categories



# The idea of higher categories



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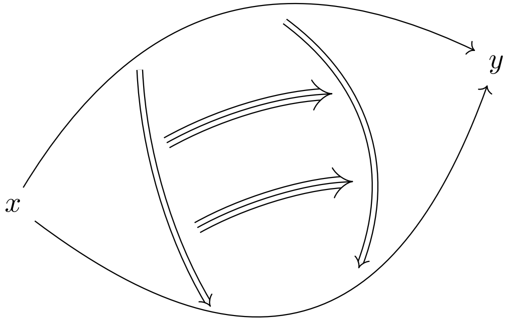

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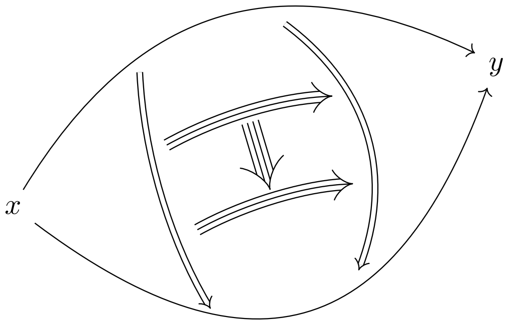

# Full dualizability I

**Definition**

An $n$-category $\mathcal{C}$ is *fully dualizable* if its morphisms have adjoints, whose structure 2-morphisms have adjoints, and so on.

Applications: topological field theory, condensed matter, homotopy theory...

# Full dualizability II

**Definition**

An $n$-category $\mathcal{C}$ is *fully dualizable* if each of its 2-truncations

$$h_2^0(\mathcal{C}), \quad h_2^1(\mathcal{C}), \quad h_2^2(\mathcal{C}), \quad \cdots, \quad h_2^{n-1}(\mathcal{C})$$

have all adjoints.

# Ladders

The sequence overlaps:

$$h_2^0(\mathcal{C}), \qquad h_2^1(\mathcal{C}), \qquad h_2^2(\mathcal{C}), \quad \cdots, \quad h_2^{n-1}(\mathcal{C})$$

The goal of this project is to study sequences of 2-categories satisfying this overlap condition:

$$\mathcal{B}_0, \qquad \mathcal{B}_1, \qquad \mathcal{B}_2, \quad \cdots$$

# Work in progress!!!

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Preliminary results:

- We constructed a category of ladders $\mathsf{Lad}_n$; and a functor $\mathcal{L} : \mathsf{Cat}_n \to \mathsf{Lad}_{n-1}$.
- We checked that certain results about $n$-categories ([Che07], [Lur09], [DSSP20]) can be proved using $\mathcal{L}$.

Open questions:

- Does $\mathcal{L}$ have an adjoint?
- What are the categorical properties of $\mathsf{Lad}_n$?
- Does it have the structure of a homotopy theory?
- What happens for $(\infty, n)$-categories?

# References

Che07 E. Cheng. *An $\omega$-category with all Duals is an $\omega$-groupoid.* Appl Categor Struct 15 (2007).

DSSP20 C.Douglas, C. Schommer-Pries and N. Snyder. *Dualizable Tensor Categories.* Memoirs of the American Mathematical Society 268 (2020).

Lur09 J. Lurie. *On the Classification of Topological Field Theories.* Preprint (2009).

# Thank you!