ACSC/STAT 3703, Actuarial Models I

WINTER 2024
Toby Kenney

Midterm Examination

Wednesday 14th February
11:35– 12:55

You should have recieved a formula sheet with the examination. **No other notes are permitted.**

1. A homeowner's house is valued at \$1,140,000 but is insured to a value of \$720,000. The insurer requires 80% coverage for full insurance. The home sustains \$9,000 of fire damage. The deductible is d, decreasing linearly to zero for losses above $2d$. The insurer reimburses \$2,700. What is d?

2. A home insurance company uses an expected loss ratio of 0.78. In accident year 2022, the earned premiums were \$34,400,000. For this accident year 2022, the insurance company made a total of \$9,140,000 in loss payments in 2022 and a total of \$11,630,000 in 2023. What loss reserves should the company hold for this accident year at the end of 2023, using the Expected Loss Ratio method?

3. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 5 years.

	Development year				
Accident year	0	1	2	3	4
2019	1819	2659	4526	4452	4927
2020	1939	3011	4256	5180	
2021	2843	4388	7043		
2022	2376	3461			
2023	3071				

The expected loss ratio is 0.78 and the earned premiums in each year are given in the following table:

Year	Earned Premiums (000's)
2019	7319
2020	7972
2021	10528
2022	8989
2023	11748

Using the mean for calculating loss development factors, esimate the total reserve needed for payments to be made in 2024 using the **chain-ladder method.** [Assume that all claims from accident year 2019 have been finalised.]

4. In 2023, an auto insurer collected \$14,800,000 in earned premiums, and paid \$11,250,000 in payments. There was a rate change on 1st May 2022. Before the rate change, the premium was \$920. After the rate change, the premium was \$980. If inflation is 5%, what should the new premium be to achieve an expense ratio of 25% in policy year 2025? [Assume policies are sold at a uniform rate.]

5. The random variable T has moment generating function $M_T(t) = \frac{0.3}{(1-4t)(1-5t)} + \frac{0.7}{(1-6t)^3}$. Calculate the skewness of $(1.03)^T$.

6. Which distribution has a heavier tail: a distribution with hazard rate $\lambda(x) = 1 - e^{-x}$ or a distribution with survival function $S(x) = e^{-x^3}$? [Use any reasonable method for comparing tail-weight.]

7. A measure of risk ρ satisfies all 4 coherence properties:

   - Subadditivity
   - Monotonicity
   - Positive homogeneity
   - Translation invariance

   If X follows an exponential distribution (gamma distribution with $\alpha = 1$) with mean $\theta = 4$ and Y follows a gamma distribution with $\alpha = 3$ and $\theta = 8$, then we have $\rho(X) = 4$ and $\rho(Y) = 23$. If Z follows a gamma distribution with $\alpha = 2$ and $\theta = 10$, which of the following is $\rho(Z)$?

   (i) $\rho(Z) = 15.6$
   (ii) $\rho(Z) = 17.9$
   (iii) $\rho(Z) = 19.3$
   (iv) $\rho(Z) = 22.1$

   Justify your answer.

   [Hint: If A follows a gamma distribution with parameters α and θ, then cA follows a gamma distribution with parameters α and $c\theta$. If A and B are independent and follow gamma distributions with parameters α_1 and θ and α_2 and θ respectively, then $A + B$ follows a gamma distribution with parameters $\alpha_1 + \alpha_2$ and θ.

   Assume X and Z are independent, and let W be an exponential distribution with mean 4, independent of X.]

8. A distribution has support $\mathbb{R}^+$ and survival function given by $S(x) = \frac{250}{(x+5)^3} - \frac{8}{(x+2)^3}$. The VaR of this distribution at the 0.99 level is 23.79973756. Calculate the TVaR of this distribution at the 0.99 level.