ACSC/STAT 3703, Actuarial Models I

WINTER 2024

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Homework Sheet 3

Due: Wednesday 7th February: 13:00

Note: This homework assignment is only valid for WINTER 2024. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution has hazard rate $\lambda(x) = \frac{3}{2+x} + \frac{9}{4+x}$ for $x \geqslant 0$. Calculate its survival function.

2. A continuous random variable has moment generating function given by $M(t) = \frac{1}{(1-2t)^2(1-\theta t)^4}$ for some parameter $\theta > 0$. What value of θ makes the coefficient of variation of the distribution equal to $\frac{1}{4}$?

3. Calculate the mean excess loss function for a distribution with survival function given by $S(x) = 2e^{-\frac{x}{4}} - e^{-\frac{x}{2}}$ for $x \geqslant 0$.

4. Calculate the probability generating function of a discrete distribution with p.m.f. given by

$$f(n) = \begin{cases} \frac{1+4e^{-4}+4e^{-8}}{9} & \text{if } n = 0 \\ \frac{4e^{-4}4^n+4e^{-8}8^n}{9(n!)} & \text{if } n > 0 \end{cases}$$

Standard Questions

5. The total cost of handling a claim is $X + Y$ where X is a discrete non-negative random variable with probability generating function $P_X(z) = \left(\frac{1+2e^{-4(1-z)}}{3}\right)^3$ and Y is a continuous non-negative random variable. X and Y are independant. The moment generating function of the total cost is

$$M_{X+Y}(t) = \frac{e^{2t}}{1-t}$$

. What is the moment generating function of Y?

6. An insurance company is trying to fit a log-logistic distribution to its claims data. The survival function for this distribution is given by

$$S(x) = \frac{\theta^\gamma}{x^\gamma + \theta^\gamma}$$

The insurance company wants to select γ and θ so that the the 5th percentile and the 95th percentile match the observed values of 826 and 43,395 respectively. What values should they choose for γ and θ to achieve this?