ACSC/STAT 3703, Actuarial Models I

WINTER 2024
Toby Kenney

Homework Sheet 4

Due: Monday 12th February: 13:00

Note: This homework assignment is only valid for WINTER 2024. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution has survival function

$$S(x) = e^{-\log(x)^{1.2}}$$

for $x \geqslant 0$. How does the tail weight of this distribution compare to that of a log-normal distribution with $\mu = 0$ and $\sigma^2 = 1$, when tail-weight is assessed by

(a) Asymptotic behaviour of hazard rate.

(b) Existence of moments.

2. Which coherence properties are satisfied by the following measure of risk?

$$\rho(X) = \frac{\mathbb{E}(X) + \sqrt[3]{\mathbb{E}(X^3)}}{2}$$

Give a proof or a counterexample for each property.

3. Calculate the TVaR at the 95% level of a distribution with survival function $S_X(x) = e^{\sqrt{3}-\sqrt{x+3}}$ for $x > 0$.

4. Which of the following density functions with parameters α, β and γ are scale distributions? Which have scale parameters?

(i) $f(x) = Ce^{-\frac{x}{\beta}-\frac{x^\alpha}{\gamma}}\left(\frac{x^{\alpha+2}}{\gamma\beta^2}\right)$

(ii) $f(x) = C\left(\frac{\beta^\alpha}{(\beta+x)^\alpha} + \frac{beta^\gamma}{\beta^\gamma+x^\gamma}\right)$

(iii) $f(x) = C(x+\alpha)^{-3}(x+\beta)^{-5}\left(x^2+\alpha\right)^{-2}$

[In each case C is a normalising constant that may depend on α, β and γ, but not on x.]

5. An insurance company observes the following sample of claims (in thousands):

```
0.8  1.7  2.6  3.6  5.5  7.1  11.4  20.6
```

They use a kernel density model with uniform kernel with bandwidth 2.

What is the TVaR at the 95% level of the fitted distribution?

Standard Questions

6. An inverse gamma distribution with α and $\theta = 1$ has mean $\frac{1}{\alpha-1}$ and variance $\frac{1}{(\alpha-1)^2(\alpha-2)}$. You can simulate n random variables following this inverse gamma distribution with the command

```
sim=1/gamma(n,shape=alpha)
```

[This is simulating a gamma distribution then taking the inverse.]

Based on the central limit theorem, if we take the average of a sample of n inverse gamma random variables, this should approximately follow a normal distribution with mean $\frac{1}{\alpha-1}$ and variance $\frac{1}{n(\alpha-1)^2(\alpha-2)}$. Plot the distribution of this sample average for $\alpha = 12$, $\alpha = 2.6$ and $\alpha = 2.1$, for sample sizes 500, 1000, and 5000, and compare it with the normal distribution.