ACSC/STAT 3703, Actuarial Models I

WINTER 2024

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Homework Sheet 5

Due: Wednesday 14th March: 13:00

Note: This homework assignment is only valid for WINTER 2024. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution of a random loss X in Canadian dollars has density function

$$f_X(x) = \begin{cases} \frac{Ce^{-\frac{x^3}{5}}}{(x+1)^3} & \text{if } x > 0 \\ \frac{4Ce^{\frac{x^3}{8}}}{(x-2)^2} & \text{if} x \leqslant 0 \end{cases}$$

   for some constant C. The loss is reinsured by a reinsurer in Europe, so needs to be converted to Euros using the conversion rate \$1 = €0.68304 What is the density function for the loss in Euros?

2. Calculate the density function of X^5 when X follows a gamma distribution with $\alpha = 4$ and $\theta = 10$.

3. An individual's expected loss depends on the number of risk factors they have (with partial risk factors being possible). Each risk factor increases the expected loss by a factor 1.07. The number of risk factors follows a continuous distribution with moment generating function $M_T(t) = \frac{1225}{(5-t)^2(7-t)^2}$ What is the variance of an individual's expected loss?

4. X is a mixture of 3 distributions:

   - With probability 0.2, X follows a Pareto distribution with $\alpha = 4.2$ and $\theta = 88$.
   - With probability 0.7, X follows a Weibull distribution with $\tau = 0.5$ and $\theta = 12$.
   - With probability 0.1, X follows a Gamma distribution with $\alpha = 2$ and $\theta = 5$.

   The moments of these distributions are given in the following table:

	Distribution 1	Distribution 2	Distribution 3
Probability	0.2	0.7	0.1
μ	27.5	24	10
μ_2	1443.75	2880	50
μ_3	344093.75	1022976	1250
μ_4	806866757.813	727584768	20000
μ'_2	2200	3456	75
μ'_3	484000	1244160	1500
μ'_4	851840000	836075520	75000

What is the skewness of X?

5. For a particular claim, an insurance company has observed the following claim sizes:

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1.9  2.6  2.9  3.3  4.7  6.9  11.8  20.2
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Using a kernel smoothing model with an exponential kernel (matching the mean of the kernel to the observed data), calculate the probability of a claim exceeding 25.

Standard Questions

6. An insurance company models the claims of an individual (in dollars) as following an inverse Pareto distribution with $\theta = 100$ and τ varying between individuals. For a random individual, τ is assumed to follow a gamma distribution with shape parameter $\alpha = 0.5$ and scale parameter θ.

   From the insurer's data, 1.2% of claims exceed \$1,000. What percentage of claims exceed \$100,000?

7. The time until failure of a product has hazard rate $\lambda(t) = \frac{a}{\sqrt{t}} + 0.1\sqrt{t}$ where a is the probability that the product is defective, and is modelled as following a distribution with moment generating function $M_A(t) = e^{e^{\frac{\sqrt{t}}{20}}-1}$. Given that a product has lasted for one year waranty, what is the probability that it will be last another year?