ACSC/STAT 3703, Actuarial Models I

WINTER 2024
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Homework Sheet 6

Due: Wednesday 21st March: 13:00

Note: This homework assignment is only valid for WINTER 2024. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. Let X follow a negative binomial distribution with $r = 4.1$ and $\beta = 1.3$. What is the probability that $X = 4$?

2. The number of claims on each insurance policy over a given time period is observed as follows:

Number of claims	Number of policies
0	492
1	374
2	251
3	148
4	74
5 or more	92

   Which distribution(s) from the $(a, b, 0)$-class and $(a, b, 1)$-class appear most appropriate for modelling this data?

3. X follows an extended modified negative binomial distribution with $r = -0.7$ and $\beta = 2.3$, and $p_0 = 0.7$. What is $P(X = 3)$?

4. Let X follow a mixed zero-modified Poisson distribution with $\lambda = 3.7$, where p_0 follows a beta distribution with $\alpha = 3$ and $\beta = 8$. What is the probability that $X = 2$?

Standard Questions

5. We can find the mean of a distribution from the $(a, b, 1)$-class as follows:

$$\mathbb{E}(X) = \sum_{n=1}^{\infty} np_n = p_1 + \sum_{n=2}^{\infty} n\left(a + \frac{b}{n}\right)p_{n-1} = p_1 + \sum_{m=2}^{\infty}(a(m+1)+b)p_m = p_1 + a\mathbb{E}(X) + (a+b)(1-p_0)$$

Thus $\mathbb{E}(X) = \frac{p_1+(a+b)(1-p_0)}{1-a}$ Use similar methods to find the raw second moment, and hence determine the value of p_0 that maximises variance for a general distribution from the $(a, b, 1)$-class.

[Hint: for a zero truncated distribution from the $(a, b, 1)$-class, the probability of 1 is $p_1^T = \frac{a+b}{(1-a)^{-1-\frac{b}{a}}-1}$.]

6. A random variable X is assumed to have distribution in the $(a, b, 1)$-class. An insurance company collects the following sample of truncated X values:

Value	Frequency
$X = 5$	1256
$X = 6$	875
$X = 7$	590

Assuming that the probabilites are proportional to these values, If the company observed all values from the sample, how many values would they expect to observe with $X = 1$?