# ACSC/STAT 3703, Actuarial Models I

## WINTER 2023
## Toby Kenney

### Practice Final Examination

This Sample examination has more questions than the actual final, in order to cover a wider range of questions. Estimated times are provided after each question to help your preparation.

1. A home insurance company uses an expected loss ratio of 0.77. In accident year 2020, the earned premiums were $3,520,000. In 2020, the insurance company made a total of $1,158,300 in loss payments for accident year 2020, a total of $752,500 in 2021. At the end of 2022, the company sets the reserves for accident year 2020 to $605,300. How much did the company pay in 2022 for losses in accident year 2020?

   [5 mins]

2. The following table shows the cumulative losses (in thousands) on claims from one line of business of an insurance company over the past 4 years.

   |  | Development year | | | |
   | Accident year | 0 | 1 | 2 | 3 |
   |---|---|---|---|---|
   | 2019 | 767 | 954 | 1388 | 1578 |
   | 2020 | 1007 | 1388 | 1757 | |
   | 2021 | 882 | 1082 | | |
   | 2022 | 1151 | | | |

   Using the mean for calculating loss development factors, esimate the total reserve needed for payments to be made in 2024 using.

   (a) The loss development triangle method [15 mins]

   (b) The Bornhuetter-Fergusson method. The expected loss ratio is 0.82 and the earned premiums in each year are given in the following table:

   | Year | Earned Premiums (000's) |
   |---|---|
   | 2019 | 1940 |
   | 2020 | 2594 |
   | 2021 | 2104 |
   | 2022 | 3194 |

   [15 mins]

3. An insurance company collected a total of $1,600,000 in premiums in 2022, and collects a total of $2,200,000 in premiums in 2023. Assume the premium was constant throughout 2022 and 2023, and new policies were sold at constant rates in 2022 and in 2023 (but the rates in 2022 and 2023 are different). The estimated incurred losses for accident year 2023 are $1,784,000. An actuary is using this data to estimate rates for premium year 2026. Claims are subject to 4% inflation per year. By what percentage should premiums increase from 2023 in order to achieve a loss ratio of 0.80? [15 mins]

4. In 2022, a home insurer collected $31,220,000 in earned premiums, and paid $24,030,000 in payments. There was a rate change on 1st September 2021. Before the rate change, the premium was $940. After the rate change, the premium was $980. Ignoring inflation, what should the new premium be to achieve an expense ratio of 20%? [15 mins]

5. An insurance company models claim sizes as following a mixture of two distributions. With probability 0.3, claims follow a Weibull distribution with $\tau = 2$ and $\theta = 100$. With probability 0.7, claims follow a Pareto distribution with $\alpha = 3$ and $\theta = 250$.

   (a) Which of the following is the VaR of the distribution at the 90% level? [10 mins]

   (i) 122.02

   (ii) 146.35

   (iii) 197.14

   (iv) 230.60

   (b) Calculate the TVaR at the 90% level. [5 mins]

6. An insurance company assigns a risk factor $\Theta$ to each individual. These $\Theta$ follow a gamma distribution with $\alpha = 2$ and $\theta = 400$. For an individual with risk factor $\Theta = \theta$, the size of a claim follows an inverse gamma distribution with $\alpha = 3$ and this value of $\theta$. What is the probability that a random individual makes a claim in excess of $3,000? [15 mins]

7. The number of policies sold in a year follows a binomial distribution with $n = 100000$ and $p = 0.002$. The number of claims resulting from each policy sold follows a Poisson distribution with $\lambda = 0.02$. Calculate the variance of the total number of claims in a year. [10 mins]

8. A random variable $X$ follows an extended truncated negative binomial distribution with $r = -0.4$ and $\beta = 0.2$. What is $P(X = 5)$? [5 mins]

9. An insurer collects the following sample of claim frequencies.

| $n$ | Frequency |
|---|---|
| 0 | 34 |
| 1 | 142 |
| 2 | 110 |
| 3 | 69 |
| 4 | 25 |
| 5 | 5 |
| 6 | 2 |
| 7 | 1 |

They make the following plot:
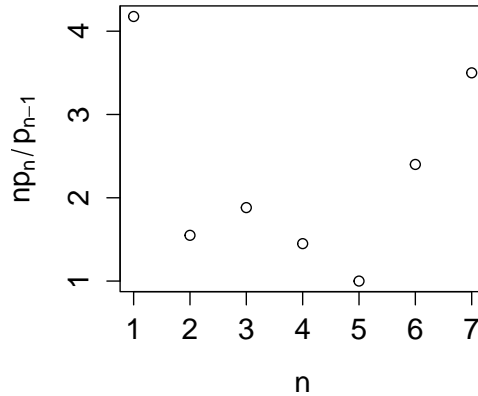
Which distribution(s) from the $(a, b, 0)$ or $(a, b, 1)$ classes might be suitable for modelling this data? Justify your answer. [5 mins]

10. The random variable $X$ follows a distribution from the $(a, b, 1)$ class. We have $P(X = 3|X \geqslant 3) = \frac{2}{5}$, $P(X = 4|X \geqslant 4) = \frac{5}{12}$ and $P(X = 5|X \geqslant 5) = \frac{3}{7}$. What is $P(X = 6|X \geqslant 6)$? [10 mins]

11. The discrete random variable $X$ satisfies the recurrence relation $P(X = n) = \left(a + \frac{b}{n+1}\right) P(X = n - 1)$ for all $n \geqslant 1$. What is the probability generating function of $X$? [10 mins]

12. An insurance company models loss size as following a Pareto distribution with $\alpha = 4$ and $\theta = 6000$. The company introduces a deductible of \$1,000. Calculate the expected payment per claim after the deductible is introduced. [10 mins]

13. An insurance company models loss size as following a Weibull distribution with $\tau = 2$ and $\theta = 2000$. The company wants to introduce a deductible so that the expected payment per loss is \$1400. What deductible should it introduce? [15 mins]

14. An insurance company models loss size as following a log-logistic distribution distribution with $\gamma = 2$ and $\theta = 2000$. The company wants to introduce a deductible with loss elimination ratio 30%.

(a) What deductible should it introduce? [15 mins]

(b) In the following years, there is uniform inflation of 4% every year. How many years does it take until the deductible calculated in (a) gives a loss elimination ratio of less than 25%? [15 mins]

15. Losses follow a generalised Pareto distribution with $\alpha = 2$, $\tau = 3$, and $\theta = 3000$. An insurance company introduces a deductible of \$600. Calculate the loss elimination ratio of this deductible after inflation of 12%. [15 mins]

16. Losses follow an inverse Pareto distribution with $\tau = 4$ and $\theta = 6000$.

(a) Calculate the expected payment per claim with a policy limit of \$1,000,000. [15 mins]

(b) Calculate the expected payment per claim if there is 15% inflation (the policy limit remains at \$1,000,000.) [10 mins]

17. Losses follow an exponential distribution with $\theta = 7000$. There is a deductible of $700, a policy limit of $25,000 and coinsurance such that the insurance pays 80% of the claim after the policy limit and deductible have been applied. Calculate the expected payment per claim and the variance of the payment per claim. [15 mins]

18. Losses follow a Pareto distribution with $\alpha = 3$ and $\theta = 5000$. There is a deductible of $1000. The insurance company wants to reduce the TVaR (per claim) for this policy at the 99.9% level to $60,000. What policy limit should they set? [15 mins]

19. Aggregate payments have a compund distribution. The frequency distribution is negative binomial with $r = 4$ and $\beta = 12$. The severity distribution is a Gamma distribution with $\alpha = 8$ and $\theta = 3000$. Use a normal approximation to aggregate payments to estimate the probability that aggregate payments are more than $2,000,000. [15 mins]

20. Claim severity is modelled as following a Pareto distribution with $\alpha = 4.4$ and $\theta = 3,200$. A reinsurer offers stop-loss reinsurance with attachment point $800,000 for a loading of 35%. Aggregate losses are modelled following a Pareto distribution with parameters fitted by matching moments. The resulting Pareto distribution has $\alpha = 4.2$. The reinsurance premium is $18,000.

    (a) What is the parameter $\theta$ for the Pareto distribution used to model aggregate losses? [15 mins]

    (i) 483029
    (ii) 614614
    (iii) 703420
    (iv) 783254

    (b) What are the mean and variance of claim frequency? [15 mins]

21. An insurance company has the following portfolio of auto insurance policies:

| Type of driver | Number | Probability claim | mean of claim | standard deviation |
|---|---|---|---|---|
| Good driver | 60 | 0.02 | $2,500 | $14,000 |
| Average driver | 140 | 0.06 | $3,800 | $19,200 |
| Bad driver | 50 | 0.13 | $7,000 | $22,600 |

    Calculate the cost of reinsuring losses above $500,000, if the loading on the reinsurance premium is one standard deviation above the expected claim payment on the reinsurance policy, using a Pareto approximation for the aggregate losses on this portfolio. [15 mins]

22. An insurance company sells home insurance. It estimates that the standard deviation of the aggregate annual claim is $5,326 and the mean is $1,804.

    (a) How many years history are needed for an individual or group to be assigned full credibility? (Use $r = 0.05$, $p = 0.95$.) [5 mins.]

    (b) What is the Credibility premium, using limited fluctuation credibility, for an individual who has claimed a total of $42,381 in the past 19 years? [5 mins.]

23. For a car insurance policy, the book premium for claim severity is $2,300. An individual has made 7 claims in the past 12 years, with average claim severity $1,074. Calculate the credibility estimate for claim severity for this individual using limited fluctuation credibility, if the standard for full credibility is:

    (a) 157 claims. [5 mins.]

    (b) 284 years. [5 mins.]