ACSC/STAT 3703, Actuarial Models I

WINTER 2025
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Homework Sheet 3

Due: Thursday 6th February: 14:30

**Note: This homework assignment is only valid for WINTER 2025. If you find this homework in a different term, please contact me to find the correct homework sheet.**

## Basic Questions

1. A distribution has hazard rate $\lambda(x) = x + \frac{3}{6+x}$ for $x \geqslant 0$. Calculate its density function.

2. A continuous random variable has moment generating function given by $M(t) = \frac{1}{(1-t)^2} + \frac{1}{(1-t^2)^4}$. What is the skewness of the distribution?

3. Calculate the mean excess loss function for a distribution with hazard rate given by $\lambda(x) = \frac{x}{x+1}$ for $x \geqslant 0$.

4. Calculate the probability generating function of a discrete distribution with p.m.f. given by
$$f(n) = \frac{e^{-1}}{2}\frac{n^2}{n!}$$

## Standard Questions

5. The total cost of handling a claim is $X_1 + X_2 + Y$ where $X_1$ and $X_2$ are i.i.d. discrete non-negative random variables with probability generating function $P_X(z) = \frac{z+1}{z+3}$ and $Y$ is a discrete non-negative random variable with probability generating function $P_Y(z) = \frac{3^z}{4(z+1)}$, independent of $X_1$ and $X_2$. What is the moment generating function of $X_1 + X_2 + Y$?

6. An insurance company is considering two models for its data. The first is a Pareto distribution with survival function
$$S(x) = \left(\frac{\theta}{\theta + x}\right)^\alpha$$
The second is a Weibull distribution with survival function
$$S(x) = e^{-\left(\frac{x}{\theta}\right)^\tau}$$

They find that for the fitted parameters, both distributions have the same values of $\theta$, and the same values for the 90th and 95th percentiles. Which distribution has a higher 99th percentile?

[You should get an equation for one of the unknown parameters $\alpha$ or $\tau$. You can numerically solve this equation by trying a range of values and seeing which satisfies the equation.]

# Bonus Questions

7. $X$ and $Y$ are continuous random variables with moment generating functions $M_X(t) = \frac{864}{(t-4)(t-6)^3}$ and $M_Y(t) = \frac{e^{-t^2}}{t+1}$. You are given that $X$ and $X+Y$ are independent. What is the probability generating function of $X+Y$?