ACSC/STAT 3703, Actuarial Models I

WINTER 2025
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Homework Sheet 4

Due: Tuesday 11th February: 14:30

Note: This homework assignment is only valid for WINTER 2025. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. A distribution has survival function

$$S(x) = e^{-e^x}$$

for $x \geqslant 0$. How does the tail weight of this distribution compare to that of a normal distribution with $\mu = 0$ and $\sigma^2 = 1$, when tail-weight is assessed by

(a) Asymptotic behaviour of hazard rate.

(b) Existence of moments.

2. Which coherence properties are satisfied by the following measure of risk?

$$\rho(X) = \sqrt{\mathbb{E}(X^2|X > \pi_{0.95}(X))}$$

Give a proof or a counterexample for each property.

[we can alternatively express this as $\rho(X) = \sqrt{\mathrm{TVaR}_{0.95}((X_+)^2)}$.]

3. Calculate the TVaR at the 99% level of a distribution with survival function $S_X(x) = e^{\sqrt{x+1}-1-x}$ for $x > 0$.

4. Which of the following distribution functions with parameters α, and β are scale distributions? Which have scale parameters?

(i) $F(x) = e^{-\beta e^{-x+\alpha}}$

(ii) $F(x) = \frac{\frac{x}{\beta}+e^{\frac{x}{\alpha}}-1}{e^{\frac{x}{\alpha}}}$

(iii) $F(x) = 1 + \frac{\beta}{\alpha} - \frac{\beta}{x+\alpha} + e^{-\frac{\beta}{x+\beta}}$

5. An insurance company observes the following sample of claims (in thousands):

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0.3 0.4 1.0 1.3 1.6 2.6 7.2 10.3
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They use a kernel density model with Gaussian kernel with standard deviation 1. What is the variance of the fitted distribution?

Standard Questions

6. An generalised Pareto distribution with $\alpha = \tau$ and $\theta = 1$ has mean $\frac{\alpha}{\alpha-1}$ and variance $\frac{\alpha(2\alpha-1)}{(\alpha-1)^2(\alpha-2)}$. You can simulate n random variables following this generalised Pareto distribution with the command

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sim=1/rbeta(n,shape1=alpha,shape2=alpha)
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[This is simulating a beta distribution then taking the inverse.]

Based on the central limit theorem, if we take the average of a sample of n generalised Pareto random variables, this should approximately follow a normal distribution with mean $\frac{\alpha}{\alpha-1}$ and variance $\frac{\alpha(2\alpha-1)}{n(\alpha-1)^2(\alpha-2)}$. Plot the distribution of this sample average for $\alpha = 10$, $\alpha = 2.5$ and $\alpha = 2.05$, for sample sizes 500, 1000, and 5000, and compare it with the normal distribution. What happens if we run the simulation with $\alpha = 1.5$?