ACSC/STAT 3703, Actuarial Models I

WINTER 2025
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Homework Sheet 5

Due: Thursday 13th March: 14:30

Note: This homework assignment is only valid for WINTER 2025. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. The time T to claim settlement, in years, has density function

$$f_T(t) = \begin{cases} \frac{Ce^{-\frac{t^2}{2}}}{(t+1)^2} & \text{if } 0 < t \leqslant 1 \\ \frac{Ce^{-e^{t-1}-\frac{1}{2}}}{4t^3} & \text{if} t > 1 \end{cases}$$

for some constant C. What is the density function for the time in days until claim settlement? [Assume all years are 365 days long.]

2. Calculate the density function of X^{-2} when X follows a Pareto distribution with $\alpha = 2$ and $\theta = 5$.

3. The time in years until a claim is processed is a random variable T with moment generating function $M_T(t) = te^t$. Inflation is at an annual rate of 4%. What is the skewness of inflation during the processing period of a random claim?

4. X is a mixture of 2 distributions:

   - With probability p, X follows a Pareto distribution with $\alpha = 2.2$ and $\theta = 48$.
   - With probability $1-p$, X follows a Weibull distribution with $\tau = 0.5$ and $\theta = 36$.

   The variance of X is 23216. What is the probability that X is more than 400?

5. For a particular claim, an insurance company has observed the following claim sizes:

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1.1 1.5 1.8 2.2 2.9 4.3 7.0 11.4
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They use a kernel smoothing model with a uniform kernel. They choose the bandwidth so that the variance of the kernel smoothing model is 17. What is the probability for this model that a random claim will exceed 7.3?

Standard Questions

6. An insurance company models the claims of an individual (in dollars) as following a Pareto distribution with $\theta = 100$ and α varying between individuals. For a random individual, $\alpha - 1$ is assumed to follow a gamma distribution with shape parameter $\alpha = 2$ and scale parameter θ. The expected claim amount is \$3,600. What is the probability that a claim exceeds \$100,000?

7. An insurance company models claims X as following the exponential of a distribution with moment generating function $M(t) = (t+1)e^{3t}$. They want to transform the distribution by raising to a power. To what power should they raise the distribution in order for the skewness to exist and be equal to 5?

   (i) 4.3252

   (ii) 6.9924

   (iii) 8.5905

   (iv) 11.9331

   Justify your answer.