ACSC/STAT 3703, Actuarial Models I

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Homework Sheet 1

Model Solutions

Basic Questions

1. *A customer has utility function $u(x) = 1 - e^{-\frac{x}{10000}}$. The customer's current wealth is \$15,000. The customer's home has the following risk of sustaining fire damage.*

Probability	*Damage*
0.93	*0*
0.05	*\$2,000*
0.02	*\$9,000*

How much would the customer be willing to pay for insurance that would cover this damage?

Without insurance, the customer has wealth \$15,000 minus the damage to the home. This gives the following:

Probability	Damage	Wealth	Utility	Expected Utility
0.93	0	\$15,000	$1 - e^{-1.5} = 0.776869839852$	0.722488951062
0.05	\$2,000	\$13,000	$1 - e^{-1.3} = 0.727468206966$	0.0363734103483
0.02	\$9,000	\$6,000	$1 - e^{-0.6} = 0.451188363906$	0.00902376727812

Thus, the customer's total expected utility is $0.722488951062+0.0363734103483+0.00902376727812 = 0.767886128688$. This is the utility of having wealth $-10000\log(1 - 0.767886128688) = 14605.2720311$ so the customer would be willing to pay $15000 - 14605.2720311 = \$394.73$.

2. *Which of the following risks are insurable? For risks which are not insurable, explain why they are not insurable. If there is not enough information to judge, explain what the insurability depends on.*

   (i) *The risk that a computer will be obsolete in 20 years.*

   (ii) *The risk that climate change will reduce the value of a house.*

   (iii) *The risk that a spaceship travelling to a distant planet in the year 2300 will be destroyed by alien attackers.*

   (iv) *The risk that a sports team will lose an important match.*

   (v) *The risk that a bottle of milk will go bad.*

(vi) The risk that an individual will be unsuccessful in a job interview.

(i) This is not insurable as it is not random.

(ii) This is not insurable. It is probably not estimable, and losses from different policyholders are certainly not independent.

(iii) This depends on the nature of space exploration in 2300. The loss may not be estimable if space exploration is not so common. We are assuming that other spaceships buying similar insurance would be travelling to different places, and so would be independent. If there were many expeditions to the same area, the risks would not be independent.

(iv) This is probably not insurable. It is not homogeneous, as different teams have different chances of losing a game. It may be possible to adjust for this. However, the loss is also probably not well-defined.

(v) This is not insurable as it is not economically feasible.

(vi) This is not insurable as the risks are not homogeneous.

3. *A homeowner's house is insured at $330,000. The insurer requires 70% coverage for full insurance. The home sustains $13,500 damage from fire. The policy has a deductible of $10,000, which decreases linearly to zero when the total cost of the loss is $20,000. The house is valued at $690,000. How much does the insurer reimburse?*

   The deductible for a loss of $13,500 is $10000\frac{20000-13500}{10000} = \$6,500$. Thus if the home were fully insured, the insurer would pay $13500-6500 = \$7,000$. The home has $\frac{330000}{690000\times 0.7} = 68.3229813665\%$ coverage. This means that the insurer reimburses $7000 \times 0.683229813665 = \$4782.61$.

4. *A worker's compensation insurance policy has a deductible of $2,000, a policy limit of $5,000,000 and co-insurance such that the injured party pays 10% of the remaining claim. How much does the insurer pay if the loss is:*

   (i) $1,400

   (ii) $2,600

   (iii) $5,142,000

   (iv) $7,330,000

   (i) This is less than the deductible, so the insurer pays $0.

   (ii) $0.9(2600 - 2000) = \$540$.

   (iii) $0.9 \times 5140000 = \$4,626,000$.

   (iv) $0.9 \times 7328000 = 6595200$. As this exceeds the policy limit, the insurance pays the policy limit of $5,000,000.

Standard Questions

5. *An insurer charges a loading of 25% on its policies with limit \$1,000,000, and a loading of 28% on its policies with limit \$1,500,000. A reinsurer offers stop-loss reinsurance of \$1,000,000 over \$1,000,000 for a loading of 35%, or stop-loss reinsurance of \$500,000 over \$1,500,000 for a loading of 45%. The second reinsurance policy costs exactly half as much as the first. Both of these reinsurance policies would result in the same premium for a policy with limit \$2,000,000. What would the overall loading of this policy be?*

   Let x be the expected losses limited to \$1,000,000, let y be the expected losses limited to \$1,500,000, and let z be the expected losses limited to \$2,000,000. The insurer's premiums are $1.25x$ for a policy with limit \$1,000,000 and $1.28y$ for a policy with limit \$1,500,000. The reinsurer's premiums are $1.35(z-x)$ for the first policy and $1.45(z-y)$ for the second policy. The resulting premiums for a policy with limit \$2,000,000 are

   $$1.25x + 1.35(z-x) = 1.35z - 0.1x$$

   and

   $$1.28y + 1.45(z-y) = 1.45z - 0.17y$$

   Setting these equal gives

   $$1.35z - 0.1x = 1.45z - 0.17y$$

   We are also given that the second reinsurance policy costs exactly one half as much as the first, meaning

   $$1.35(z-x) = 2.9(z-y)$$

   Substituting this into the previous equation gives

   $$\begin{aligned}
   1.25z + 0.1\frac{2.9(z-y)}{1.35} &= 1.28z + 0.17(z-y)\\
   0.03z &= \left(\frac{0.29}{1.35} - 0.17\right)(z-y)\\
   &= 0.044814814815(z-y)\\
   0.014814814815z &= 0.044814814815y\\
   \frac{y}{z} &= \frac{0.014814814815}{0.044814814815}\\
   &= 0.330578512399
   \end{aligned}$$

   The overall loading is

   $$\frac{1.45z - 0.17y}{z} - 1 = 1.45 - 0.17\frac{y}{z} - 1 = 1.45 - 0.17\times 0.330578512399 - 1 = 39.38\%$$

[Note that there is a typo in the question, meaning that the ratio $\frac{y}{z}$ is not actually possible - The minimum possible value for $\frac{y}{z}$ is $\frac{1{,}500{,}000}{2{,}000{,}000} = 0.75$.]

6. *Policyholders are assumed to have a utility function* $u(x) = \log(x)$*, and wealth varies between policyholders following a distribution with survival function* $S(w) = \frac{\theta}{\theta+w-4000}$ *for* $w > 4000$*, for some unknown parameter* θ*.*

   An insurance company sells an insurance policy which covers a risk which causes a loss of $4,000 with probability 0.5. The insurer finds that 20% of potential customers would buy the policy for a premium of $2,500. What premium should they set to maximise expected profits? Justify your answer. [Remember the average payment of $2,000 per policy needs to be subtracted from the premium to get profit per policy.]

   (i) $2,106

   (ii) $2,352

   (iii) $2,603

   (iv) $2,913

   For an individual with wealth w, the expected utility without insurance is $\mathbb{E}(u(x)) = 0.5\log(w) + 0.5\log(w-4000)$

   The individual would buy the policy if this is less than the utility from wealth $w - 2500$. That is, if

$$\begin{aligned}
0.5\log(w) + 0.5\log(w-4000) &< \log(w-2500)\\
\sqrt{w(w-4000)} &< w-2500\\
w(w-4000) &< (w-2500)^2\\
w^2 - 4000w &< w^2 - 5000w + 6250000\\
1000w &< 6250000\\
w &< 6250
\end{aligned}$$

   Since 20% of potential customers would buy the policy, this gives $S(6250) = 0.8$, so

$$\begin{aligned}
\frac{\theta}{\theta+2250} &= 0.8\\
\theta &= 0.8\theta + 1800\\
\theta &= 9000
\end{aligned}$$

If the policy has premium $p < 4000$, then a policyholder will buy it if

$$\begin{aligned} 0.5\log(w) + 0.5\log(w-4000) &< \log(w-p) \\ \sqrt{w(w-4000)} &< w-p \\ w(w-4000) &< (w-p)^2 \\ w^2 - 4000w &< w^2 - 2pw + p^2 \\ (2p-4000)w &< p^2 \\ w &< \frac{p^2}{2p-4000} \end{aligned}$$

provided $w > p$. The probability that a policyholder will buy the policy with premium $1000p$ is therefore

$$1 - S\left(\frac{1000^2p^2}{2000p-4000}\right) = 1 - \frac{9000}{5000 + \frac{1000p^2}{2p-4}} = 1 - \frac{18p-36}{p^2+10p-20} = \frac{p^2-8p+16}{p^2+10p-20} = \frac{(p-4)^2}{p^2+10p-20}$$

for $p < 4000$ The expected profit for a policy with premium $1000p$ is $1000(p-2)$. The insurer therefore aims to set the premium to maximise

$$\frac{(p-2)(p-4)^2}{p^2+10p-20}$$

We evaluate the given values:

p	$\frac{(p-2)(p-4)^2}{p^2+10p-20}$
2.106	0.06919576
2.352	0.10561294
2.603	0.09189881
2.913	0.06123972

We see that a premium of \$2,352 maximises the expected profit.