ACSC/STAT 3703, Actuarial Models I

WINTER 2025
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Homework Sheet 2

Model Solutions

Basic Questions

1. *An insurer collects \$16,400,000 in earned premiums for accident year 2024. The total loss payments are \$11,932,000. Payments are subject to inflation of 5%, and policies are sold uniformly throughout the year. If the insurer's permissible loss ratio is 75%, by how much should the premium be changed for policy year 2028?*

   The loss ratio in 2024 is $\frac{11932000}{16400000} = 0.72756097561$. Without inflation, the premium should be adjusted by a factor of $\frac{0.72756097561}{0.75} = 0.970081300813$. Inflation from the start of 2024 to a random claim in accident year 2024 is

   $$\int_0^1 (1.05)^t\,dt = \left[\frac{(1.05)^t}{\log(1.05)}\right]_0^1 = \frac{0.05}{\log(1.05)} = 1.02479671572$$

   Inflation from the start of 2028 to a random claim time for policy year 2028 is

$$\begin{aligned}
\int_0^1 t(1.05)^t\,dt + \int_1^2 (2-t)(1.05)^t\,dt &= \left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) + 1.05\int_0^1 (1-t)(1.05)^t\,dt \\
&= \left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) + 1.05\left(\int_0^1 1(1.05)^t\,dt - \int_0^1 t(1.05)^t\,dt\right) \\
&= 1.05\left(\frac{0.05}{\log(1.05}\right) - 0.05\left(\frac{1.05}{\log(1.05)} - \frac{0.05}{\log(1.05)^2}\right) \\
&= \frac{0.05^2}{\log(1.05)^2} \\
&= 1.05020830854
\end{aligned}$$

   Therefore, the premium should be adjusted by a factor

   $$\frac{0.970081300813 \times 1.05^4 \times 1.05020830854}{1.02479671572} = 1.20837868063$$

   This is an increase of 20.84%.

2. *An insurer is reviewing claims for a certain line of insurance from Accident Year 2024. The earned premiums in 2024 were \$11.2 million. The base premium in 2024 was \$930. However there was a rate change from the old premium of \$895 on 1st October 2023, which affects some policies in Accident Year 2024. The total losses in Accident Year 2024 were \$9.43 million. What should the new premium for Policy Year 2026 be if the permissible loss ratio is 0.8 and annual inflation is 6%?*

   [Assume policies are sold and losses occur uniformly through the year.]

   We first adjust the earned premiums to the current premium. The rate change happened 3 months before the start of 2024, so the old premium applied to $\frac{1}{2}\times\left(\frac{9}{12}\right)^2=\frac{9}{32}$ of policy-years in accident year 2024. Therefore, the adjusted earned premiums are $11.2\times\frac{930}{\frac{9}{32}\times 895+\frac{23}{32}\times 930}=11.3198166072$ million. The loss ratio is therefore $\frac{9.43}{11.3198166072}=0.833052365354$, so without inflation, the premiums should be adjusted by a factor $\frac{0.833052365354}{0.8}=1.04131545669$.

   Inflation from the start of 2024 to a random loss time in accident year 2024 is

$$\int_0^1 (1.06)^t\,dt=\left[\frac{1.06^t}{\log(1.06)}\right]_0^1=\frac{0.06}{\log(1.06)}=1.02970867194$$

   Inflation from the start of 2026 to a random loss time in Policy year 2026 is

$$\begin{aligned}\int_0^1 t(1.06)^t\,dt+\int_1^2(2-t)(1.06)^t\,dt &= \int_0^1 t(1.06)^t\,dt+1.06\int_0^1(1-t)(1.06)^t\,dt1.06\int_0^1(1.06)^t\,dt-0.06\int_0^1 t(1.06)^t\,dt\\ &=1.06\frac{0.06}{\log(1.06)}-0.06\left(\left[\frac{t1.06^t}{\log(1.06)}\right]_0^1-\int_0^1\frac{(1.06)^t}{\log(1.06)}\right)\\ &=\frac{0.06^2}{\log(1.06)^2}\\ &=1.06029994908\end{aligned}$$

   The premium for policy year 2026 is therefore

$$930\times 1.04131545669\times 1.06^2\times\frac{1.06029994908}{1.02970867194}=\$1,120.45$$

3. *An insurance company has two lines of coverage in its tenant's insurance packages, with different expected loss ratios, and has the following data on recent claims:*

Policy Type	*Policy Year*	*Earned Premiums*	*Expected Loss Ratio*	*Losses paid to date*
	2022	*$14,400,000*	*0.77*	*$9,300,000*
House	*2023*	*$15,000,000*	*0.75*	*$8,400,000*
	2024	*$15,700,000*	*0.76*	*$7,700,000*
	2022	*$15,400,000*	*0.81*	*$12,100,000*
Apartment	*2023*	*$14,900,000*	*0.82*	*$11,600,000*
	2024	*$16,300,000*	*0.82*	*$10,900,000*

Calculate the loss reserves at the end of 2024.

We calculate the expected losses and the expected unpaid losses.

Policy Type	Policy Year	Expected total Losses	Losses paid to date	Reserves Needed
	2022	$11,088,000	$9,300,000	$1,788,000
House	2023	$11,250,000	$8,400,000	$2,850,000
	2024	$11,932,000	$7,700,000	$4,232,000
	2022	$12,474,000	$12,100,000	$374,000
Apartment	2023	$12,218,000	$11,600,000	$618,000
	2024	$13,366,000	$10,900,000	$2,466,000
Total				$12,328,000

So the total loss reserves needed at the end of 2024 are $12,328,000.

4. *The following table shows the cumulative paid losses (in thousands) on claims from one line of business of an insurance company over the past 5 years.*

Accident	*Earned*	*Development year*				
year	*premiums*	*0*	*1*	*2*	*3*	*4*
2020	*50889*	*7145*	*19180*	*33387*	*36966*	*39648*
2021	*57723*	*9705*	*23004*	*39189*	*42420*	
2022	*55395*	*8894*	*19217*	*37796*		
2023	*61480*	*9712*	*22537*			
2024	*58319*	*6665*				

Assume that all payments on claims arising from accidents in 2020 have now been settled. Estimate the future payments arising each year from open claims arising from accidents in each calendar year using

(a) The loss development triangle method

First we compute the loss development factors:

$$\begin{array}{ll} 0/1 & \frac{83938}{35456} = 2.3673849278 \\ 1/2 & \frac{110372}{61401} = 1.79756030032 \\ 2/3 & \frac{79386}{72576} = 1.09383267196 \\ 3/4 & \frac{39648}{36966} = 1.07255315696 \end{array}$$

Using these values to complete the table gives the following cumulative losses:

Accident year	Development year 0	1	2	3	4	
LDF		2.3673849278	1.79756030032	1.09383267196	1.07255315696	
2021					42420	45498
2022				37796	41342	44342
2023			22537	40512	44313	47528
2024		6665	15779	28363	31024	33275

The future payments are the differences between consecutive years:

Accident year	Development year 0	1	2	3	4
2021					3078
2022				3546	3000
2023			17975	3801	3215
2024		9114	12584	2661	2251

The total reserves needed are the sum of these, or 61,225.

(b) The Bornhuetter-Ferguson method with expected loss ratio 0.81.

From the LDFs calculated in (a), we get the following proportions of losses paid.

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	$\frac{1}{2.3673849278\times 1.79756030032\times 1.09383267196\times 1.07255315696} = 0.200298610667$	0.200298610667
1	$\frac{1}{1.79756030032\times 1.09383267196\times 1.07255315696} = 0.474183911953$	0.273885301286
2	$\frac{1}{1.09383267196\times 1.07255315696} = 0.852374175176$	0.378190263223
3	$\frac{1}{1.07255315696} = 0.932354721545$	0.079980546369
4	1	0.067645278455

This gives the following reserves:

Accident year	Earned premiums	Expected Total claims	Development year 0	1	2	3	4
2021	57723	46756					3163
2022	55395	44870				3589	3035
2023	61480	49799			18833	3983	3369
2024	58319	47238		12938	17865	3778	3195

The total outstanding reserves are the sum of these payments, or 73,748.

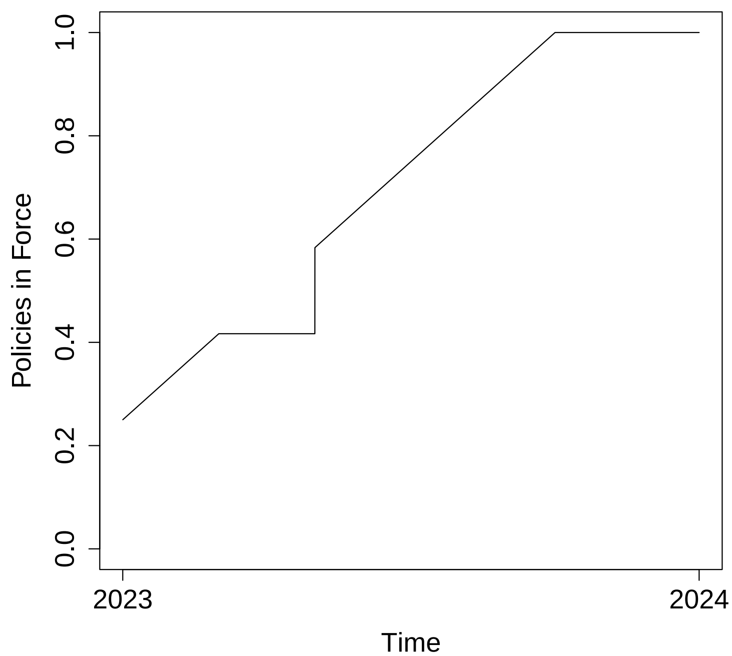
Standard Questions

5. *An insurance company starts a new line of insurance at the start of October 2022. It sells policies at a uniform rate throughout the remainder of 2022, and for the first two months of 2023. Because of technical difficulties, it cannot sell policies in March or April 2023. The policies that it would*

have sold in March and April are all sold on 1st May 2023. It then sells policies at the same uniform rate throughout the remainder of 2023.

It finds that by increasing its premium by 8%, it would have achieved the desired loss ratio for accident year 2023. The actuary estimates inflation is 4%. By how much should the premiums increase for policy year 2025, assuming policies are sold uniformly during 2025?

Policies are sold at a uniform rate for the last 3 months of 2022 and throughout 2023, except for the period March and April, when no policies are sold. The number of policies in force therefore grows linearly until October 2023, except that during the period March and April 2023, the number of policies in force remains at $\frac{5}{12}$.



Thus the average inflation in accident year 2022 is

$$\frac{\int_0^{0.75}(0.25+t)(1.04)^t\,dt+\int_{0.75}^1(1.04)^t\,dt-\int_{\frac{2}{12}}^{\frac{4}{12}}\left(t-\frac{2}{12}\right)(1.04)^t\,dt}{\int_0^{0.75}(0.25+t)\,dt+\int_{0.75}^1 1\,dt-\int_{\frac{2}{12}}^{\frac{4}{12}}\left(t-\frac{2}{12}\right)\,dt}$$

$$=\frac{0.25\int_0^{0.75}(1.04)^t\,dt+\int_{0.75}^1(1.05)^t\,dt+\frac{2}{12}\int_{\frac{2}{12}}^{\frac{4}{12}}(1.04)^t\,dt+\int_0^{\frac{2}{12}}t(1.04)^t\,dt+\int_{\frac{4}{12}}^{0.75}t(1.04)^t\,dt}{0.46875+0.25-\frac{1}{72}}$$

$$=\frac{0.25((1.04)^{0.75}-1)+1.04-(1.04)^{0.75}+\frac{2}{12}\left((1.04)^{\frac{4}{12}}-(1.04)^{\frac{2}{12}}\right)+\left[t(1.04)^t\right]_0^{\frac{2}{12}}-\int_0^{\frac{2}{12}}(1.04)^t\,dt+\left[t(1.04)^t\right]_{\frac{4}{12}}^{0.75}-\int_{\frac{4}{12}}^{0.75}t(1.04)^t\,d}{0.704861111111\log(1.04)}$$

$$=\frac{1.04-\frac{2}{12}(1.04)^{\frac{4}{12}}-0.25-\frac{(1.04)^{\frac{2}{12}}-1+1.04^{0.75}-(1.04)^{\frac{4}{12}}}{\log(1.04)}}{0.704861111111\log(1.04)}$$

$$=1.02403167866$$

The inflation from the start of 2025 to a random claim in policy year 2025 is $\frac{0.04^2}{\log(1.04)^2}=1.04013332308$.

Thus the premium needs to be increased by a factor $1.08\times1.04^2\times\frac{1.04013332308}{1.02403167866}=1.18649538266$ or 18.65%.

6. *An insurance company has the following cumulative aggregate loss development data:*

Accident year	*Earned Premium*	*Development year 0*	*1*	*2*	*3*	*4*
2020	*19291*	*4467*	*9866*	*14433*	*15351*	*15802*
2021	*23607*	*7331*	*12180*	*16834*	*17985*	
2022	*22464*	*7957*	*12041*	*15462*		
2023	*23343*	*5801*	*12259*			
2024	*19574*	*5340*				

From this table, it calculates the following mean loss development factors:

Development year	*LDF*
0/1	*1.813508*
1/2	*1.370875*
2/3	*1.066172*
3/4	*1.029379*

and the following cumulative reserves:

Accident year	*Development year 0*	*1*	*2*	*3*	*4*	
2021						*18513*
2022					*16485*	*16969*
2023				*16806*	*17918*	*18444*
2024			*9684*	*13276*	*14154*	*14570*

The total reserves at the end of 2024 are therefore

$18513 - 17985 + 16969 - 15462 + 18444 - 12259 + 14570 - 5340 = 17450$

After a correction to a payment made in Accident year 2020, Development year 3, the cumulative losses are increased by 2000 in Development years 3 and 4 for that accident year.

(a) By how much do the necessary reserves at the end of 2024 change?

Changing the losses for 2020, Development Year 3 to 5621 changes the 2/3 LDF to $\frac{35336}{31267} = 1.13013720536$ and the 3/4 LDF to $\frac{17802}{17351} = 1.02599273817$. With these new LDFs, the new estimated cumulative losses through Development Year 4 are

Accident Year	Ultimate Losses
2021	$17985 \times 1.02599273817 = 18452.479396$
2022	$15462 \times 1.13013720536 \times 1.02599273817 = 17928.383293$
2023	$16806 \times 1.13013720536 \times 1.02599273817 = 19486.7681815$
2024	$13276 \times 1.13013720536 \times 1.02599273817 = 15393.6888241$

The total reserves are therefore $18452.479396 - 17985 + 17928.383293 - 15462 + 19486.7681815 - 12259 + 15393.6888241 - 5340 = 20215.3196946$

This is an increase of $20215.3196946 - 17450 = 2765.3196946$

(b)

Using the Bornhuetter-Fergusson method with expected loss ratio 0.81, the reserves before the correction were

Accident	*Expected*	*Development year*				
year	*Claims*	*0*	*1*	*2*	*3*	*4*
2021	*19121.67*					*545.7457*
2022	*18195.84*				*1097.0937*	*519.3218*
2023	*18907.83*			*4660.884*	*1140.0222*	*539.6425*
2024	*15854.94*		*4727.227*	*3908.330*	*955.9523*	*452.5109*

meaning that the total reserves are 18546.73. What will the new reserves be after the correction?

Replacing the LDFs for 2/3 and 3/4 by the corrected LDFs, 1.13013720536 and 1.02599273817, the proportion of total losses in each year is given by

Development Year	Cumulative proportion of losses paid	Proportion of losses paid
0	$\frac{1}{1.813508 \times 1.370875 \times 1.13013720536 \times 1.02599273817} = 0.346902323152$	0.346902323152
1	$\frac{1}{1.370875 \times 1.13013720536 \times 1.02599273817} = 0.629110138254$	0.282207815102
2	$\frac{1}{1.13013720536 \times 1.02599273817} = 0.862431360781$	0.233321222527
3	$\frac{1}{1.02599273817} = 0.974665767892$	0.112234407111
4	1	0.025334232108

The expected payments are then

Accident year	Expected Claims	Development year				
		0	1	2	3	4
2021	19121.67					484
2022	18195.84				2042	461
2023	18907.83			4412	2122	479
2024	15854.94		4474	3699	1779	402

Thus, the total reserves needed are 20355.15. This is an increase of 1808.42.