ACSC/STAT 4703, Actuarial Models II

FALL 2024
Toby Kenney

Homework Sheet 5

Due: Thursday 31st October: 11:30

1. A home insurance company classifies policyholders as high-risk, medium-risk or low-risk . Annual claims from high-risk policyholders follow a Pareto distribution with $\alpha = 2.4$ and $\theta = 830$. Annual claims from medium-risk policyholders follow a Pareto distribution with $\alpha = 4.6$ and $\theta = 960$. Annual claims from low-risk policyholders follow a gamma distribution with $\alpha = 4$ and $\theta = 59$. 16% of policyholders are high risk, 68% are medium risk and 16% are low risk.

   (a) Calculate the expectation and variance of the aggregate annual claims from a randomly chosen policyholder.

   (b) Given that a policyholder's annual claims over the past 2 years were $1,421 and $119, what are the expectation and variance of the policyholder's claims next year?

2. An insurance company sets the book pure premium for its auto insurance at $630. The expected process variance is 17,215,000 and the variance of hypothetical means is 196,000. If a policyholder has aggregate claims of $15,400 over the past 12 years, calculate the credibility premium for this policyholder's next year's insurance using the Bühlmann model.

3. An insurance company has the following data on its auto insurance policies for a certain model of car.

Year	1	2	3	4	5
Distance driven (1,000 km)	6,221	4,495	7,251	6,304	7,554
Aggregate claims	$1,484,100	$1,226,000	$1,609,300	$1,355,300	$1,664,700

   The book premium is $142 per 1000km. The variance of hypothetical means per 1000km is 842. The expected process variance per 1000km is 6,237,157,440. Using a Bühlmann-Straub model, calculate the credibility premium for Year 6 for a car that is expected to be driven 14,623km.

Standard Questions

4. A workers' compensation insurer classifies workplaces as "low-risk" and "high-risk". It estimates that 84% of workplaces are low-risk. Annual claims from low-risk workplaces are modelled as following a Pareto distribution with $\alpha = 6.6$ and $\theta = 7585$. Annual claims from high-risk

workplaces are modelled as following a Pareto distribution with unkown α and θ. A company is initially charged the book premium of $1,192, then after claiming $19,521 in its first year, it is charged a new premium of $1,284. Which of the following is the value of the unknown parameter α, and what is the corresponding value of the unknown parameter θ?

(i) 1.94

(ii) 2.77

(iii) 4.22

(iv) 6.50

5. An insurance company uses the Bühlmann-Straub model to calculate credibility. A new customer pays the book premium for 223 units of exposure, paying a total net premium of $78,719 in its first year. It claims a total of $92,218. In the second year, the customer pays a credibility premium of $60,332 and claims a total of $34,902. In the third year, the customer has 353 units of exposure, and pays a premium of $121,666. How many units of exposure did the customer have in the second year?

6. A health insurance company is pricing its policies for an individual. It has 4 years of past history for this individual, and the annual claims from year i are denoted X_i. It uses the formula $\hat{X}_5 = \alpha_0 + \sum_{i=1}^{4} \alpha_i X_i$. It makes the following assumptions about the claims each year:

• The expected aggregate claims is $417 in each year.

• The variance of the annual claims is 2,241,433.

• The correlation between claims in Years i and j for $i \neq j$ is 0.55 if i and j are consecutive, and 0.40 otherwise.

Find a set of equations which can determine the values of α_i for $i = 0, 1, \ldots, 5$. [You do not need to solve these equations.]