

ACSC/STAT 4703, Actuarial Models II

FALL 2024

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Homework Sheet 6

Due: Thursday 7th November: 11:30

Note: This homework assignment is only valid for FALL 2024. If you find this homework in a different term, please contact me to find the correct homework sheet.

Basic Questions

1. An insurance company has the following previous data on aggregate claims:

Policyholder	Year 1	Year 2	Year 3	Year 4	Year 5	Mean	Variance
1	0.00	37.42	0.00	0.00	10.74	9.632	262.9317
2	245.20	2033.31	462.66	94.24	1388.56	844.794	694280.6027
3	695.47	1655.90	1.97	106.79	40.06	500.038	497126.5281
4	924.94	75.32	0.13	85.97	101.15	237.502	149193.3216
5	326.55	0.05	183.39	48.46	38.82	119.454	18193.9676

Calculate the Bühlmann credibility premium for each policyholder in Year 6.

2. The file `HW6_data.txt` contains aggregate claim data from 100 policyholders over the past 10 years. Use this data to estimate the book premium and the credibility of 10 years' experience.
3. An insurance company collects the following numbers of claims from five policyholders over a 5-year period.

Policyholder	Year 1	Year 2	Year 3	Year 4	Year 5
1	7	12	9	8	9
2	5	2	2	3	4
3	2	8	7	6	4
4	11	4	9	6	3
5	2	2	2	5	1

The company assumes that the number of claims for each policyholder follows a Poisson distribution. Use Bühlmann credibility to estimate the average number of claims for Policyholder 2 in Year 6.

Standard Questions

4. The file `HW6_data2.txt` contains aggregate claim data from 100 policyholders over the past 10 years. The data are assumed to come from a

gamma distribution with shape $\alpha = 4$ and scale parameter varying between policyholders. Calculate the credibility of 10 years of experience, and the premium in Year 11 for Policyholder 15.

5. Aggregate claims per unit of exposure for a given individual policy are modelled as following a certain parametric distribution. Each policyholder has a risk parameter Θ . For a policyholder with risk parameter Θ and exposure m , the expected value of total annual claims is Θm , and the variance is $m(\Theta^2 + 8)$.

From a dataset of 100 policyholders with different exposures, they find that the total aggregate claim is \$432,228 from a total of 1,220 units of exposure. They also calculate:

$$\begin{aligned}\sum m_i^2 &= 19,145 \\ \sum m_i^3 &= 410,153 \\ \sum m_i X_i^2 &= 316,484,432\end{aligned}$$

where X_i is the aggregate claims per unit of exposure for Policyholder i (so $\sum m_i X_i = 432228$). Estimate the EPV and VHM from this data.

[Hint: calculate the expectation of $\sum_{i=1}^{100} m_i \left(X_i - \widehat{\mathbb{E}(\Theta)} \right)^2$.]