ACSC/STAT 4703, Actuarial Models II

FALL 2024
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Homework Sheet 8

Model Solutions

Basic Questions

1. *The file* `HW8_data.txt` *contains a run-off triangle. Fit an overdispersed Poisson model to this data and use it to find the 95th percentile of estimated outstanding claims.*

   [The easiest way to do this is by simulation. The parameter estimates should be approximated by a multivariate normal distribution. You can use `vcov` *to find the variance matrix of this distribution, and use the* `mvrnorm` *function in the* `MASS` *package to simulate the parameter estimates.]*

We first fit the overdispersed Poisson model.

```
HW8Q1<-read.table("HW8_data.txt")

### Convert to long table format
ODP.data<-data.frame(Acc.Year=as.factor(rep(seq_len(10),10)),
                     Dev.Year=as.factor(rep(seq_len(10),each=10)),
                     value=as.vector(as.matrix(HW8Q1)))

### Fit an overdispersed Poisson model.
ODP.model<-glm(value~Acc.Year+Dev.Year,ODP.data,family="quasipoisson")
```

This fits the following model:

Parameter	estimate	p-value	Parameter	estimate	p-value
(Intercept)	9.86621	$< 2\times 10^{-16}$			
Acc.Year2	0.03757	0.6814	Dev.Year2	0.02787	0.5988
Acc.Year3	-0.07005	0.4606	Dev.Year3	-0.72816	1.67×10^{-12}
Acc.Year4	-0.26386	0.0120	Dev.Year4	-1.68290	$< 2\times 10^{-16}$
Acc.Year5	0.02015	0.8297	Dev.Year5	-1.89206	$< 2\times 10^{-16}$
Acc.Year6	0.17093	0.0672	Dev.Year6	-2.37550	1.55×10^{-15}
Acc.Year7	0.16856	0.0753	Dev.Year7	-2.81417	1.56×10^{-13}
Acc.Year8	0.06069	0.5337	Dev.Year8	-2.72895	2.72×10^{-12}
Acc.Year9	0.17022	0.0976	Dev.Year9	-2.88302	5.39×10^{-10}
Acc.Year10	-0.03411	0.8051	Dev.Year10	-3.19291	2.34×10^{-06}

The dispersion parameter is fitted as 252.5111.

We then simulate 10000 sets of parameter values from the sampling distribution.

```
library(MASS)

### Simulate Coefficients from the sampling distribution.
CoeffValues<-mvrnorm(10000,ODP.model$coefficients,vcov(ODP.model))

### Construct the model matrix for these coefficients.

model.mat.NA<-t(cbind(rep(1,45),
                  diag(rep(1,10))[
                     ODP.model$data[is.na(ODP.model$data$value),1],-1],
                  diag(rep(1,10))[
                     ODP.model$data[is.na(ODP.model$data$value),2],-1]))

### Now the transformed predictors can be found by matrix multiplication.

SimReserves<-rowSums(exp(CoeffValues%*%model.mat.NA))

### Take the 95th percentile.
quantile(SimReserves,0.95)
```

[These are the Poisson means. We could also include the process variance by simulating payments from a Poisson distribution with the simulated means.]

For my simulation, the 95th percentile is $126,589.

2. *A health insurance company classifies policyholders as "Young", "Middle-aged" and "Elderly. The experience from policy year 2023 is:*

Policyholder	*Current differential*	*Earned premiums (000s)*	*Loss payments (000s)*
Young	*0.47*	*9,600*	*8,040*
Middle-aged	*1*	*14,400*	*9,330*
Elderly	*1.82*	*11,700*	*8,720*

The base premium was $750. If the expense ratio is 25%, calculate the new premiums for each type of policyholder (ignoring inflation) for policy year 2025.

We calculate the observed loss ratio and new differential for each class.

Policyholder	Old differential	Loss Ratio	New differential
Young	0.47	$\frac{8040}{9600} = 0.8375$	$0.47 \times \frac{0.8375}{0.647916666667} = 0.607524115757$
Middle-aged	1	$\frac{9330}{14400} = 0.647916666667$	1
Elderly	1.82	$\frac{8720}{11700} = 0.745299145299$	$1.82 \times \frac{0.745299145299}{0.647916666667} = 2.0935476956$

With these differentials, the adjusted total earned premium is

$$14400+\frac{9600\times 0.607524115757}{0.47}+\frac{11700\times 2.0935476956}{1.82}=40267.5241158$$

The overall loss ratio is $\frac{26090}{40267.5241158}=0.647916666666$, so before inflation, the premium needs to be adjusted by a factor of $\frac{0.647916666666}{0.75}=0.863888888888$.

The new base premium is therefore $750\times 0.863888888888=647.916666666$. The premiums are therefore:

type	New premium
Young	$0.607524115757\times 647.916666666=\$393.625$
Middle-aged	\$647.92
Elderly	$2.0935476956\times 647.916666666=\$1,356.44$

3. *An insurer uses two variables to classify companies. The categories and differentials are given in the following table:*

Size Category		*Industry*	
Small (1–30 employees)	*0.89*	*Manufacture*	*1.85*
Medium (31-100 employees)	*1*	*Resources*	*2.21*
Large (> 100 employees)	*1.30*	*Services*	*1*

The earned premiums from accident year 2023 are:

	Industry			
Size Category	*Manufacture*	*Resources*	*Services*	*Total*
Small	*4,201*	*3,521*	*9,105*	*16,827*
Medium	*7,367*	*8,202*	*12,494*	*28,063*
Large	*9,242*	*7,339*	*7,322*	*23,903*
Total	*20,810*	*19,062*	*28,921*	*68,793*

And the claims are:

Category	*Total Claims*
Small	*12,593*
Medium	*23,312*
Large	*19,690*
Manufacture	*15,379*
Resources	*16,022*
Services	*24,104*
Total	*55,505*

The expense ratio is 0.2. Ignoring inflation, by what factor should they increase the base premium in future years?

We first calculate the loss ratios and adjusted differentials for each class:

Category	Current differential	Loss ratio	New differential
Small	0.89	$\frac{12593}{16827} = 0.748380578832$	$0.89 \times \frac{0.748380578832}{0.830702348288} = 0.801801892739$
Medium	1	$\frac{23312}{28063} = 0.830702348288$	1
Large	1.30	$\frac{19690}{23903} = 0.823745973309$	$1.30 \times \frac{0.823745973309}{0.830702348288} = 1.28911368495$
Manufacture	1.85	$\frac{15379}{20810} = 0.739019702066$	$1.85 \times \frac{0.739019702066}{0.833442827012} = 1.64040820139$
Resources	2.21	$\frac{16022}{19062} = 0.840520407093$	$2.21 \times \frac{0.840520407093}{0.833442827012} = 2.22876727651$
Services	1	$\frac{24104}{28921} = 0.833442827012$	1

At these differentials, the adjusted total earned premiums are:

$$\begin{aligned}
4201 \times \frac{0.801801892739}{0.89} \times \frac{1.64040820139}{1.85} + 3521 \times \frac{0.801801892739}{0.89} \times \frac{2.22876727651}{2.21} + 9105 \times \frac{0.801801892739}{0.89} \\
+7367 \times \frac{1.64040820139}{1.85} + 8202 \times \frac{2.22876727651}{2.21} + 12494 + 9242 \times \frac{1.28911368495}{1.30} \times \frac{1.64040820139}{1.85} \\
+ 7339 \times \frac{1.28911368495}{1.30} \times \frac{2.22876727651}{2.21} + 7322 \times \frac{1.28911368495}{1.30} \\
= 64781.9936041
\end{aligned}$$

With these adjusted premium, the base premium needs to be adjusted by a factor

$$\frac{55505}{0.8 \times 64781.9936041} = 1.07099590704$$

Standard Questions

4. *An insurance company uses three variables: sex, age category and vehicle type to distinguish policyholders. The base classes are "Female", "Young" and "Car". The total losses from policy year 2023 were $5,305,444.*

   The total earned premiums for each combination of age category and vehicle type were as given in the following table:

	Car	*Motorcycle*	*Total*
Young	*2,895,987*	*675,686*	*3,571,673*
Elderly	*2,557,859*	*249,931*	*2,807,790*
Total	*5,453,846*	*925,617*	*6,379,463*

   The total earned premiums for each combination of age category and sex were as given in the following table:

	Female	*Male*	*Total*
Young	*1,750,810*	*1,820,863*	*3,571,673*
Elderly	*1,548,972*	*1,258,818*	*2,807,790*
Total	*3,299,782*	*3,079,681*	*6,379,463*

   The total earned premiums for each combination of vehicle type and sex were as given in the following table:

	Car	*Motorcycle*		*Total*
Female	*2,978,702*	*321,080*		*3,299,782*
Male	*2,475,144*	*604,537*		*3,079,681*
Total	*5,453,846*	*925,617*	*6,379,463*	*6,119,219*

After reviewing the data for 2023, they calculate the following new differentials:

Class	*Old differential*	*New differential*
Male	*1.201*	*1.323*
Elderly	*1.103*	*1.053*
Motorcycle	*0.536*	*0.564*

With an expense ratio of 0.2, The base premium is adjusted by a factor 1.003274. What were the total earned premiums from elderly female motorcyclists?

Since the base premium is adjusted by a factor of 1.003274, the adjusted earned premiums must be $\frac{5305444}{1.003274\times 0.8} = 6610163.32527$. Let x be the total earned premiums from elderly female motorcyclists. We compute the adjusted total earned premiums from each combination of classes:

Class	Earned Premiums	Adjustment factor	Adjusted Earned premiums
Elderly Female Motorcyclists	x	$\frac{1.053}{1.103} \times \frac{0.564}{0.536} = 1.00453985738$	$1.00453985738x$
Elderly Female Cars	$1548972 - x$	$\frac{1.053}{1.103} = 0.954669084316$	$1478755.68087 - 0.954669084316x$
Elderly Male Motorcyclists	$249931 - x$	$\frac{1.053}{1.103} \times \frac{1.323}{1.201} \times \frac{0.564}{0.536} = 1.10658304023$	$276569.405828 - 1.10658304023x$
Elderly Male Cars	$1008887 + x$	$\frac{1.053}{1.103} \times \frac{1.323}{1.201} = 1.05164629355$	$1060992.27416 + 1.05164629355x$
Young Female Motorcyclists	$321080 - x$	$\frac{0.564}{0.536} = 1.05223880597$	$337852.835821 - 1.05223880597x$
Young Female Cars	$1429730 + x$	1	$1429730 + x$
Young Male Motorcyclists	$354606 + x$	$\frac{1.323}{1.201} \times \frac{0.564}{0.536} = 1.15912734413$	$411033.510993 + 1.15912734413x$
Young Male Cars	$1466257 - x$	$\frac{1.323}{1.201} = 1.10158201499$	$1615202.34055 - 1.10158201499x$
Total			$6610136.04822 + 0.00024054955x$

Thus, we need to solve $6610136.04822 + 0.00024054955x = 6610163.32527$ which gives

$$x = \frac{6610163.32527 - 6610136.04822}{0.00024054955} = \$113,395$$